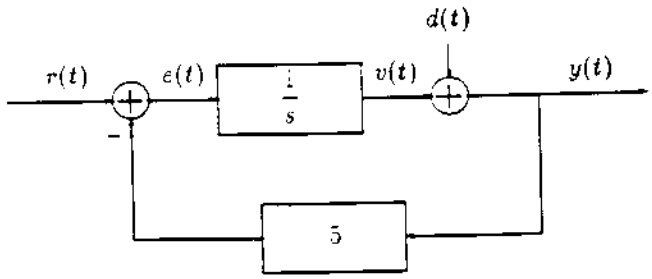
EE 231 **Exam#1** **Feb. 25, 1992**

75 minutes

1. In the following block diagram,



the Laplace transform of the output,

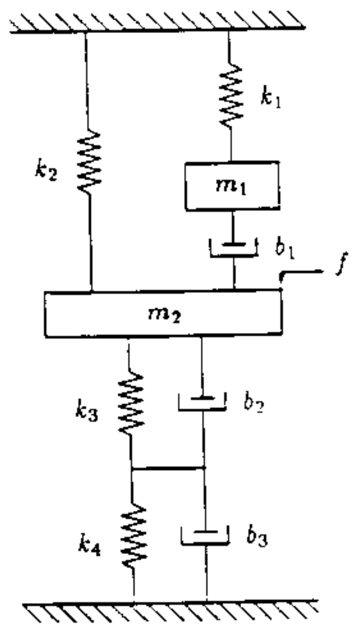
$$\mathcal{L}[y(t)] = Y(s) = \frac{2}{s+5}\left[\frac{6s}{s^2+9} + \frac{1}{s+1}\right],$$

and the reference input,

$$r(t) = 2e^{-t}\,1(t).$$

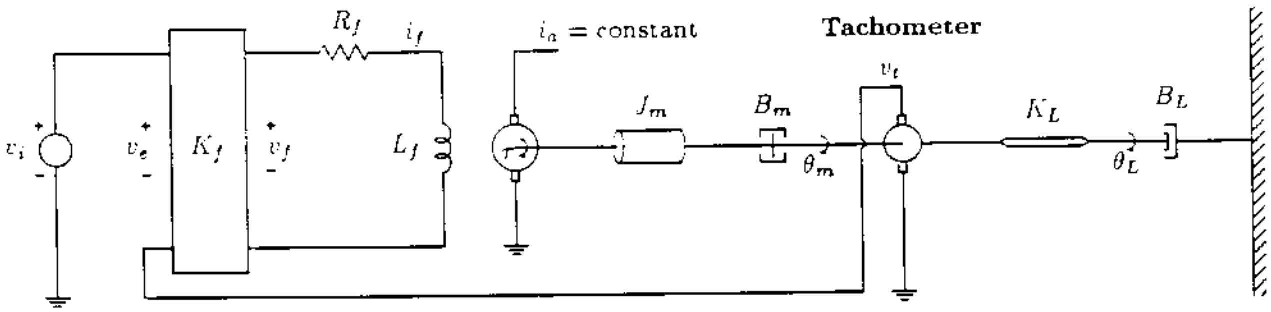
Determine the disturbance signal $d(t)$. *Hint: Obtain $Y(s)$ in terms of $D(s)$ and $R(s)$ first, then solve for $D(s)$.* (20pts)

2. For the mechanical system shown below,



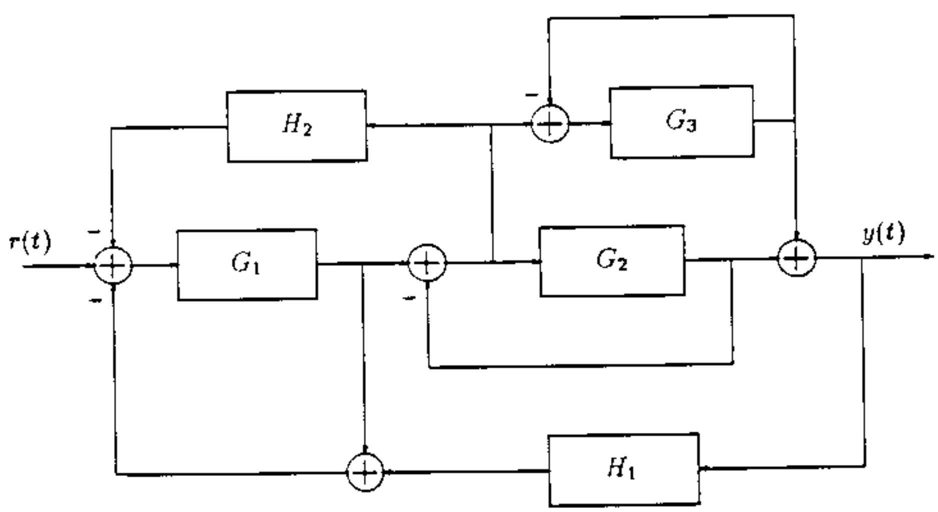
find the differential equations describing the motion, and obtain *either* the force-voltage *or* the force-current analog of the system. (20pts)

3. The block diagram of an electronic drill which adjust its output speed under different loads is shown below.



Here, the ideal amplifier at the field winding of the motor has a gain of K_f. Assuming that the output voltage of the tachometer is proportional to the angular speed, such that $v_t = K_t\omega_m$, draw the most detailed block diagram of the system. Also assume v_i is the input, θ_L is the output, and show all the variables v_i, v_e, v_f, i_f, τ, ω_m, θ_m and θ_L. (30pts)

4. For the block diagram given below,



determine the transfer function *either* by block diagram reduction, *or* by Mason's formula. Show your work clearly. (30pts)

Feb. 25, 1992

EE 231

Exam#1
Solutions

1. From the block diagram

$$\begin{aligned} Y(s) &= D(s) + V(s) \\ &= D(s) + \frac{1}{s}E(s) \\ &= D(s) + \frac{1}{s}\left(R(s) - 5Y(s)\right), \end{aligned}$$

or

$$\begin{aligned} D(s) &= Y(s) + \frac{5}{s}Y(s) - \frac{1}{s}R(s) \\ &= \frac{s+5}{s}Y(s) - \frac{1}{s}R(s). \end{aligned}$$

Since,

$$Y(s) = \frac{2}{s+5}\left[\frac{6s}{s^2+9} + \frac{1}{s+1}\right],$$

and

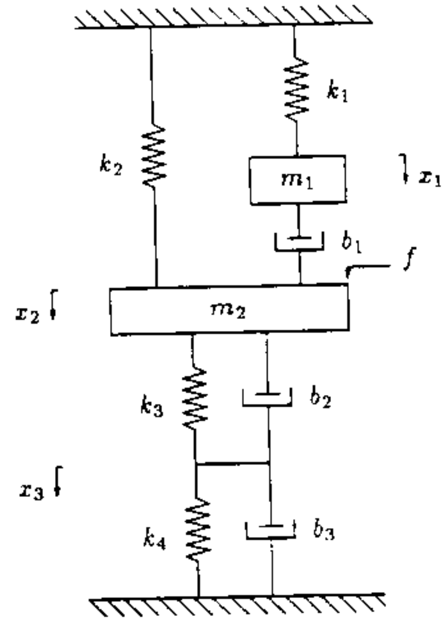
$$r(t) = 2e^{-t}\,1(t) \longleftrightarrow 2\frac{1}{s+1} = R(s),$$

$$\begin{aligned} D(s) &= \frac{s+5}{s}\frac{2}{s+5}\left[\frac{6s}{s^2+9} + \frac{1}{s+1}\right] - \frac{1}{s}\frac{2}{s+1} \\ &= \frac{12}{s^2+9} + \frac{2}{s(s+1)} - \frac{2}{s(s+1)} \\ &= \frac{12}{s^2+9} \\ &= 4\frac{3}{s^2+3^2}. \end{aligned}$$

Therefore,

$$d(t) = 4\sin 3t\,\mathbf{1}(t) \longleftrightarrow 4\frac{3}{s^2+3^2} = D(s).$$

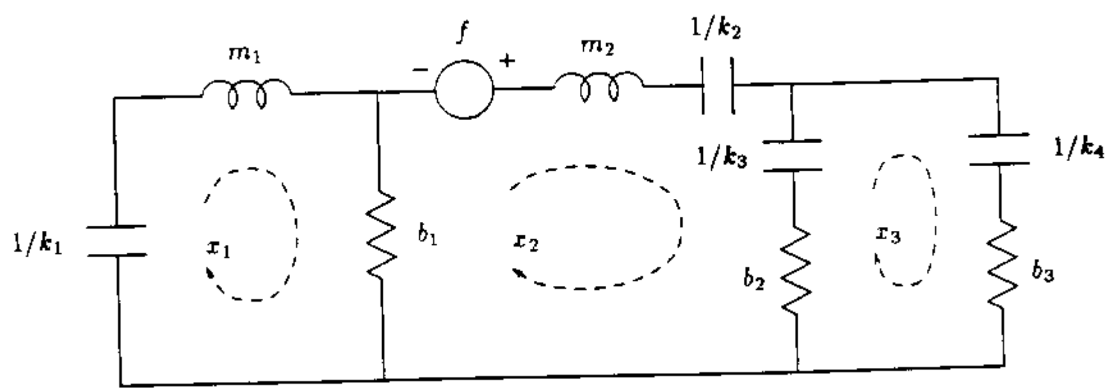
2. We first denote all the displacements of the system.



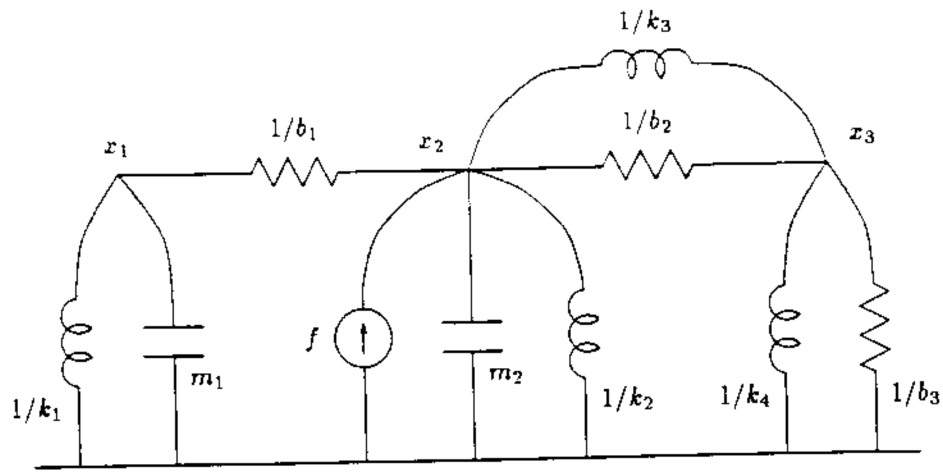
Then, we can write the differential equations describing the motion directly from the mechanical system.

$$m_1\ddot{x}_1 = -k_1x_1 - b_1(\dot{x}_1 - \dot{x}_2),$$
$$m_2\ddot{x}_2 = f - k_2x_2 - b_1(\dot{x}_2 - \dot{x}_1) - b_2(\dot{x}_2 - \dot{x}_3) - k_3(x_2 - x_3),$$
$$0 = -b_3\dot{x}_3 - k_4x_3 - b_2(\dot{x}_3 - \dot{x}_2) - k_3(x_3 - x_2).$$

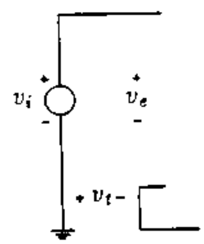
If you choose to obtain the force-voltage equivalent of the above system, then you form loops to represent the equations.



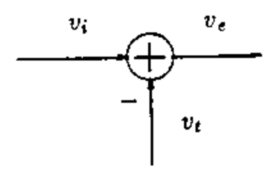
If you choose to obtain the force-voltage equivalent of the system, then you form nodes to represent the equations.

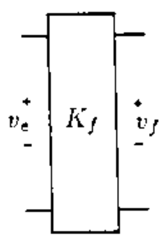


3. We first separate the components of the system.

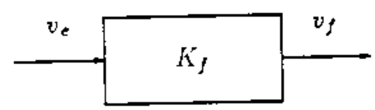


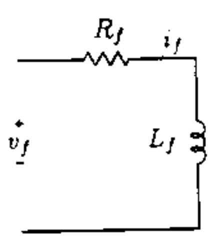
$$v_e = v_i - v_t$$



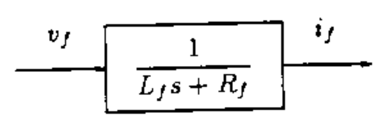


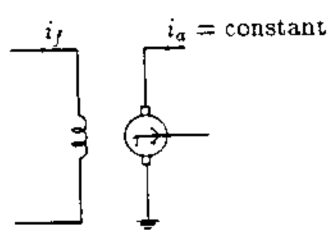
$$v_f = K_f v_e$$



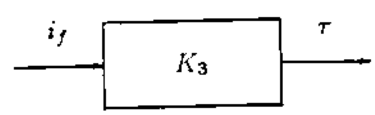


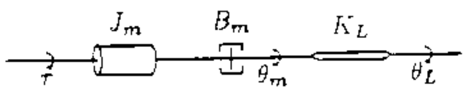
$$L_f \frac{di_f}{dt} + R_f i_f = v_f$$
$$(L_f s + R_f) I_f(s) = V_f(s)$$
$$I_f(s) = \frac{1}{L_f s + R_f} V_f(s)$$



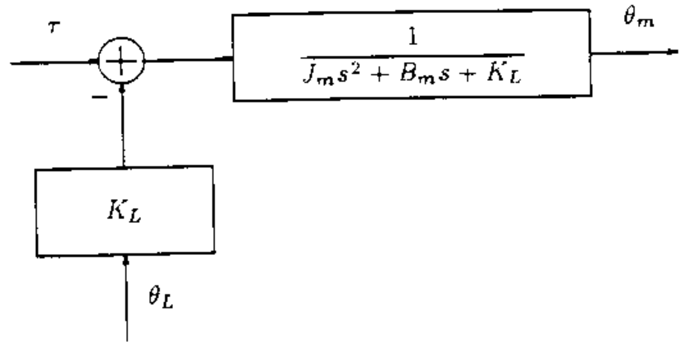


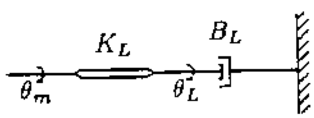
$$\tau = K_3 i_f$$



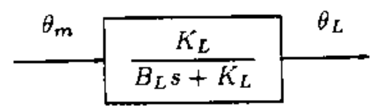


$$J_m\ddot{\theta}_m = \tau - B_m\dot{\theta}_m - K_L(\theta_m - \theta_L)$$
$$(J_m s^2 + B_m s + K_L)\Theta_m(s) = \mathcal{T}(s) + K_L\Theta_L(s)$$
$$\Theta_m(s) = \frac{1}{J_m s^2 + B_m s + K_L}(\mathcal{T}(s) + K_L\Theta_L(s))$$

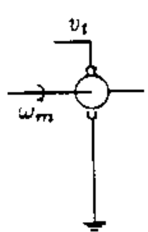




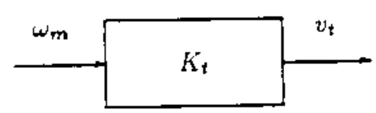
$$0 = -B_L\dot{\theta}_L - K_L(\theta_L - \theta_m)$$
$$(B_L s + K_L)\Theta_L(s) = K_L\Theta_m(s)$$
$$\Theta_L(s) = \frac{K_L}{B_L s + K_L}\Theta_m(s)$$



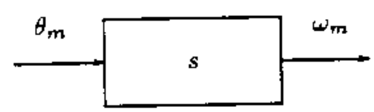
Tachometer



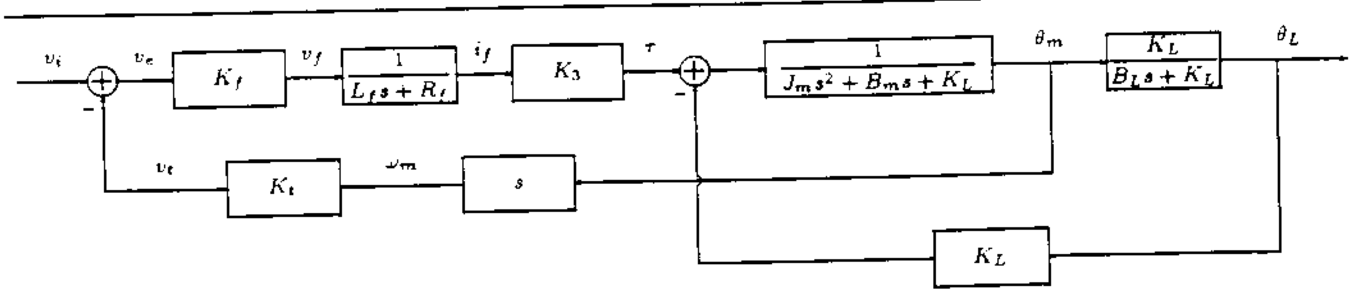
$$v_t = K_t\omega_m$$



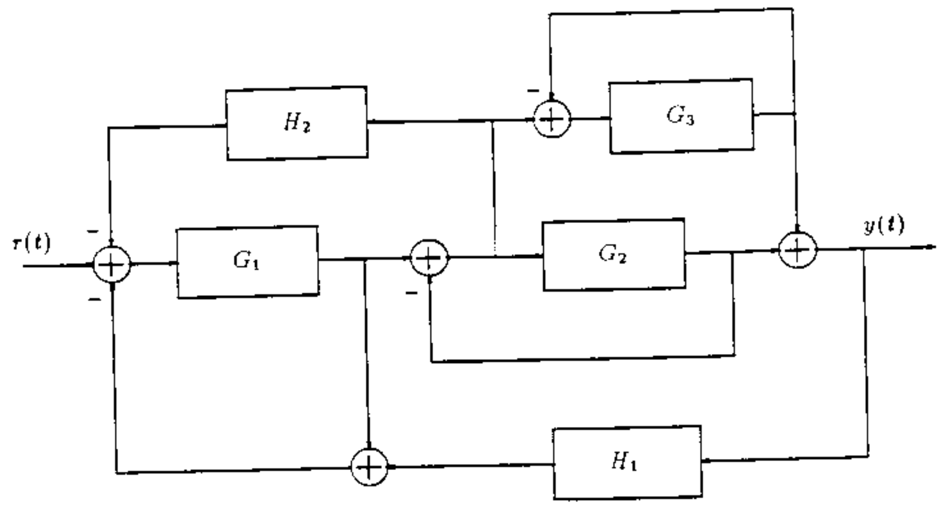
$$\omega_m = \dot{\theta}_m$$
$$\Omega_m = s\Theta_m$$

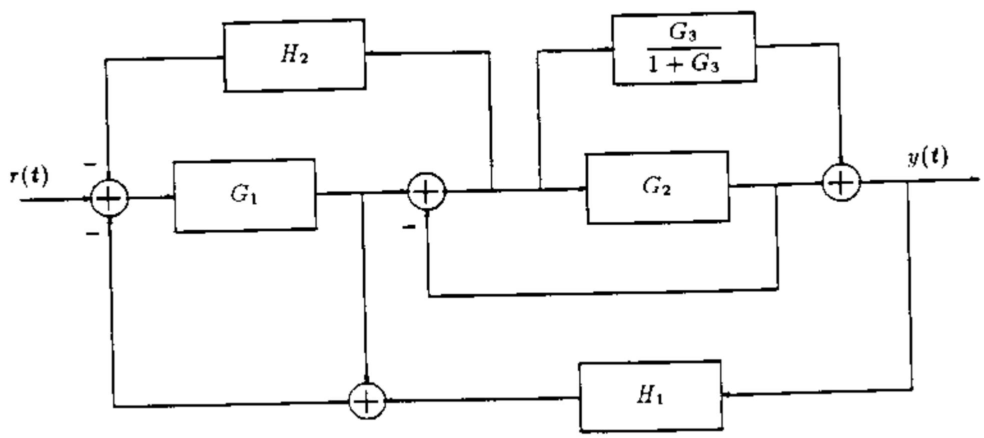


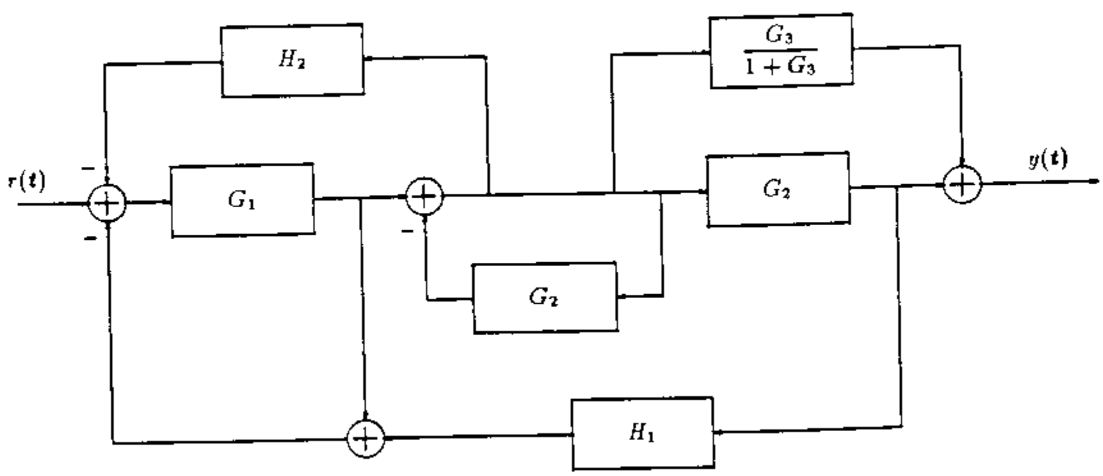
Then, we put all these blocks together to form the complete block diagram.

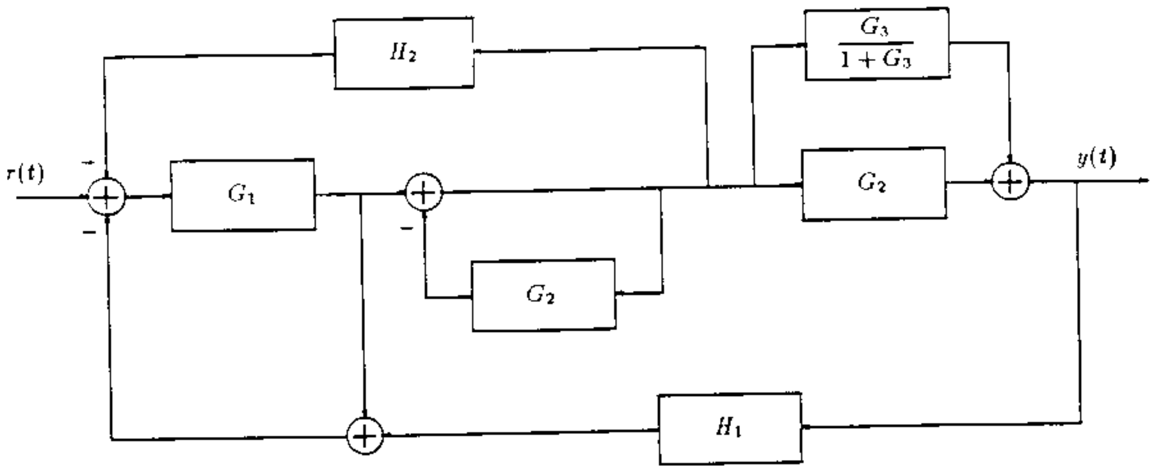


4. If you choose to use the block diagram reduction, best approach is to reduce the block diagram step by step, until you obtain the transfer function.

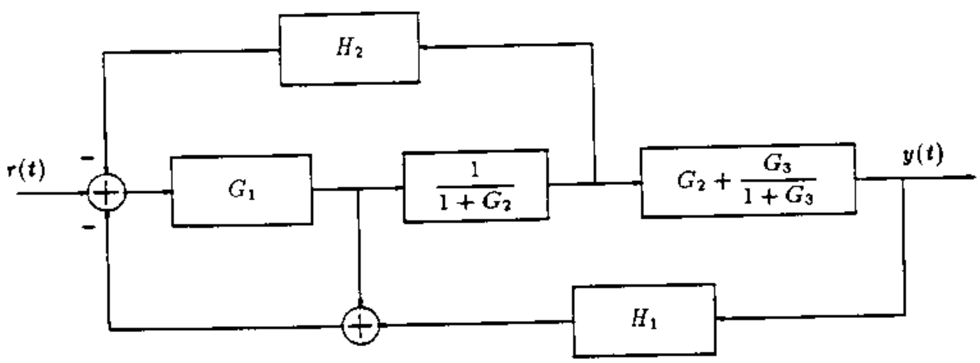




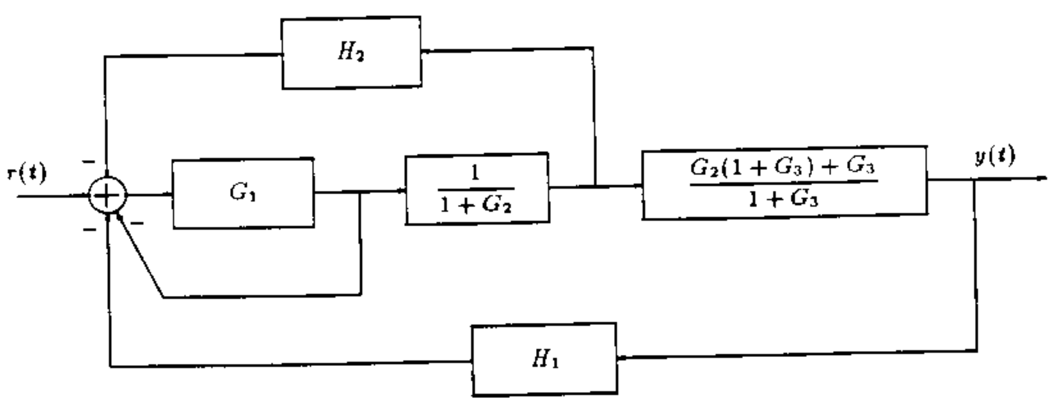




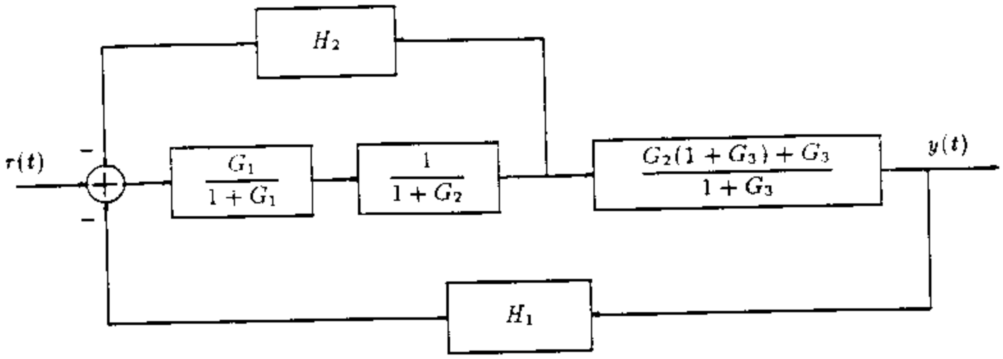
H_2
$\frac{G_3}{1+G_3}$
$r(t)$
G_1
G_2
$y(t)$
G_2
H_1

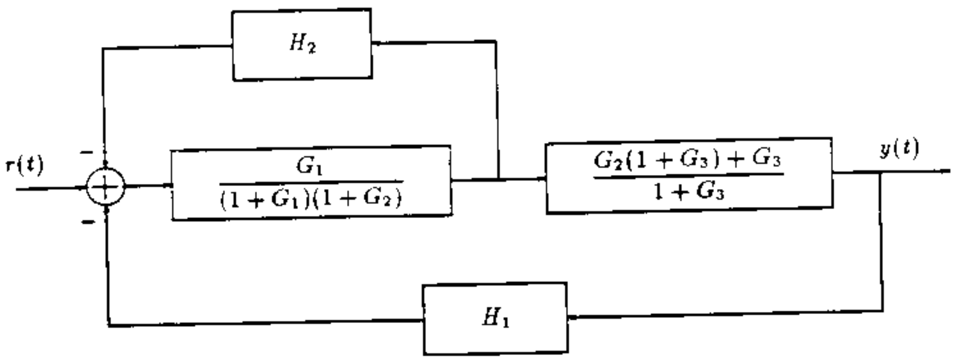


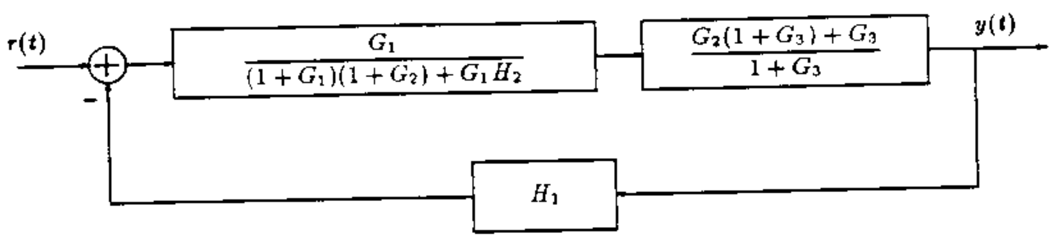
H_2
$r(t)$
G_1
$\frac{1}{1+G_2}$
$G_2 + \frac{G_3}{1+G_3}$
$y(t)$
H_1



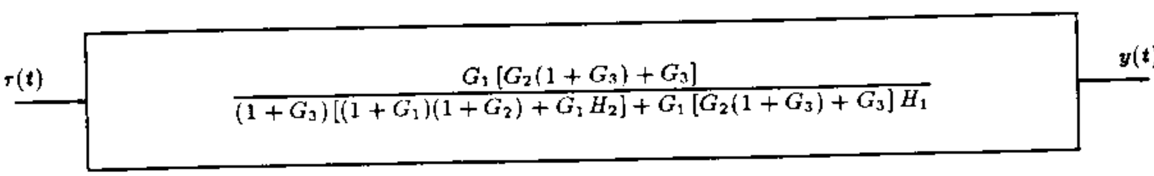
H_2
$r(t)$
G_1
$\frac{1}{1+G_2}$
$\frac{G_2(1+G_3)+G_3}{1+G_3}$
$y(t)$
H_1



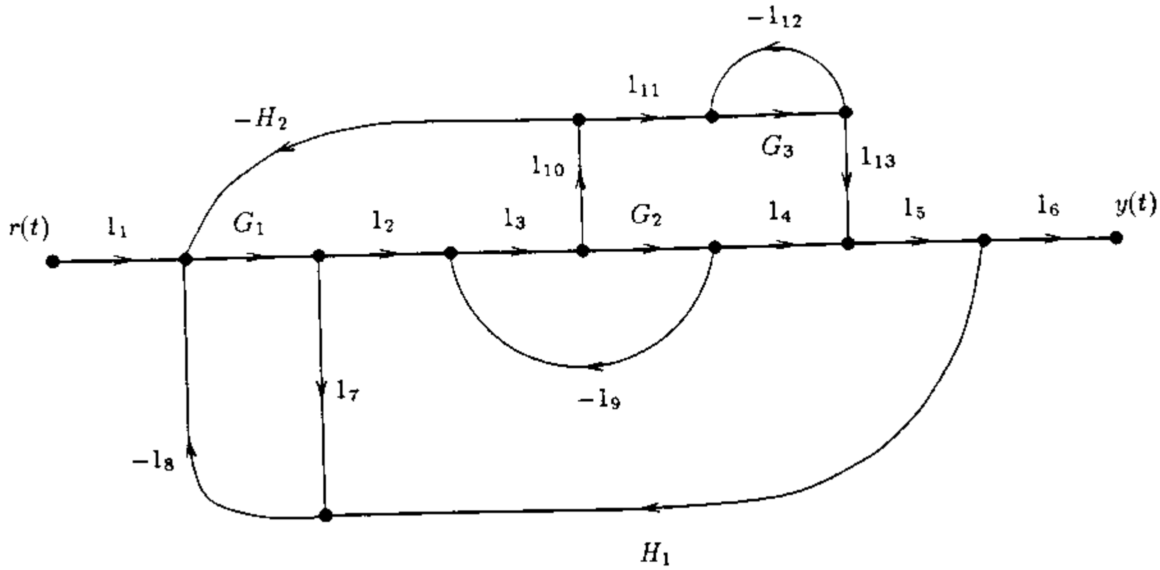




Finally,



If you choose to use Mason's formula, you need to draw the signal flow graph of the block diagram.



In drawing the signal flow graph, the unity gains are subscribed for your convenience. This way you can easily follow the forward paths and the loops.

The forward paths are:

$$F_1 = 1_1 G_1 1_2 1_3 G_2 1_4 1_5 1_6 = G_1 G_2,$$
$$F_2 = 1_1 G_1 1_2 1_3 1_{10} 1_{11} G_3 1_{13} 1_5 1_6 = G_1 G_3.$$

The loops are:

$$L_1 = G_1 1_7(-1_8) = -G_1,$$
$$L_2 = G_1 1_2 1_3 1_{10}(-H_2) = -G_1 H_2,$$
$$L_3 = 1_3 G_2(-1_9) = -G_2,$$
$$L_4 = G_1 1_2 1_3 G_2 1_4 1_5 H_1(-1_8) = -G_1 G_2 H_1,$$
$$L_5 = G_1 1_2 1_3 1_{10} 1_{11} G_3 1_{13} 1_5 H_1(-1_8) = -G_1 G_3 H_1,$$
$$L_6 = G_3(-1_{12}) = -G_3.$$

Non-touching loops are:

Two Loops	Three Loops
$L_1 \& L_3$	
$L_1 \& L_6$	
$L_2 \& L_6$	$L_1 \& L_3 \& L_6$
$L_3 \& L_6$	
$L_4 \& L_6$	

The loops on the forward paths are such that:

F_1 touches $L_1, L_2, L_3, L_4, L_5,$

F_2 touches $L_1, L_2, L_3, L_4, L_5, L_6.$

As a result.

$$\begin{aligned}\Delta = 1 &- (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)\\ &+(L_1L_3 + L_1L_6 + L_2L_6 + L_3L_6 + L_4L_6)\\ &-(L_1L_3L_6)\\ = 1 &+ G_1 + G_1H_2 + G_2 + G_1G_2H_1 + G_1G_3H_1 + G_3\\ &+G_1G_2 + G_1G_3 + G_1G_3H_2 + G_2G_3 + G_1G_2G_3H_1\\ &+G_1G_2G_3,\end{aligned}$$

and

$$\begin{aligned}\Delta_1 &= \Delta|_{L_2=\cdots=L_6=0} = 1 + G_3,\\ \Delta_2 &= \Delta|_{L_1=\cdots=L_6=0} = 1.\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{Y(s)}{R(s)} = F(s) &= \frac{1}{\Delta}(F_1\Delta_1 + F_2\Delta_2)\\ &= \frac{G_1G_2(1+G_3) + G_1G_3}{\Delta}.\end{aligned}$$