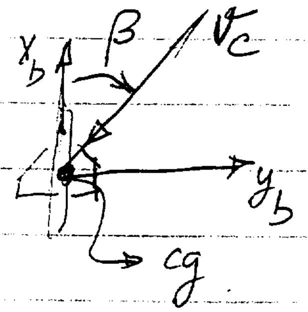
DIRECTIONAL STABILITY AND CONTROL

— THIS is about motion in $X_b Y_b$ plane



DEFINITIONS

- Sideslip angle, β

$$\beta \equiv \sin^{-1}\left(\frac{V}{V_c}\right) \approx \frac{v}{v_c} > 0 \quad \text{if } V > 0$$

$V \equiv$ velocity in Y_b-direction

- Side Wash angle, σ
  THIS is caused due to sideslip

$$\sigma = \sigma_0 + \sigma_\beta \beta \qquad \text{where } \sigma_\beta \equiv \frac{\partial \sigma}{\partial \beta}$$

- Yawing Moment Coefficient, C_n

$$C_n \equiv \frac{N}{\bar{q} S b}$$

$b \equiv$ wing span

• DIRECTIONAL STABILITY

Tendency of an aircraft to turn into the wind.

sideslip to right ($\beta > 0$), we want $N > 0$.
sideslip to left ($\beta < 0$), we want $N < 0$

$$\Rightarrow \cancel{\Delta C_n} \ \frac{\Delta N}{\Delta \beta} > 0 \Rightarrow \frac{\Delta C_n}{\Delta \beta} > 0 \Rightarrow \cancel{C_n}$$

$\boxed{C_{n_\beta} > 0}$ criterion for directional stability.

• Estimation of C_{n_β}

The main contributor to C_{n_β} of aircraft is the vertical tail (or Fin)

• Angle of attack at the fin, α_V

$$\alpha_V = \beta + \sigma \quad = \beta + \sigma_0 + \sigma_\beta \beta$$

$$= \sigma_0 + \beta(1 + \sigma_\beta)$$

Lift due to vertical Tail

$$(Lift)_V = \bar{q}_V S_V C_{L_V}$$

$\bar{q}_V \equiv$ Dynamic pressure at vertical Tail

Sideforce, $Y = -(\text{lift})_V$

Yawing moment due to Sideforce

$$N_V = C_{n_V} \bar{q} S b$$

$$= -Y * l_V \qquad (1)$$

where

$l_V \equiv$ distance between the Fin aerodynamic center and the a/c cg

(1) can be expanded as

$$C_{n_V} \bar{q} S b = + \bar{q}_V S_V C_{L_V} * l_V$$

$$\Rightarrow C_{n_V} = \left(\frac{\bar{q}_V}{\bar{q}}\right) \boxed{\frac{S_V}{S} \frac{l_V}{b}} C_{L_V}$$

Define:

$$\eta_V \equiv \frac{\bar{q}_V}{\bar{q}} \equiv \text{ratio of dynamic pressures at fin to a/c (wing)}$$

$V_V \equiv$ Vertical Tail Volume Ratio

$$\Rightarrow C_{n_V} \equiv \eta V_V C_{L_V}$$

$$= \eta V_V C_{L_{\alpha_V}} \alpha_V$$

$$= \eta V_V C_{L_{\alpha_V}} \left(\beta (1 + \sigma_\beta) + \sigma_0 \right)$$

$$\Rightarrow \frac{\partial C_{n_V}}{\partial \beta} = \boxed{(C_{n_\beta})_V = C_{L_{\alpha_V}} \eta_V V_V (1+\sigma_\beta)}$$

Notes

(1) $(C_{n_\beta})_V$ is the largest part of C_{n_β}. This value depends on V_V which in turn depends on $(S_V l_V)$ size of the vertical tail.

(2) Other contributors to C_{n_β}

a) Fuselage (book)
b) propellers at large sideslip produce side forces affecting C_{n_β}

c) Dihedral of the wing

d) Sweep of the wing with a sidewind creates yawing moment that affects C_{n_β}

Directional Control

- This is the ability to yaw at will and is provided by the rudder.

Definitions

- $\delta_r \equiv$ rudder deflection
  > 0 if it produces positive side force

- $\boxed{C_{n_{\delta_r}}}$ — similar to $C_{m_{\delta_e}}$

Let

$\Delta N \equiv$ change in the yawing moment due to rudder deflection.

$$\Delta N = l_V \Delta Y \qquad (2)$$

$\Delta Y \equiv$ change in side force due to δ_r

$$\Delta Y = \Delta C_{L_V} \bar{q}_V S_V \qquad (3)$$

In (3), $$\Delta C_{L_V} = \frac{\partial C_{L_V}}{\partial \delta_r} \delta_r \qquad (4)$$

$$\frac{\partial C_{L_V}}{\partial \delta_r} = \frac{\partial C_{L_V}}{\partial \alpha_V}\frac{\partial \alpha_V}{\partial \delta_r} = C_{L_{\alpha_V}}\tau_V = C_{L_{\alpha_V}}\tau \quad (5)$$

$$\Rightarrow \Delta C_{L_V} = C_{L_{\alpha_V}}\tau\,\delta_r$$

$$\Rightarrow \Delta Y = C_{L_{\alpha_V}}\tau\,\bar{q}_V S_V \delta_r$$

$$\Rightarrow \Delta N = -l_V S_V \bar{q}_V \tau\, C_{L_{\alpha_V}} \delta_r$$

$$\cancel{\frac{\partial \Delta N}{\partial \delta_r} = C_{n_{\delta_r}}}$$

$$\Delta C_n = \frac{\Delta N}{\bar{q} S b}$$

$$C_{n_{\delta_r}} = \frac{\Delta C_n}{\Delta \delta_r} = \frac{\partial}{\partial r}\left(\frac{-l_V S_V}{S b}\frac{\bar{q}_V}{\bar{q}}\tau\, C_{L_{\alpha_V}}\delta_r\right)$$

$$\boxed{C_{n_{\delta_r}} = -V_V\,\eta\,\tau\, C_{L_\alpha}}$$