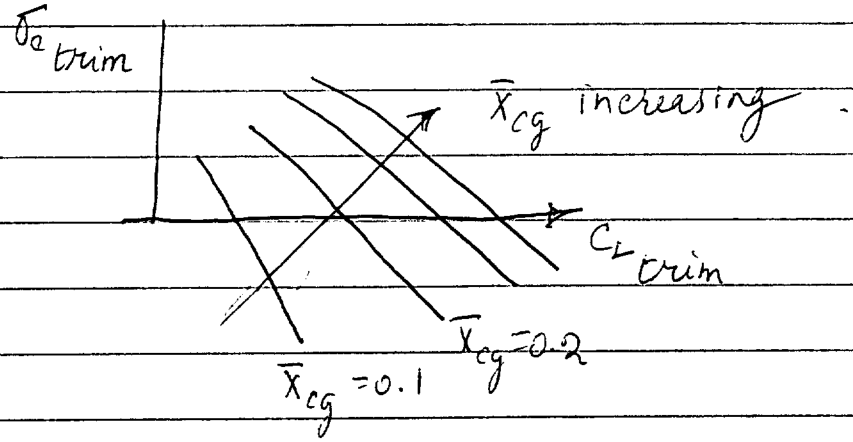
Flight Measurement of x_{NP}

How do we determine the stick-fixed neutral point from flight tests?

First find the elevator angle to trim, that is, $\delta_{e_{trim}}$ at various positions of cg and plot δ_{trim} Vs $C_{L_{trim}}$.



observe

$$\delta_{e_{trim}} = \frac{C_{m_0} C_{L_\alpha} + C_{m_\alpha} C_{L_{trim}}}{C_{m_{\delta_e}} C_{L_\alpha} - C_{m_\alpha} C_{L_{\delta_e}}} \qquad (1)$$

Differentiate (1) with respect to $C_{L_{trim}}$ to get

$$\frac{\partial \delta_{e_{trim}}}{\partial C_{L_{trim}}} = \frac{C_{m_\alpha}}{C_{m_{\delta_e}} C_{L_\alpha} - C_{L_{\delta_e}}} \qquad (2)$$

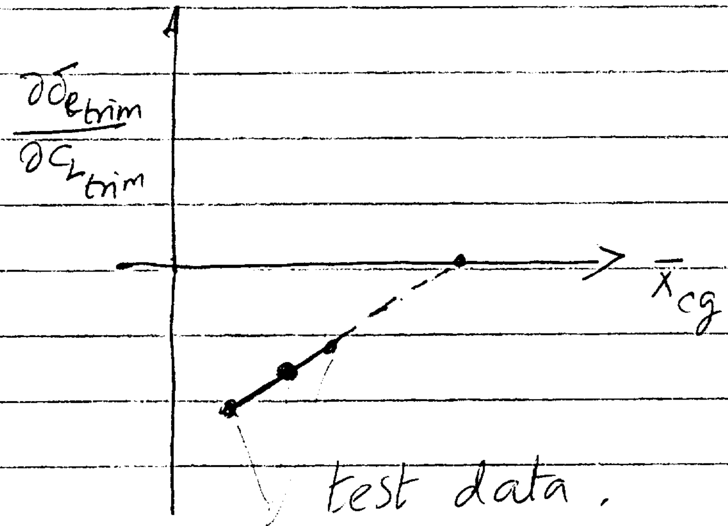
Note that $\dfrac{\partial \delta_{e_{trim}}}{\partial C_{L_{trim}}} = 0$ when $C_{m_\alpha} = 0$

This means, this slope is zero

at $C_{m_\alpha} = 0 \Rightarrow X_{cg} = X_{NP}$!

[Since we don't want to fly at $X_{cg} = X_{NP}$!!]

Plot the $\boxed{\text{slope}}$ of $\delta_{e_{trim}}$ vs $C_{L_{trim}}$ against $\bar{X}_{cg}$.

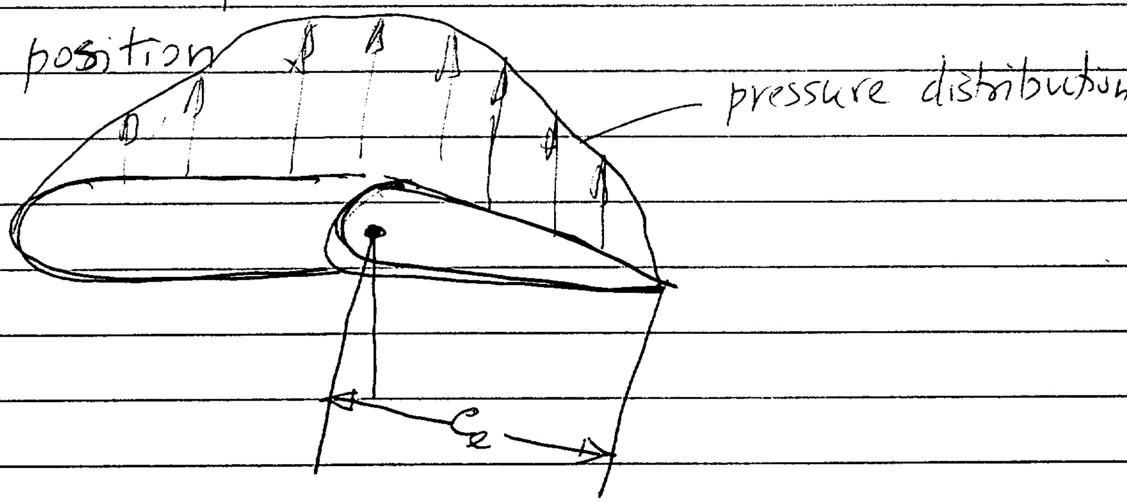


The $\bar{X}_{cg}$ intercept will give the location of the neutral point.

Elevator Hinge Moment

Definition: Elevator Hinge Moment is the hinge moment acting at the hingeline of the elevator.

Importance: This is the moment that the pilot has to overcome (to keep the elevator at certain deflection) to maintain some stick position



$$H_e = C_{h_e} \frac{1}{2} \rho V_c^2 S_e c_e \equiv \text{Hinge Moment}$$

where $S_e \equiv$ Area aft of the hingeline

$c_e \equiv$ chord measured from the hinge line to the trailing edge.

$C_{h_e} \equiv$ Hinge Moment coefficient

Since S_e, C_e are fixed, for a given dynamic pressure, $\underline{H_e \propto C_{h_e}}$. So, we need to study what affects C_{h_e}.

[Remember Taylor series $f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x$]

$$C_{h_e} = C_{h_0} + C_{h_{\alpha_t}} \alpha_t + C_{h_{\delta_e}} \delta_e + C_{h_{\delta_t}} \delta_t$$

where

$$\frac{\partial C_{h_e}}{\partial \alpha_t} \equiv C_{h_{\alpha_t}} \; ; \; \frac{\partial C_{h_e}}{\partial \delta_e} \equiv C_{h_{\delta_e}} \text{ and } \frac{\partial C_{h_e}}{\partial \delta_t} \equiv C_{h_{\delta_t}}$$

$$\delta_t \equiv \text{Tab deflection.}$$

First we want to study about a "stick-free" flight. So, assume $\delta_t = 0$. 'Stick-free' flight refers to flight where we take the hands off the control stick. When this happens, $C_{h_e} = 0$ since we do not exert any force on the stick! Can we still fly a steady flight? yes!

$\boxed{\text{But}}$ it will result in a different steady state!

Analysis

- Assume $\delta_t = 0$
- Assume symmetric airfoil, $C_{h_0} = 0$

Then,

$$C_{h_e} = 0 \Rightarrow C_{h_{\alpha_t}} \alpha_t + C_{h_{\delta_e}} \delta_e$$

$$\Rightarrow (\delta_e)_{free} = -\left(\frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}}\right) \alpha_t \qquad (1)$$

What this means is that (δ_e) is no longer an independent quantity (to be calculated for trim). It is $\boxed{\text{set}}$ by what the α_t is for stick-free conditions.

- usually, $C_{h_{\alpha_t}} < 0$ $\quad C_{h_{\delta_e}} < 0 \Rightarrow$

δ_e floats up if α_t is positive

δ_e floats down if α_t is negative

Furthermore,

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta_e}} \delta_{e_{free}}$$

$$= \left[C_{L_{\alpha_t}} - C_{L_{\delta_e}} \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \right] \alpha_t \qquad (2)$$

by using (1)

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t \left[1 - \frac{C_{L_{\delta_e}} C_{h_{\alpha_t}}}{C_{L_{\alpha_t}} C_{h_{\delta_e}}}\right] = C'_{L_{\alpha_t}} \alpha_t$$

$$\Rightarrow \quad C'_{L_{\alpha_t}} = C_{L_{\alpha_t}} \left[1 - \frac{C_{L_{\delta_e}} C_{h_{\alpha_t}}}{C_{L_{\alpha_t}} C_{h_{\delta_e}}}\right] = C_{L_{\alpha_t}} f$$

~~usually~~ $\Rightarrow$ $\cancel{C_{L_{\alpha_t}}}$

- f determines whether $C_{L_{\alpha_t}} \gtreqless C'_{L_{\alpha_t}}$

The net effect of this $C_{L_{\alpha_t}}$ change in other stability related quantities can be expressed by:

$$C'_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + C'_{L_{\alpha_t}} \eta V_H (\epsilon_0 + i_w - i_t)$$

$$C'_{m_\alpha} = C_{L_{\alpha_w}} (\bar{X}_{cg} - \bar{X}_{ac_w}) + C_{m_{\alpha_f}} - C'_{L_{\alpha_t}} \eta V_H (1 - \epsilon_\alpha)$$

and

$$\bar{X}'_{NP} = \bar{X}_{ac_w} + V_H \eta \frac{C'_{L_{\alpha_t}}}{C_{L_{\alpha_w}}} (1 - \epsilon_\alpha) - \frac{C_{m_{\alpha_f}}}{C_{m_{\alpha_w}}}$$

Stick-free static margin

$$= \bar{X}'_{NP} - \bar{X}_{cg}$$