# STATIC MANEUVERS

## DEFINITIONS

1. Static MANEUVERS
   - a maneuver with constant turn rate with constant speed

$\Rightarrow$

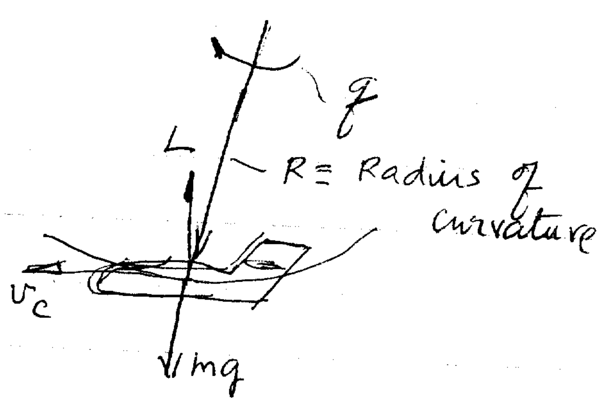
$$\Sigma \bar{F} = 0 \qquad \Sigma \bar{M} = 0$$

2. LOAD Factor, n

$$n \equiv \text{ratio of a/c lift to a/c weight} = \frac{L}{W}$$

Level Flight $\Rightarrow n = 1$
Pull-up $\Rightarrow n > 1$
Push-over $\Rightarrow n < 1$

## Basic Relations




Consider the a/c at the bottom of a shallow pull-up.

$$\text{Net force} = L - mg$$
$$= L - W$$
$$= nW - W = (n-1)W$$

$$\text{Centrifugal acceleration} = q^2 R = q v_c$$

$q \equiv$ pitch rate (turn rate) ; $R \equiv$ Radius of curvature

For a constant turn

$$m q v_c = (n-1) mg$$

$$\boxed{q = \frac{(n-1)g}{v_c}}$$

- For a large turn rate (climb rate)

  i) low speed (limited by stall)
  ii) large load factor (limited by pilot and ~~load factor~~ structures)

<u>Measures of maneuverability</u>

Let $\Delta P$, the additional stick force to be exerted by the pilot for a pull-up and let this lead to an additional elevator deflection, $\Delta\delta_e$ (upwards for a pull-up).

Define:

Stick force per $g$ $\equiv \frac{\Delta P}{n-1}$

Elevator angle per $g$ $\equiv \frac{\Delta \delta_e}{n-1}$

Smaller the values of these quantities, more maneverable is the aircraft.

Expression for elevator angle per $g$, $\frac{\Delta\delta_e}{n-1}$

<u>In level flight,</u>

$$C_{m_{level}} = C_m(\alpha, \delta_e)$$

<u>During steady pull-up</u>

$$C_{m_{pull\text{-}up}} = C_m(\alpha, \delta_e, q)$$

$$C_{m_{pull\ up}} = C_{m_{level}} + \frac{\partial C_m}{\partial \alpha}\Delta\alpha + \frac{\partial C_m}{\partial \delta_e}\Delta\delta_e + \frac{\partial C_m}{\partial q}\Delta q$$

[Taylor series again!]

2.

$$\Rightarrow \quad \boxed{\Delta\delta_e = -\frac{C_{m_\alpha}\Delta\alpha + \frac{\partial C_m}{\partial q} q}{C_{m_{\delta_e}}}} \quad (1)$$

Now, we want to find $\frac{\Delta\delta_e}{n-1}$; so,

we need to express the numerator of RHS in terms of $(n-1)$

Note, $\boxed{q = \frac{(n-1)g}{V_c}}$ . . . (good!) (2)

What is $\Delta\alpha$ in terms of $(n-1)$?

Consider

$$\Delta C_L = C_{L_{pull\text{-}up}} - C_{L_{level}}.$$

$$= C_{L_\alpha}\Delta\alpha + C_{L_{\delta_e}}\Delta\delta_e$$

$$\cancel{\Rightarrow C_{L_\alpha}\Delta\alpha}$$

Now, $C_{L_{pull\text{-}up}} = \frac{nW}{\bar{q}S}$

$$C_{L_{level}} = \frac{W}{\bar{q}S} = C_{L_{trim}}$$

$$\Rightarrow \Delta C_L = \frac{(n-1)W}{\bar{q}S}$$

$$\Delta C_L = (n-1)\, C_{L_{trim}}$$

Therefore,

$$\Delta\alpha = \frac{\Delta C_L - C_{L_{\delta_e}}\Delta\delta_e}{C_{L_\alpha}}$$

$$\boxed{\Delta\alpha = \frac{(n-1)C_{L_{trim}} - C_{L_{\delta_e}}\Delta\delta_e}{C_{L_\alpha}}} \quad (3)$$

use (3) and (2) in (1) and ~~solve~~

solve to get

$$\frac{\Delta\delta_e}{n-1} = -\left[\frac{C_{m_\alpha} C_{L_{trim}} + \frac{\partial C_m}{\partial q}\frac{C_{L_\alpha} g}{V_e}}{C_{m_{\delta_e}} C_{L_\alpha} - C_{m_\alpha} C_{L_{\delta_e}}}\right] \quad (4)$$

for $n > 1$

This gives the additional elevator deflection needed for a pull-up.

• Usually, this is written in terms of the pitch damping derivative, $C_{m_q}$.

What is $C_{m_q}$?

$$C_{m_q} \neq \frac{\partial C_m}{\partial q}.$$

$$\boxed{C_{m_q} = \frac{\partial C_m}{\partial\left(\frac{q\bar{c}}{2v_c}\right)} = \frac{2v_c}{\bar{c}}\frac{\partial C_m}{\partial q}} \qquad (5)$$

• $C_{m_q}$ is non-dimensional.

Define

• $\mu \equiv$ relative mass parameter

$$= \frac{2m}{\rho S\bar{c}}. \qquad (6)$$

Recall,

$$C_{m_\alpha} = C_{L_\alpha}(\bar{X}_{cg} - \bar{X}_{NP}). \qquad (7)$$

using (6), (7) and $C_{m_q}$ from (5) in (4)

$$\bar{X}_{cg} - \bar{X}_{NP} + \frac{C_{m_q}}{2\mu} = 0 \quad \text{at} \quad \bar{X}_{cg} = \bar{X}_{MP}$$

or
$$\boxed{\bar{X}_{MP} = \bar{X}_{NP} - \frac{C_{m_q}}{2\mu}}$$

Notes:

1. $C_{m_q} < 0 \Rightarrow$ Maneuver Point is behind the neutral point.

2. Stick-fixed Maneuver Margin $= \bar{X}_{MP} - \bar{X}_{cg}$

3. ~~$\bar{X}_{cg}$ is~~ $\bar{X}_{NP}$ is a function of geometry only but $\bar{X}_{MP}$ varies with altitude.

$$\mu = \frac{2m}{\rho S \bar{c}} \Rightarrow \text{altitude} \uparrow \Rightarrow \mu \uparrow$$

$$\Rightarrow \bar{X}_{MP} \rightarrow \bar{X}_{NP}$$

4. $$\boxed{C_{m_q} = -2K C_{L_{\alpha_t}} \frac{l_t}{\bar{c}} V_H \eta}$$

$K \equiv$ fudge factor $\approx 1.1$ for a conventional a/c.