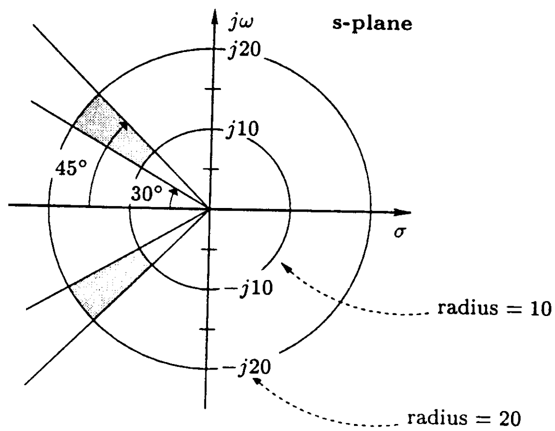
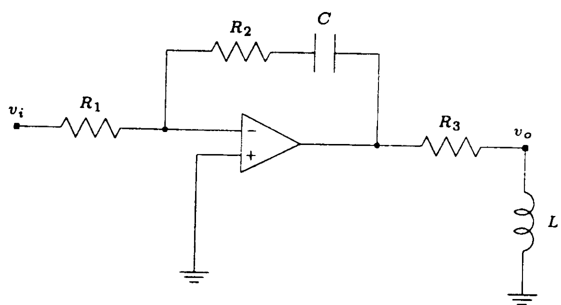
EE 231 **Exam#2** **Apr. 29, 2003**

75 minutes

1. Obtain the necessary inequalities to describe the strictly complex poles in the shaded region below in terms of only ζ and ω_n of a second-order system described by $Y(s)/U(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. (20pts)

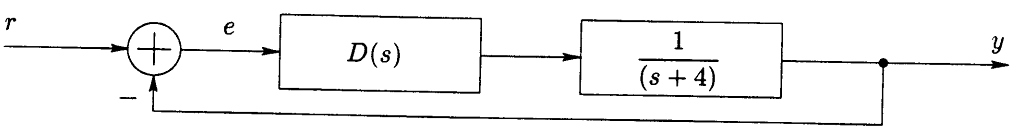


2. Consider the following control system.

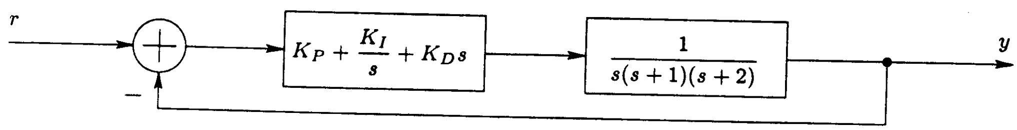


Assume that $R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$, $R_3 = 4\,\Omega$, $C = 1\,\mathrm{F}$, and $L = 2\,\mathrm{H}$. Only the capacitor and the inductor are sensitive to temperature changes; such that the sensitivity of the capacitance with respect to temperature $S_T^C = 5$, and the sensitivity of the inductance with respect to temperature $S_T^L = 4$. Determine the sensitivity of the transfer function $V_o(s)/V_i(s)$ with respect to the temperature. (20pts)

3. Design the simplest controller $D(s)$, such that the steady-state error for a unit-step reference input is zero, and the maximum percent overshoot is about 10%. Show all your work clearly. (15pts)



4. A PID controller is to be designed for the following control system.



(a) Determine the requirements for the PID constants K_P, K_I, and K_D, such that the closed-loop system is asymptotically stable. (15pts)

(b) Determine whether or not the values $K_P = 1$, $K_I = 0$, and $K_D = 1$ result in an asymptotically-stable system. (05pts)

5. Consider a negative unity-feedback control system with the open-loop transfer function

$$G(s) = K\frac{s+1}{(s-1)s(s^2+4s+16)} = K\frac{s+1}{s^4+3s^3+12s^2-16s}.$$

(a) Determine the values of K such that the closed-loop system is asymptotically stable. (15pts)

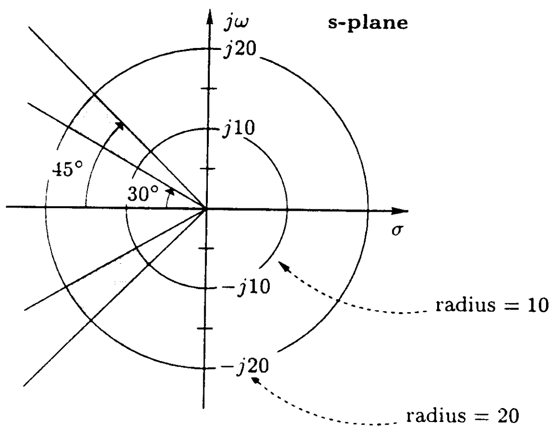
(b) Determine the value (or values) of K and the natural frequency (or frequencies), such that the closed-loop system would have sustained oscillations. (10pts)

EE 231

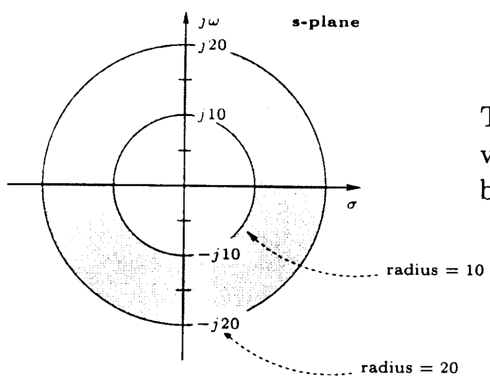
Exam#2
Solutions

Apr. 29, 2003

1. Obtain the necessary inequalities to describe the strictly complex poles in the shaded region below in terms of only ζ and ω_n of a second-order system described by $Y(s)/U(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$.

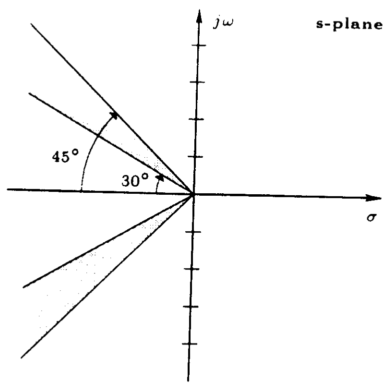


Solution: To be able to describe the shaded region, we need to separate it into unions or intersections of simpler regions.



The equi-distance points from the origin designate constant value for ω_n. As a result, the shown shaded area is represented by

$$10 \le \omega_n \le 20.$$



A straight line originating from the origin designates a constant ζ value, where $\cos^{-1}(\zeta)$ is the acute angle between the line and the negative real axis. So for the shaded area shown, we have

$$30^\circ \leq \cos^{-1}(\zeta) \leq 45^\circ,$$

or

$$\cos(30^\circ) \geq \zeta \geq \cos(45^\circ),$$

since $\cos(\theta)$ is a monotonically decreasing function for $0 < \theta < 180^\circ$. So, we have

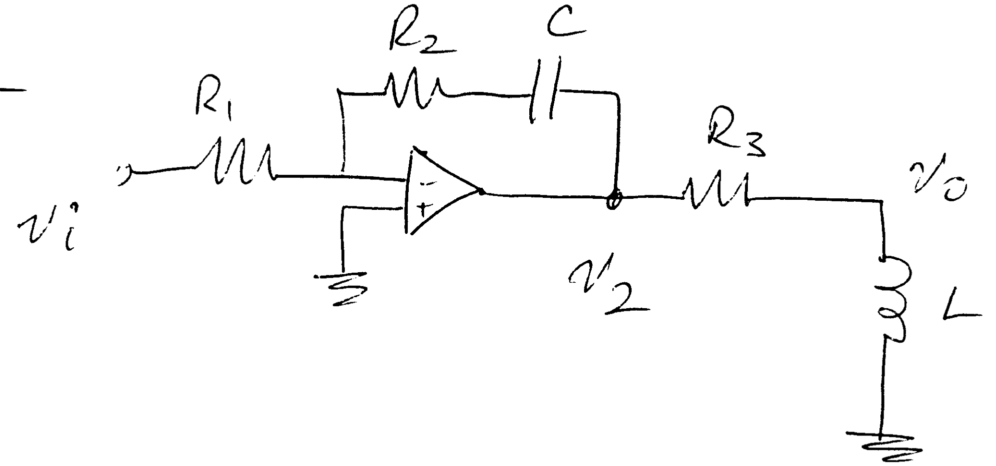
$$\frac{\sqrt{2}}{2} \leq \zeta \leq \frac{\sqrt{3}}{2}.$$

Therefore, the shaded area given in the problem is the intersection of the individual shaded areas, and it can be represented by

$$10 \leq \omega_n \leq 20,$$

$$\sqrt{2}/2 \leq \zeta \leq \sqrt{3}/2.$$

#2



Ideal opamp $\Rightarrow$ $\dfrac{v_2}{v_i} = -\dfrac{R_2 + 1/Cs}{R_1}$

Voltage division $\Rightarrow$ $\dfrac{v_o}{v_2} = \dfrac{Ls}{R_3 + Ls}$

$$\frac{V_o}{V_i} = \frac{V_o}{V_2} \cdot \frac{V_2}{V_i} = -\left(\frac{R_2}{R_1} + \frac{1}{R_1 C_1 s}\right)\left(\frac{Ls}{Ls + R_3}\right)$$

$$= -\frac{R_2 Cs + 1}{R_1 C s} \cdot \frac{Ls}{Ls + R_3} = -\frac{L}{R_1 C} \frac{R_2 C s + 1}{(Ls + R_3)}$$

$$= -\frac{R_2/R_1\, s + 1/R_1 C}{s + R_3/L}$$

Sensitivity

$$S_T^{V_o/V_i} = S_C^{V_o/V_i} S_T^C + S_L^{V_o/V_i} S_T^L$$

$$S_C^{V_0/V_i} = \frac{C}{V_0/V_i} \frac{\partial (V_0/V_i)}{\partial C}$$

where $$\frac{\partial (V_0/V_i)}{\partial C} = - \frac{\left(-\frac{1}{R_1 C^2}\right)}{s + R_3/L} = \frac{1}{R_1 C^2 (s + R_3/L)}$$

$$\Rightarrow S_C^{V_0/V_i} = \frac{C}{-\frac{(R_2/R_1 s + 1/R_1 C)}{(s + R_3/L)}} \cdot \frac{1}{R_1 C^2 (s + R_3/L)}$$

$$= - \frac{1}{CR_2 s + 1}$$

$$S_L^{V_0/V_i} = \frac{L}{V_0/V_i} \frac{\partial (V_0/V_i)}{\partial L}$$

where $$\frac{\partial (V_0/V_i)}{\partial L} = - \frac{-(R_2/R_1 s + 1/R_1 C)(-R_3/L^2)}{(s + R_3/L)^2}$$

$$\Rightarrow S_L^{V_0/V_i} = \left(\frac{L}{-\frac{(R_2/R_1 s + 1/R_1 C)}{(s + R_3/L)}} \right) \left(- \frac{-(R_2/R_1 s + 1/R_1 C)(-R_3/L^2)}{(s + R_3/L)^2} \right)$$

$$S_L^{v_o/v_i} = \frac{R_3}{L} \frac{1}{s + R_3/L} = \frac{1}{L/R_3 s + 1}$$

5

$$\Rightarrow S_T^{v_o/v_i} = -\frac{1}{CR_2 s + 1} S_T^C + \frac{1}{L/R_3 s + 1} S_T^L$$

$$= -\frac{1}{2s+1}(5) + \frac{1}{2/4 s + 1}(4)$$

$$= -\frac{5}{2s+1} + \frac{8}{s+2} = \frac{11s - 2}{(2s+1)(s+2)}$$

#3 $M_p = 10\% = 0.1 = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}$ (6)

$$\Rightarrow \zeta = \frac{|\ln(M_p)|}{\sqrt{\pi^2 + \ln^2(M_p)}} = 0.59$$

$e_{ss} = 0$ for a step input $\Rightarrow D(s) = \frac{1}{s} D'(s)$

Simplest $D'(s) = K$

$$\Rightarrow \quad 1 + D(s)G(s) = 0$$

$$1 + \frac{K}{s} \frac{1}{s+4} = 0$$

$$s(s+4) + K = 0$$

$$s^2 + 4s + K = 0$$

For a second-order system with poles at

$$s = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\,\omega_n$$

the char. equation is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Comparing the terms in

$$s^2+4s+K=0$$

and

$$s^2+2\zeta\omega_n s+\omega_n^2=0$$

we get $2\zeta\omega_n=4 \Longrightarrow \omega_n=\frac{4}{2\zeta}=\frac{4}{2(0.59)}$

$\omega_n^2=K$ $\qquad\qquad\qquad\qquad = 3.38$

so if $K=\omega_n^2=(3.38)^2$, then we satisfy all the constraints.

$$K=(3.38)^2=11.49$$

or $\boxed{D(s)=\frac{11.49}{s}}$

#4 (a) The characteristic equation is 8

$$1 + D(s)\,G(s) = 0$$

$$1 + \frac{K_P s + K_I + K_D s^2}{s} \cdot \frac{1}{s(s+1)(s+2)} = 0$$

or

$$s^2(s+1)(s+2) + K_P s + K_I + K_D s^2 = 0$$

$$s^4 + 3s^3 + (K_D+2)s^2 + K_P s + K_I = 0$$

R-H table

s^4	1	K_D+2	K_I
s^3	3	K_P	
$\cancel{s^2}$	$\cancel{\dfrac{3K_D+6-K_P}{3}}$	$\cancel{K_I}$	
s^2	$3K_D+6-K_P$	$3K_I$	
s	$\dfrac{(3K_D+6-K_P)K_P - 9K_I}{3K_D+6-K_P}$		
1	$3K_I$		

For asymptotical stability

(i) $\boxed{3K_D+6-K_P > 0}$

(ii) $\boxed{(3K_D+6-K_P)K_P-9K_I > 0}$

(iii) $\boxed{K_I > 0}$

(b) For $K_P=1$, $K_I=0$, $K_D=1$

(i) $3(1)+6-(1) \overset{?}{>} 0$

$8>0$ ✓

(ii) $(3(1)+6-(1))(1)-9(0) \overset{?}{>} 0$

$8>0$ ✓

(iii) $0 \overset{?}{>} 0$ ✗

Not asympt. stable

(Actually, if $K_I=0$ the order of the system will be reduced.)

For the reduced order system, we get

$$1 + (K_p + K_D s) \cdot \frac{1}{s(s+1)(s+2)} = 0$$

$$s(s+1)(s+2) + K_p + K_D s = 0$$

$$s^3 + 3s^2 + (K_D + 2)s + K_p = 0$$

R-H table

$$\begin{array}{c|cc}
s^3 & 1 & K_D + 2 \\
s^2 & 3 & K_p \\
s & \frac{3(K_D+2) - K_p}{3} & \\
1 & K_p &
\end{array}$$

For asympt. stability

(i) $3(K_D+2)-K_P>0$

$3((1)+2)-(1)>0$ ✓

(ii) $K_P>0$

$(1)>0$ ✓

$\boxed{\text{So system is asympt. stable.}}$

#5 The char. eqn. is

a) $$1+K\frac{s+1}{s^4+3s^3+12s^2-16s}=0$$

$$s^4+3s^3+12s^2-16s+Ks+K=0$$

R-H Table

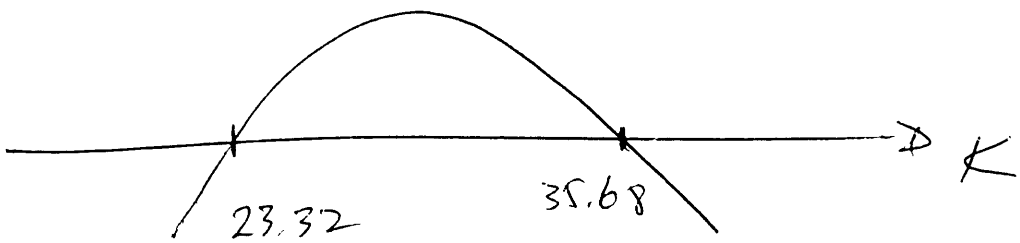
s^4	1	12	K
s^3	3	$K-16$	
$\cancel{s^2}$	$\cancel{\frac{52-K}{3}}$	$\cancel{K}$	
s^2	$52-K$	$3K$	
s	$\frac{-K^2+59K-832}{52-K}$		
1	$3K$		

Condition

(i) $\boxed{K < 52}$

(ii) $-K^2 + 59K - 832 > 0$

$-(K - 23.32)(K - 35.68) = 0$



$\Rightarrow \boxed{23.32 < K < 35.68}$

(iii) $\boxed{K > 0}$

All three cond. are satisfied

iff $\boxed{23.32 < K < 35.68}$

(b) Setting a row of zero

(the s-term row)

$\Rightarrow \quad -K^2+59K-832=0$

$-(K-23.32)(K-35.68)=0$

s^2-term

when $\boxed{K=23.32} \longrightarrow (52-K)s^2+3K=0$

$\Rightarrow 28.68s^2+69.96=0$

$s=\pm j\,1.5618$

$\Rightarrow \boxed{\omega=1.5618}$

when $\boxed{K=35.68} \longrightarrow (52-K)s^2+3K=0$

$\Rightarrow 16.32s^2+107.04=0$

$s=\pm j\,2.5610$

$\boxed{\omega=2.5610}$