EE 231 | **Exam#2** | Dec. 08, 1999

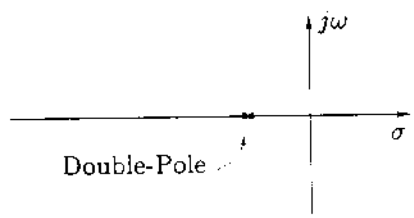
50 minutes

1. For the following open-loop pole/zero locations, sketch *expected* root-locus diagrams

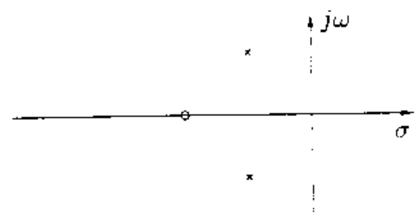
   (a) for $K > 0$, and

   (b) for $K < 0$.

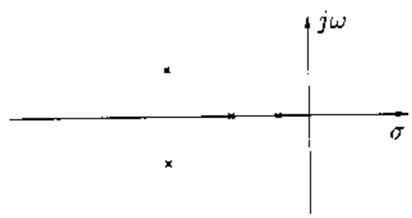
   Do not determine any features of the diagram, except the asymptote angles. Make reasonable guesses for the other features, and show the expected shapes of all the root-locus branches. (24pts)



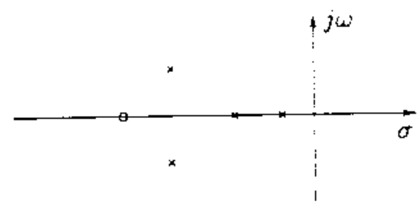
(i)



(ii)



(iii)



(iv)

2. Consider a unity-feedback control system with the open-loop transfer function

$$G(s) = K\frac{(s+2)^2}{s(s^2+1)} = K\frac{s^2+4s+4}{s^3+s}.$$

   (a) Construct the root-locus diagram. Determine all the important features like asymptotes, imaginary-axis crossings, angle of arrivals and departures; however *do not* determine the break-away and/or break-in points explicitly. Obtain only the equation whose solutions would give those points i.e., *do not solve that equation.* (22pts)

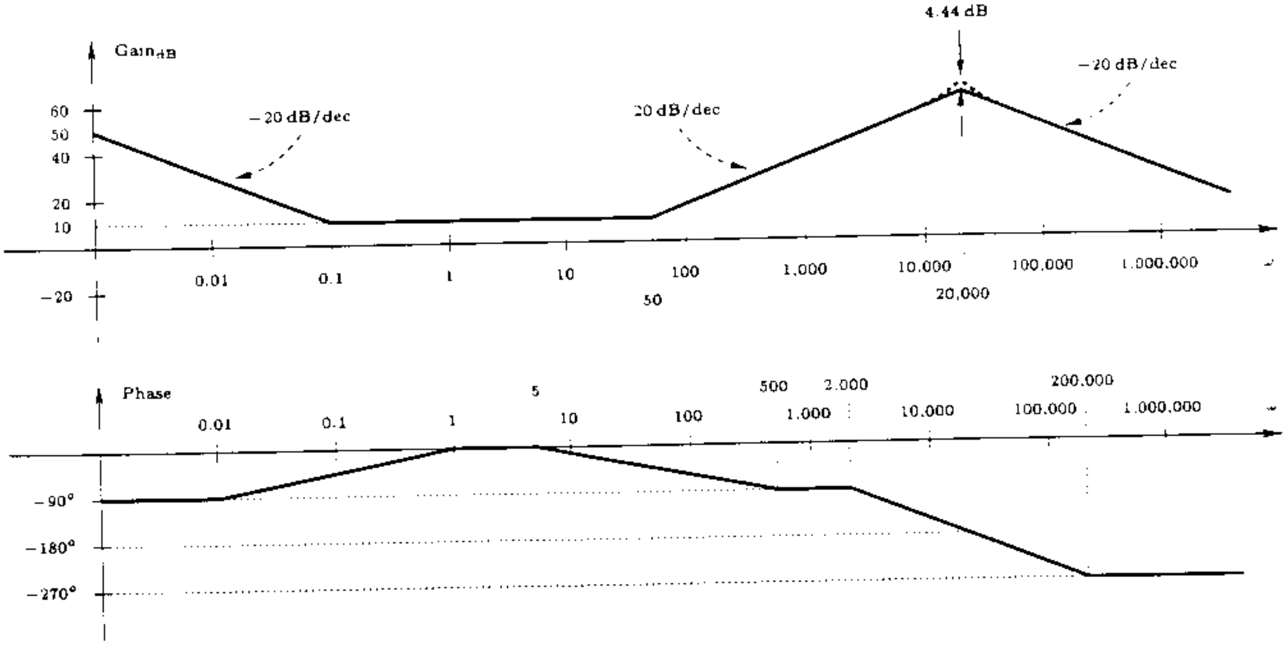
   (b) Determine all the values of K such that the closed-loop system is asymptotically stable. (10pts)

3. Consider a unity-feedback system with the open-loop transfer function

$$D(s)G(s) = D(s)\frac{s+5}{s}.$$

   Design the simplest controller $D(s)$, such that the steady state errors for a step input and a sinusoidal input with frequency 1 rad/s are zero. (22pts)

4. The frequency response of a control system has been obtained experimentally, and the following asymptotes have been fitted.

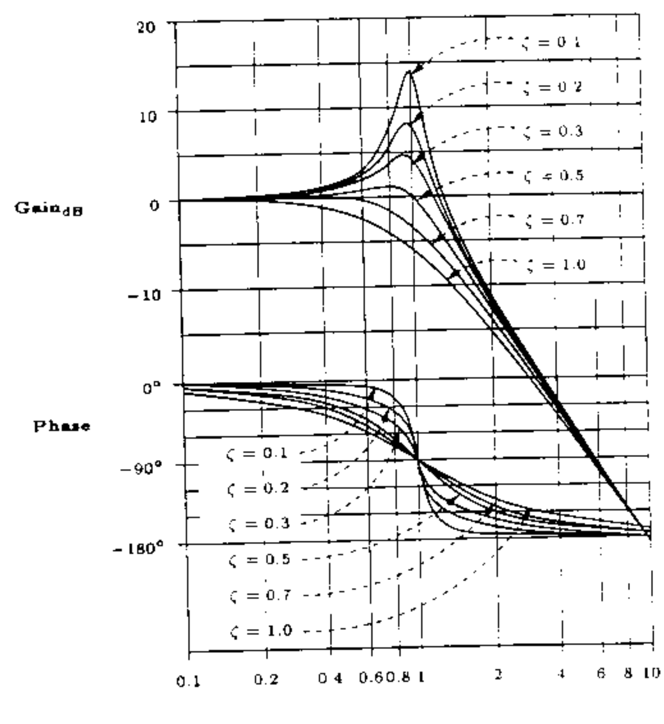


Determine the transfer function from the asymptotes. (22pts)

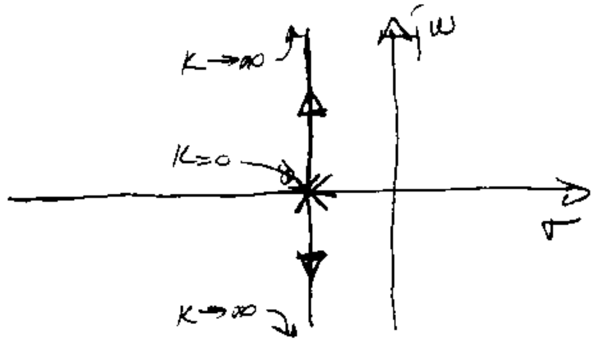
Make intelligent use of the figure on the right which shows the magnitude and phase versus frequency plots of

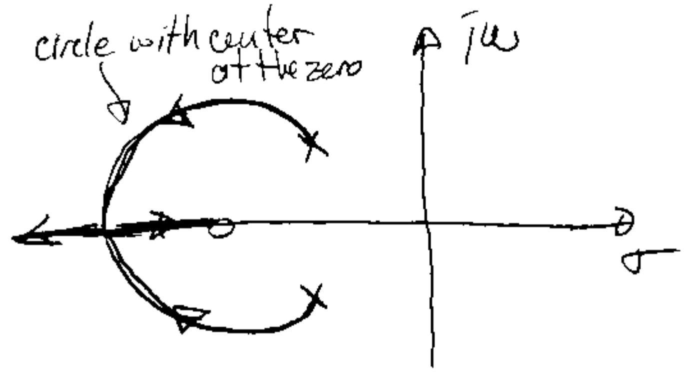
$$\frac{1}{(j(\omega/\omega_n))^2 + 2\zeta\,(j(\omega/\omega_n)) + 1}$$

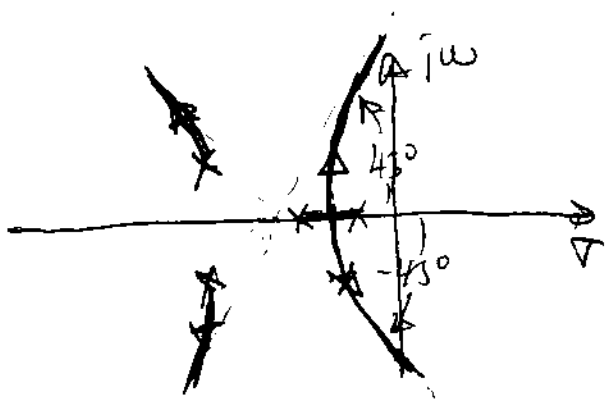
for different values of ζ.

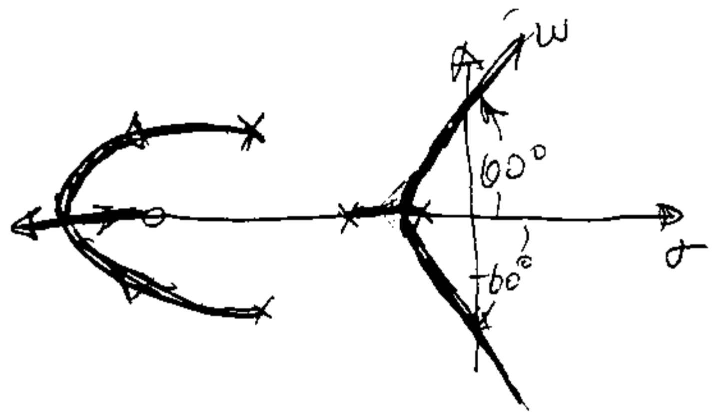


#1 (a) K>0

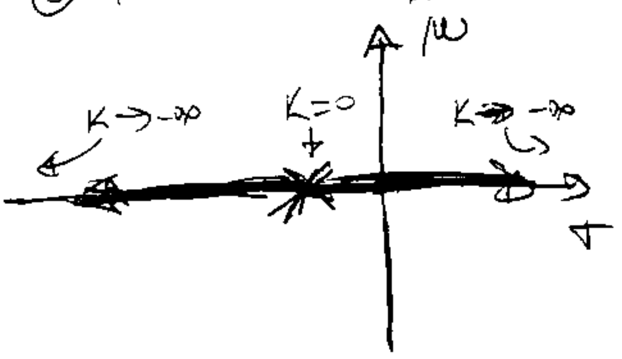

K→∞
K=0
K→∞
jω
σ

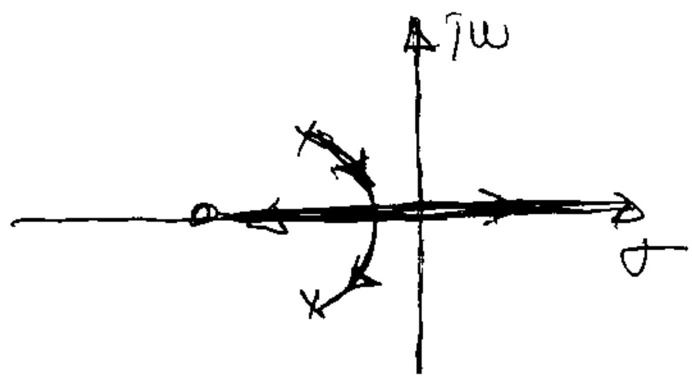

circle with center at the zero
jω
σ


jω
45°
-45°
σ

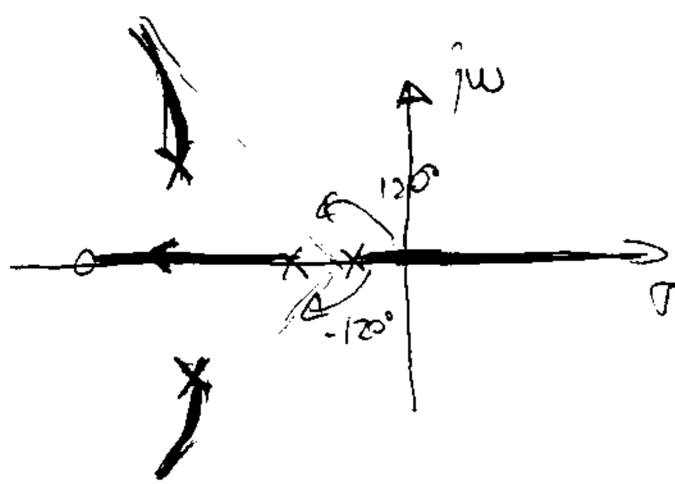

jω
60°
-60°
σ

(b) K<0


jω
K→-∞
K=0
K→-∞
σ

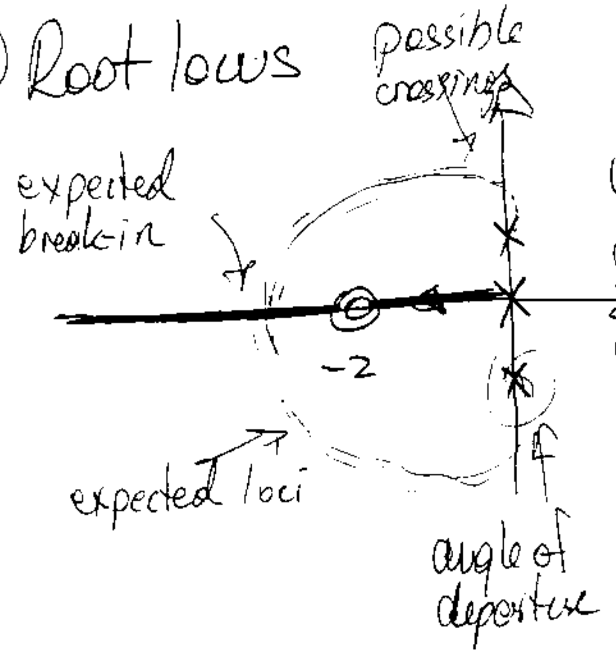

jω
σ


jω
σ


jω
120°
-120°
σ

#2 $G(s) = K \dfrac{(s+2)^2}{s(s^2+1)} = K \dfrac{s^2+4s+4}{s^3+s}$

(a) Root locus



Need to determine
- (*) Break-in pt
- (*) Imaginary axis crossings
- (*) Angle of departure

(*) Break-in pt. from $\dfrac{dK}{ds} = 0$

Since $1 + K \dfrac{s^2+4s+4}{s^3+s} = 0$

$$-K = \frac{s^3+s}{s^2+4s+4}$$

$$-\frac{dK}{ds} = \frac{(3s^2+1)(s^2+4s+4) - (s^3+s)(2s+4)}{(s^2+4s+4)^2}$$

Break away when $\boxed{(3s^2+1)(s^2+4s+4) - (s^3+s)(2s+4) = 0}$

or when
$s = -6.2128$
$s = -2$
$s = 0.1064 \pm j0.5573$

no need to compute in this problem

(*) Imag-axis crossings from R-H table

$$1+K\frac{s^2+4s+4}{s^3+s}=0$$

$$s^3+s+K(s^2+4s+4)=0$$

$$s^3+Ks^2+(4K+1)s+4K=0$$

$$\begin{array}{c|cc} s^3 & 1 & 4K+1 \\ s^2 & K & 4K \\ s & \frac{4K^2+K-4K}{K} & \\ 1 & 4K & \end{array}$$

$\frac{4K^2+K-4K}{K} \rightarrow \frac{4K^2+K-4K}{K} = 4K-3 \rightarrow K=\frac{3}{4}$

$4K \rightarrow K=0 \rightarrow$ trivial crossing at $s=0$

$$(Ks^2+4K=0)_{K=3/4}$$

$$s^2+4=0$$

Imag. axis crossings $\longrightarrow \boxed{s=\pm j2}$

(*) Angle of departure from the ang. cond. about $s=j1$ (or $s=-j1$)

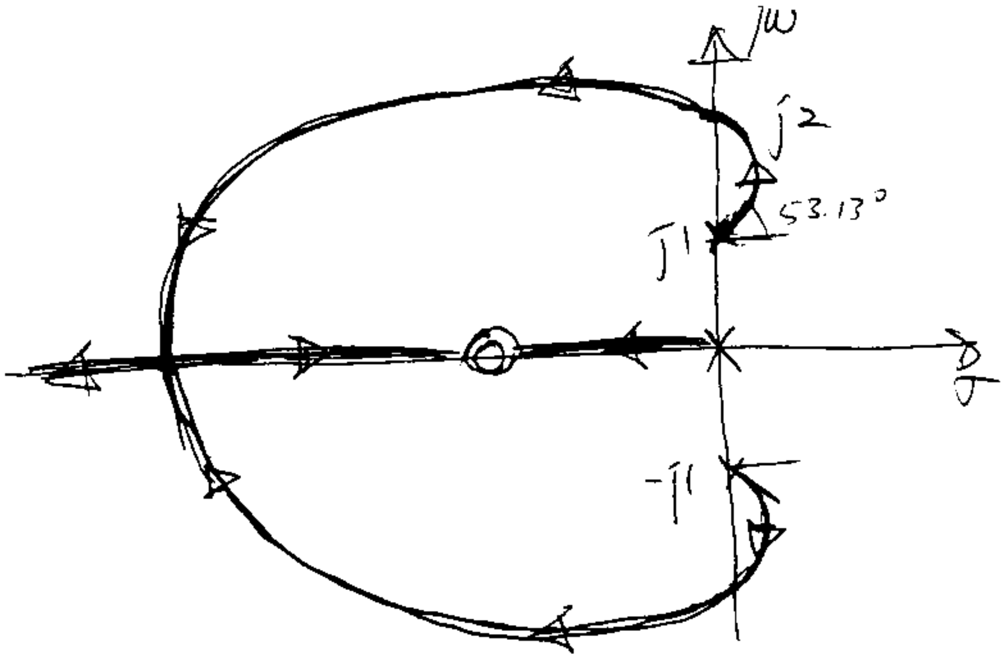
$$2\angle(s+2)-\angle s-\angle(s-j1)-\angle(s+j1)=180^\circ+k360^\circ$$

$$2\tan^{-1}\left(\frac{1-(0)}{0-(-2)}\right)-\tan^{-1}\left(\frac{1-(0)}{0-(0)}\right)-\theta_{dept.}-\tan^{-1}\left(\frac{1-(-1)}{0-(0)}\right)$$

$$=180^\circ+k360^\circ$$

$$2 \times 26.57^\circ - 90^\circ - \theta_{dept} - 90^\circ = 180^\circ + k360^\circ$$

$$\boxed{\theta_{dept} = 53.13^\circ}$$



(b) Stable values of K from the R-H table as well

We had

s^3	1	$4K+1$
s^2	K	$4K$
s	$4K-3$	
1	$4K$	

$\Rightarrow$ $K > 0$ (s^2 and 1)
$4K-3 > 0$ (s) or $K > \frac{3}{4}$ $\Big\} \Rightarrow K > 3/4$

So system is stable for $K > 3/4$

#3 $D(s)G(s) = D(s)\,\frac{s+5}{s}$

Steady state error is zero for step input $\Rightarrow D(s)G(s) = \frac{1}{s} H(s)$

for some

and sinusoidal input with freq. 1 rad/sc $\Rightarrow D(s)G(s) = \frac{1}{s} \cdot \frac{1}{s^2+1} H'(s)$

open-loop gain should have the poles of the non-disappearing components

since $D(s)G(s) = D(s)\,\frac{s+5}{s}$

$$= \frac{1}{s(s^2+1)} H'(s)$$

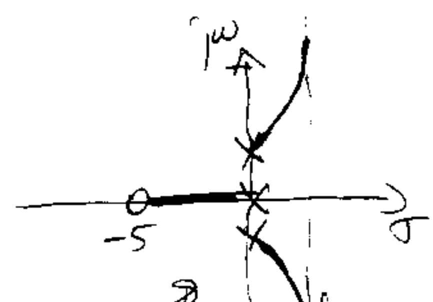
The simplest $D(s) = \frac{1}{(s^2+1)} D'(s)$ where $D'(s)$ is s.t. the closed-loop system is stable

With this $D(s)$, the openloop gain is

$$D(s)G(s) = \frac{s+5}{s(s^2+1)} D'(s)$$

- If $D'(s) = K$, then we have

$$\sigma_a = \frac{(0 + j - j) - (-5)}{3 - 1} = \frac{5}{2}$$

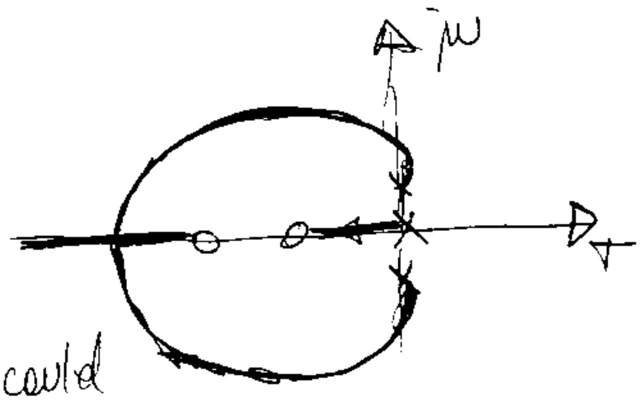


always unstable

So we need a zero to pull the poles in to the LHP

- If $D'(s) = K(s+a)$, $a > 0$ $\left(D(s) = K \frac{s+a}{s^2+1} \right)$

then we have



and the system could be stable

$\boxed{\text{any } a>0 \text{ is o.K}}$

So let $a = 10$ and let's check stab. from the R-H criterion

$$D(s)G(s) = K\frac{(s+5)(s+10)}{s(s^2+1)} = K\frac{s^2+15s+50}{s^3+s}$$

$$1 + D(s)G(s) = 1 + K\frac{s^2+15s+50}{s^3+s} = 0$$

$$\Longrightarrow \quad s^3 + s + K(s^2+15s+50) = 0$$

$$s^3 + Ks^2 + (15K+1)s + 50K = 0$$

$$\begin{array}{c|cc}
s^3 & 1 & 15K+1 \\
s^2 & K & 50K \\
s & \frac{15K^2+K-50K}{K} & \\
1 & 50K &
\end{array}$$

For stability $K>0$ (s^2 & 1)

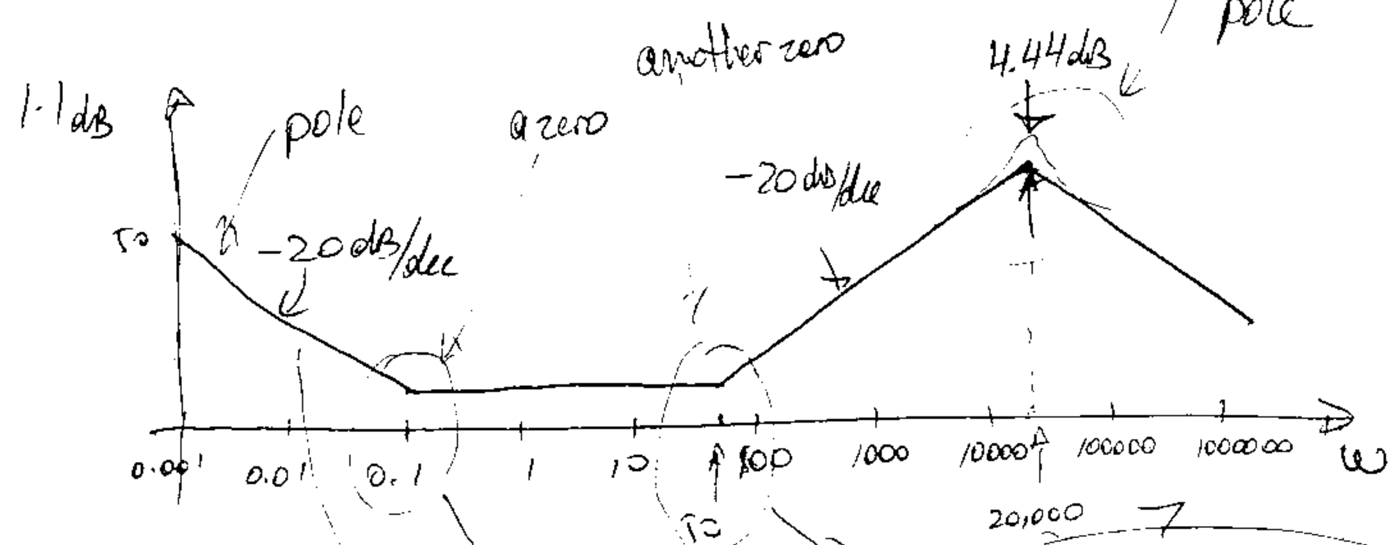
$$\frac{15K^2+K-50K}{K} = 15K-49>0 \quad (s)$$

$$\text{or} \quad K>\frac{49}{15}=3.2667$$

So $\boxed{D(s)=4\,\frac{s+10}{s^2+1}}$ is one choice

any value greater than 49/5 is O.K. for $a=10$
this value depends on your choice of a

#4

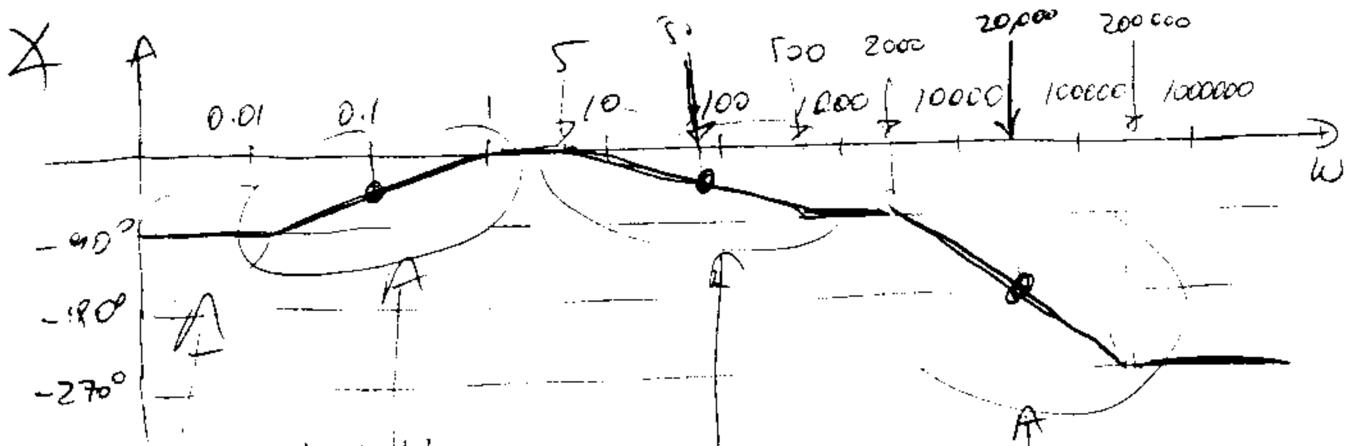


$$G(s)=K\,\frac{1}{s}\,(s/0.1\pm1)\,(s/50\pm1)\left(\frac{1}{\left(\frac{s}{20{,}000}\right)^2\pm2\zeta\left(\frac{s}{20{,}000}\right)+1}\right)$$

ζ from the extra dB at the corner freq of 20,000
and from the given graph 4.44dB results in for $\zeta=0.3$

So $G(s) = K \frac{1}{s} \cdot \left(\frac{s}{0.1} \pm 1\right)\left(\frac{s}{50} \pm 1\right)\left(\frac{1}{\left(\frac{s}{20{,}000}\right)^2 \pm 0.6\left(\frac{s}{20{,}000}\right)+1}\right)$

the signs need to be determined from the phase plot



looks like a pole at $s=0$ ✓

looks like a zero at $s=0.1$ ✓

looks like a pole at $s=50$ X

looks like a second order pole at $s=20{,}000$ ✓

This is a zero (from the gain plot) that looks like a pole in the phase plot
$\Rightarrow$ It is a non-min. phase zero
So the sign for this zero is negative

$$\Rightarrow G(s) = K \frac{\left(\frac{s}{0.1}+1\right)\left(\frac{s}{50}-1\right)}{s\left(\left(\frac{s}{20{,}000}\right)^2 + 0.6\left(\frac{s}{20{,}000}\right)+1\right)}$$

Last thing is to determine K perhaps from low frequencies

At low freq. we have $\frac{K}{s}$ term and

at $\omega = 0.001$, $|G|_{dB} = 50\,dB$

or $20\log\left(\frac{K}{\omega}\right)\Big|_{\omega=0.001} = 50$

$$\Rightarrow \log\left(\frac{K}{0.001}\right) = 2.5$$

$$\frac{K}{0.001} = 10^{2.5}$$

$$K = 0.3162$$

or $$\boxed{G(s) = 0.3162 \frac{\left(\frac{s}{0.1}+1\right)\left(\frac{s}{50}-1\right)}{s\left(\left(\frac{s}{20{,}000}\right)^2 + 0.6\left(\frac{s}{20{,}000}\right)+1\right)}}$$