1. In the following block diagram.

\[ r(t) \quad e(t) \quad \frac{1}{s} \quad v(t) \quad y(t) \]

the Laplace transform of the output.

\[ \mathcal{L}\{y(t)\} = Y(s) = \frac{2}{s + 5} \left[ \frac{6s}{s^2 + 9} + \frac{1}{s + 1} \right], \]

and the reference input.

\[ r(t) = 2e^{-t} 1(t). \]

Determine the disturbance signal \( d(t) \). \textit{Hint: Obtain} \( Y(s) \) \textit{in terms of} \( D(s) \) \textit{and} \( R(s) \) \textit{first, then solve for} \( D(s) \). (20pts)

2. For the mechanical system shown below.

\[ k_2 \quad k_1 \quad m_4 \quad k_3 \quad k_4 \quad m_2 \quad f \]

find the differential equations describing the motion, and obtain either the force-voltage or the force-current analog of the system. (20pts)
3. The block diagram of an electronic drill which adjusts its output speed under different loads is shown below.

Here, the ideal amplifier at the field winding of the motor has a gain of $K_f$. Assuming that the output voltage of the tachometer is proportional to the angular speed, such that $v_t = K\omega$, draw the most detailed block diagram of the system. Also assume $v_i$ is the input, $\theta_L$ is the output, and show all the variables $v_i, v_e, v_f, i_f, \tau, \omega_m, \theta_m$ and $\theta_L$.

4. For the block diagram given below.

Determine the transfer function either by block diagram reduction, or by Mason’s formula. Show your work clearly.
1. From the block diagram

\[ Y(s) = D(s) + V(s) \]
\[ = D(s) + \frac{1}{s} E(s) \]
\[ = D(s) + \frac{1}{s} (R(s) - 5Y(s)), \]

or

\[ D(s) = Y(s) + \frac{5}{s} Y(s) - \frac{1}{s} R(s) \]
\[ = \frac{s + 5}{s} Y(s) - \frac{1}{s} R(s). \]

Since,

\[ Y(s) = \frac{2}{s + 5} \left[ \frac{6s}{s^2 + 9} + \frac{1}{s + 1} \right], \]

and

\[ r(t) = 2e^{-t} 1(t) \quad \Rightarrow \quad 2 \frac{1}{s + 1} = R(s), \]

\[ D(s) = \frac{s + 5}{s} \cdot 2 \left[ \frac{6s}{s^2 + 9} + \frac{1}{s + 1} \right] - \frac{2}{s + 1} \]
\[ = \frac{12}{s^2 + 9} + 2 \frac{2}{s(s + 1)} - \frac{2}{s(s + 1)} \]
\[ = \frac{12}{s^2 + 9} \]
\[ = \frac{3}{s^2 + 3^2}. \]

Therefore,

\[ d(t) = 4 \sin 3t 1(t) \quad \Rightarrow \quad 4 \frac{3}{s^2 + 3^2} = D(s). \]

2. We first denote all the displacements of the system.
Then, we can write the differential equations describing the motion directly from the mechanical system.

\[
\begin{align*}
    m_1 \ddot{x}_1 &= -k_1 x_1 - b_1 (\dot{x}_1 - \dot{x}_2), \\
    m_2 \ddot{x}_2 &= f - k_2 x_2 - b_1 (\dot{x}_2 - \dot{x}_1) - b_2 (\dot{x}_2 - \dot{x}_3) - k_3 (x_2 - x_3), \\
    0 &= -b_3 \ddot{x}_3 - k_4 x_3 - b_2 (\dot{x}_3 - \dot{x}_2) - k_3 (x_3 - x_2).
\end{align*}
\]

If you choose to obtain the force-voltage equivalent of the above system, then you form loops to represent the equations.

If you choose to obtain the force-voltage equivalent of the system, then you form nodes to represent the equations.
3. We first separate the components of the system.

\[ v_e = v_1 - v_t \]

\[ v_f = K_f v_e \]

\[ L_f \frac{di_f}{dt} + R_f i_f = v_f \]

\[ (L_f s + R_f)I_f(s) = V_f(s) \]

\[ I_f(s) = \frac{1}{L_f s + R_f} V_f(s) \]

\[ r = K_3 i_f \]
\[
J_m \dot{\theta}_m = \tau - B_m \dot{\theta}_m - K_L (\theta_m - \theta_L)
\]
\[
(J_m s^2 + B_m s + K_L) \Theta_m(s) = \tau(s) + K_L \Theta_L(s)
\]
\[
\Theta_m(s) = \frac{1}{J_m s^2 + B_m s + K_L} \left( \tau(s) + K_L \Theta_L(s) \right)
\]
\[
0 = -B_L \dot{\theta}_L - K_L (\theta_L - \theta_m)
\]
\[
(B_L s + K_L) \Theta_L(s) = K_L \Theta_m(s)
\]
\[
\Theta_L(s) = \frac{K_L}{B_L s + K_L} \Theta_m(s)
\]

Tachometer
\[
v_t = K_t \omega_m
\]
\[
\omega_m = \dot{\theta}_m
\]
\[
\Omega_m = s \Theta_m
\]

Then, we put all these blocks together to form the complete block diagram.
4. If you choose to use the block diagram reduction, the best approach is to reduce the block diagram step by step, until you obtain the transfer function.
Finally,

\[
\begin{align*}
\frac{G_1 [G_2 (1 + G_3) + G_3]}{(1 + G_3) [(1 + G_1)(1 + G_2) + G_3 H_2] + G_1 [G_2 (1 + G_3) + G_3] H_1}
\end{align*}
\]

If you choose to use Mason's formula, you need to draw the signal flow graph of the block diagram.
In drawing the signal flow graph, the unity gains are subscribed for your convenience. This way you can easily follow the forward paths and the loops.

The forward paths are:

\[ F_1 = l_1 G_1 l_2 l_3 G_2 l_4 l_5 l_6 = G_1 G_2, \]
\[ F_2 = l_1 G_1 l_2 l_3 l_10 l_11 G_3 l_3 l_5 l_6 = G_1 G_3. \]

The loops are:

\[ L_1 = G_1 l_7 (-l_8) = -G_1, \]
\[ L_2 = G_1 l_2 l_3 l_10 (-H_2) = -G_1 H_2, \]
\[ L_3 = l_3 G_2 (-l_9) = -G_2, \]
\[ L_4 = G_1 l_2 l_3 G_2 l_4 l_5 H_1 (-l_8) = -G_1 G_2 H_1, \]
\[ L_5 = G_1 l_2 l_3 l_10 l_11 G_3 l_3 l_5 H_1 (-l_8) = -G_1 G_3 H_1, \]
\[ L_6 = G_3 (-l_12) = -G_3. \]

Non-touching loops are:

Two Loops
- \( L_1 \& L_3 \)
- \( L_1 \& L_6 \)
- \( L_2 \& L_5 \)
- \( L_3 \& L_6 \)
- \( L_4 \& L_5 \)

Three Loops
- \( L_1 \& L_3 \& L_6 \)

The loops on the forward paths are such that:

\[ F_1 \text{ touches } L_1, L_2, L_3, L_4, L_5, \]
\[ F_2 \text{ touches } L_1, L_2, L_3, L_4, L_5, L_6. \]
As a result,

\[
\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6) \\
+ (L_1L_3 + L_1L_6 + L_2L_6 + L_3L_6 + L_4L_6) \\
- (L_1L_3L_6) \\
= 1 + G_1 + G_1H_2 + G_2 + G_1G_2H_1 + G_1G_3H_1 + G_3 \\
+ G_1G_2 + G_1G_3 + G_1G_3H_2 + G_2G_3 + G_1G_2G_3H_1 \\
+ G_1G_2G_3,
\]

and

\[
\Delta_1 = \Delta_{|L_2=\ldots=L_6=0} = 1 + G_3, \\
\Delta_2 = \Delta_{|L_1=\ldots=L_6=0} = 1.
\]

Therefore,

\[
\frac{Y(s)}{R(s)} = F(s) = \frac{1}{\Delta} (F_1\Delta_1 + F_2\Delta_2) \\
= \frac{G_1G_2(1 + G_3) + G_1G_3}{\Delta}.
\]