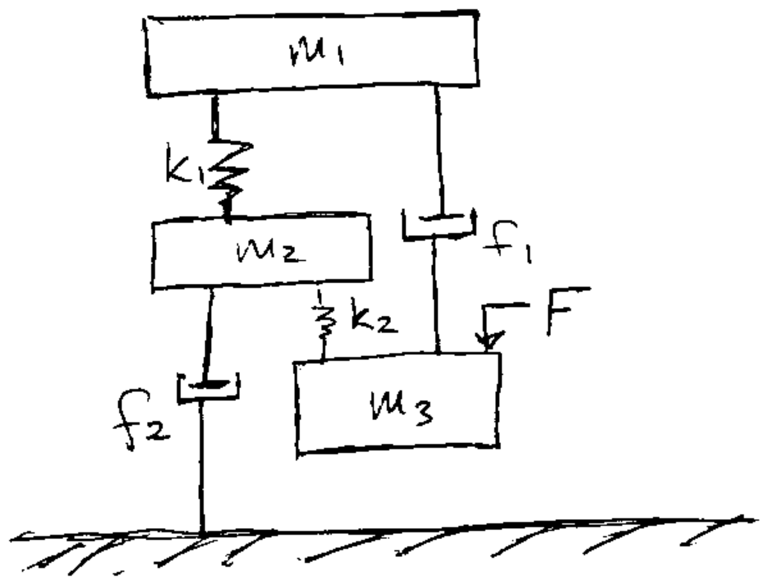
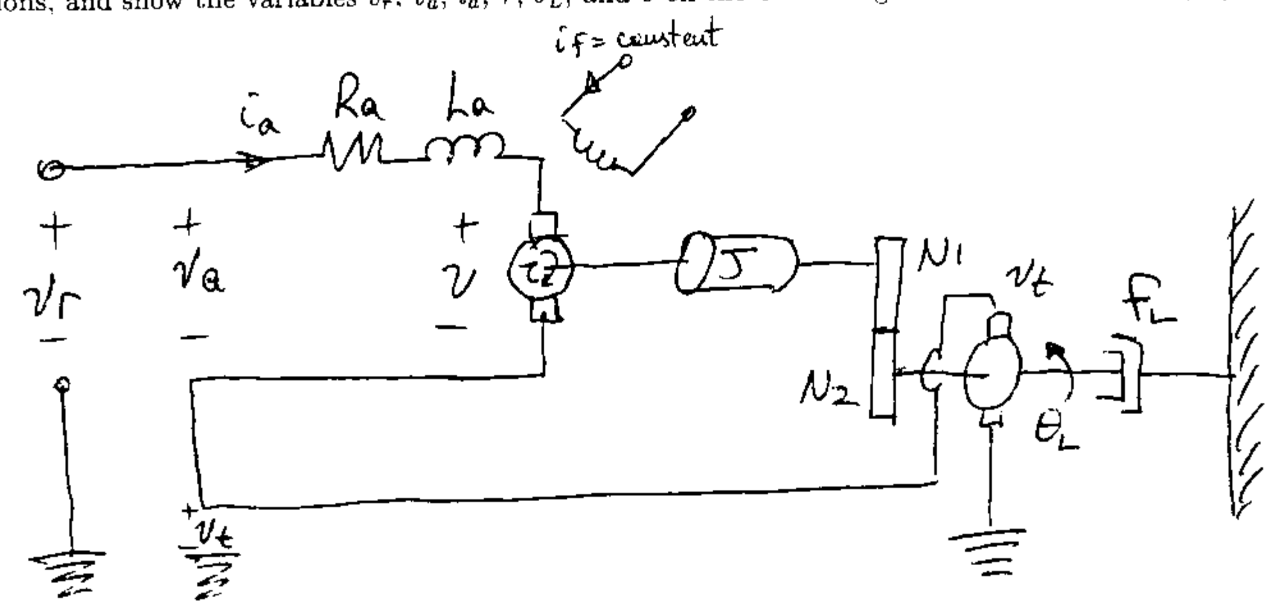
EE 231 | Exam#1 | Oct. 18, 1995
50 minutes

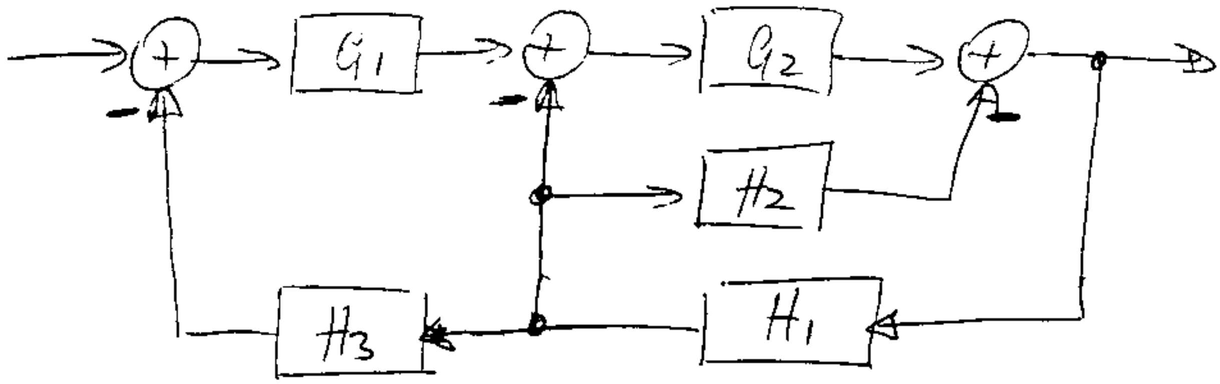
1. For the mechanical system shown below, obtain *either* the force-voltage *or* the force-current analog of the system. (25pts)



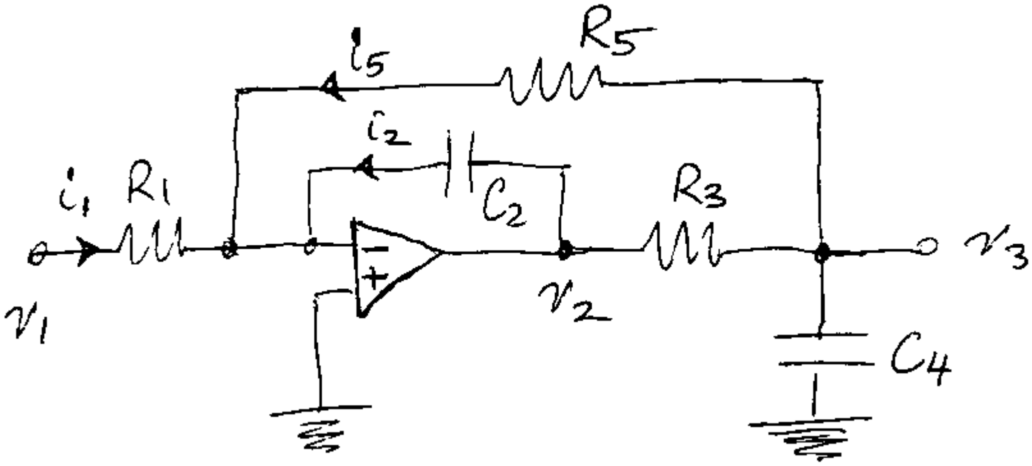
2. In the following system, the output speed of a motor is detected by a tachometer that generates a voltage proportional to the angular speed, such that $v_t = K_t\dot{\theta}_L$. Assuming that the input and the output are v_r and $\dot{\theta}_L$, respectively; obtain a detailed block diagram of the system without reducing or combining the equations, and show the variables v_r, v_a, i_a, τ, $\dot{\theta}_L$, and v on the block diagram. (25pts)



3. For the block diagram given below, determine the transfer function *either* by block diagram reduction, *or* by Mason's formula. Show your work clearly. (25pts)



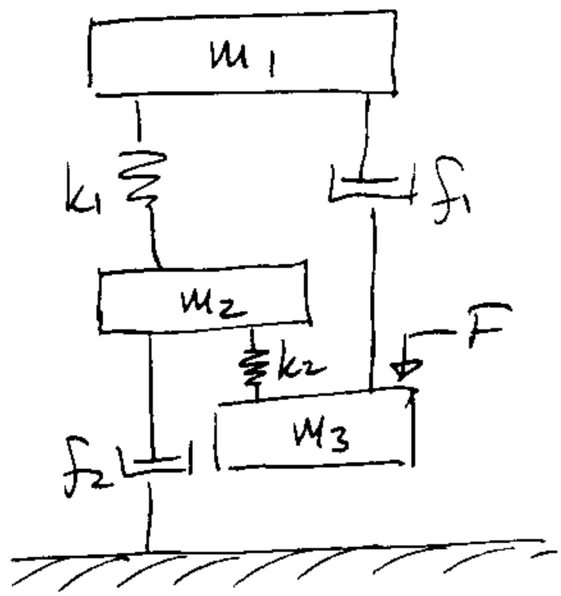
4. A feedback controller is to be designed for the following circuit.



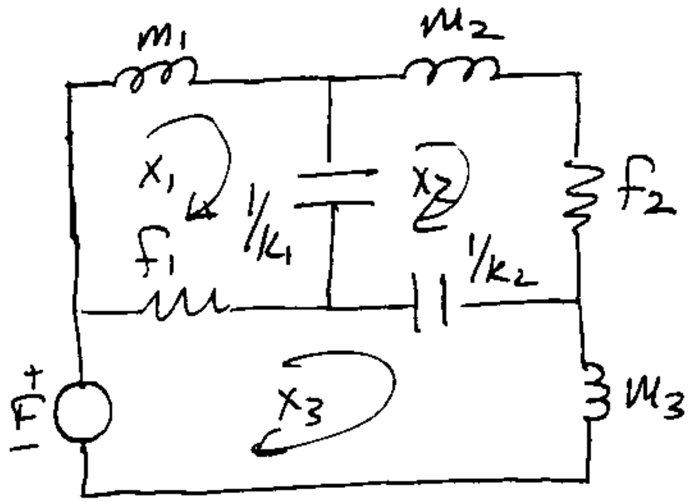
(a) First, determine its block diagram, such that the variables v_1, i_1, i_2, v_2, v_3, and i_5 are clearly shown. Then, obtain its transfer function from the block diagram. (25pts)

(b) Assuming that $R_1 = 100\,\mathrm{k\Omega}$, $R_3 = 10\,\mathrm{k\Omega}$, and $C_4 = 50\,\mu\mathrm{F}$, design for C_2 and R_5, such that the percent maximum overshoot, $M_p \approx 9.5\%$, and the 2% settling time, $t_{2\%s} \approx 2/3\,\mathrm{s}$. (+25pts)

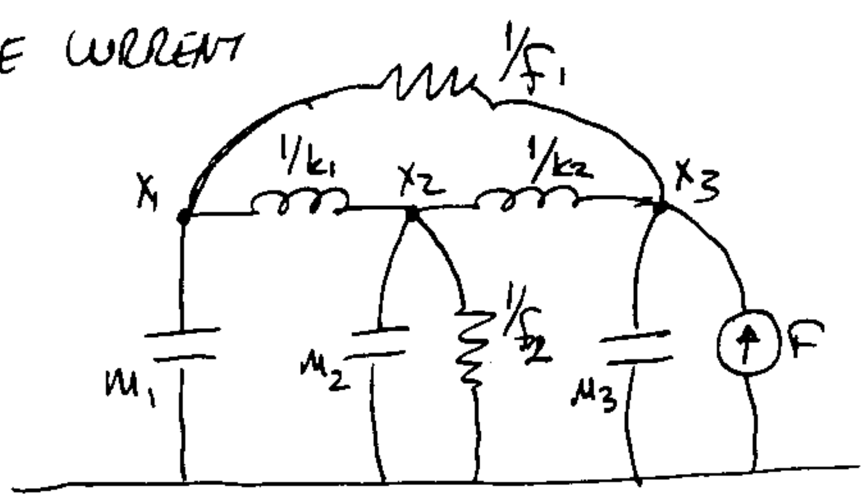
#1



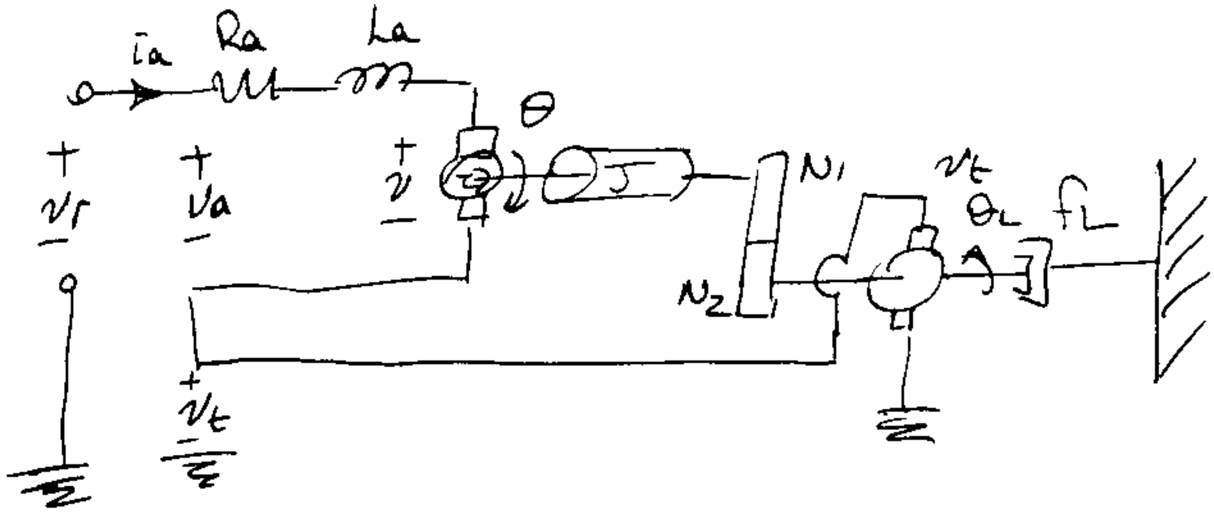
FORCE VOLTAGE



FORCE CURRENT



#2



$v_r = v_a + v_t$, $v_a = v_r - v_t$ $\qquad v_r \to \oplus \xrightarrow{v_a}$, $v_t \uparrow$

$$i_a = \frac{v_a - v}{R_a + sL_a} \qquad v_a \to \oplus \,(-v) \to \boxed{\frac{1}{R_a + sL_a}} \xrightarrow{i_a}$$

$$\tau = K_m i_a \qquad \xrightarrow{i_a} \boxed{K_m} \xrightarrow{\tau}$$

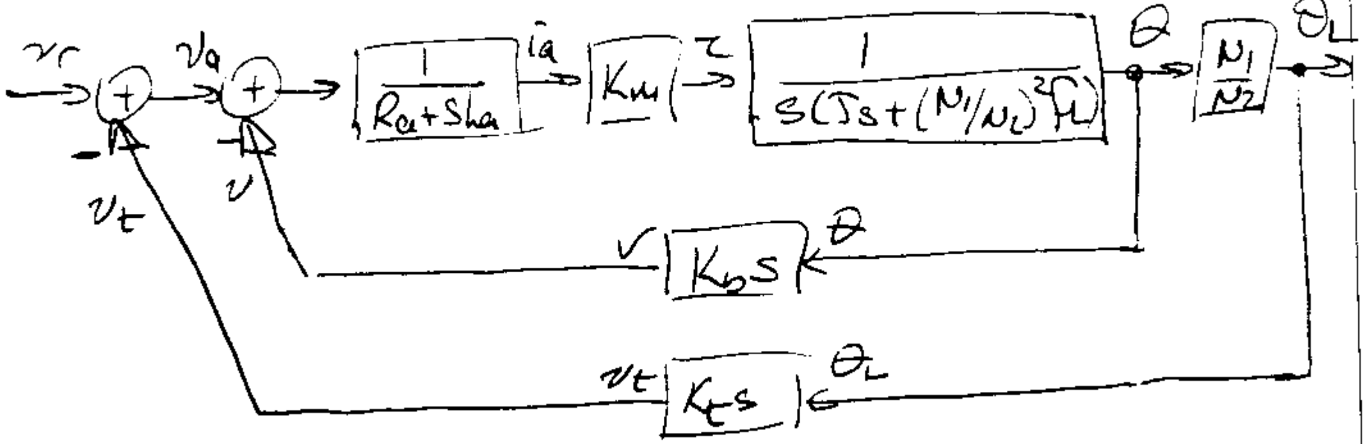
$$J\ddot{\theta} = \tau - \left(\frac{N_1}{N_2}\right)^2 f_L \dot{\theta} \qquad s\left(Js + \left(\frac{N_1}{N_2}\right)^2 f_L\right)\Theta = \tau$$

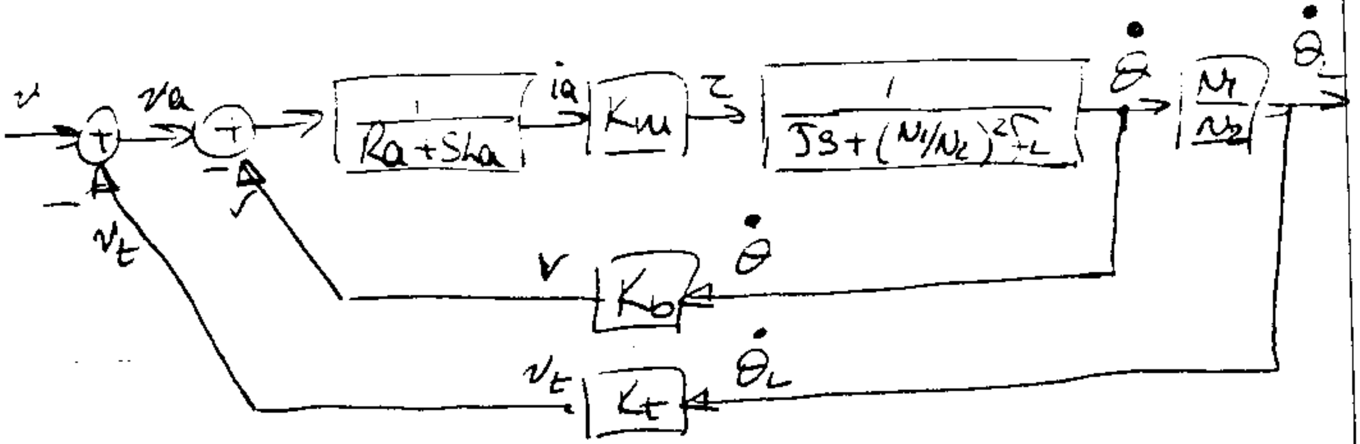
$$\Theta = \frac{1}{s\left(Js + \left(\frac{N_1}{N_2}\right)^2 f_L\right)}\tau \qquad \xrightarrow{\tau} \boxed{\frac{1}{s\left(Js + \left(\frac{N_1}{N_2}\right)^2 f_L\right)}} \xrightarrow{\Theta}$$

$$\theta_L = \frac{N_1}{N_2}\theta \qquad \xrightarrow{\theta} \boxed{\frac{N_1}{N_2}} \xrightarrow{\theta_L}$$

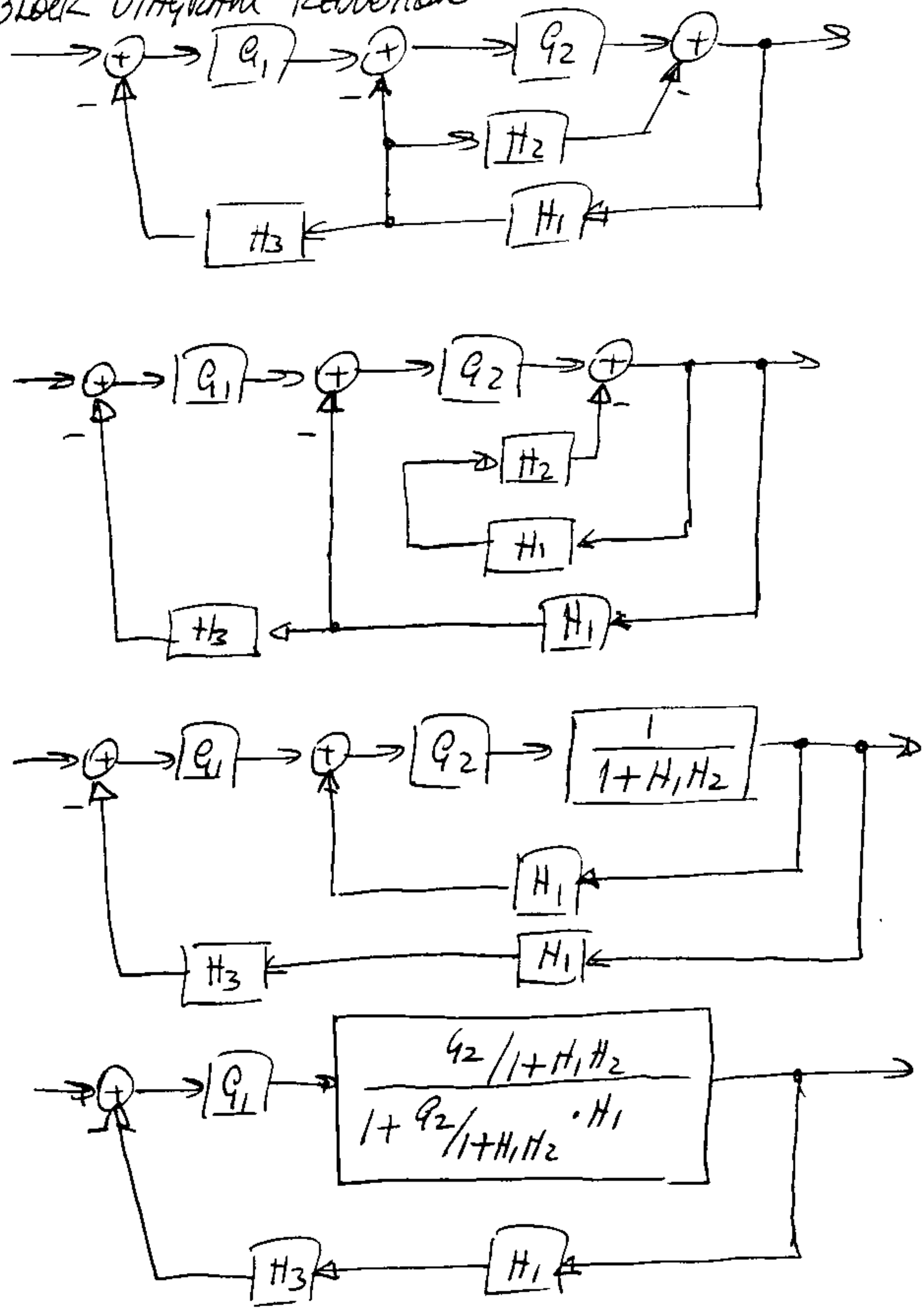
$$v_t = K_t \dot{\theta} \qquad \xrightarrow{\theta} \boxed{sK_t} \xrightarrow{v_t}$$

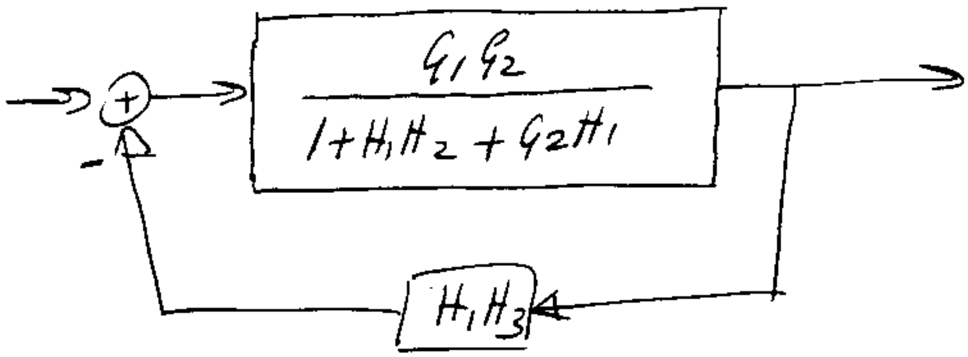
also $v = K_b \dot{\theta}$ $\qquad \xrightarrow{\theta} \boxed{sK_b} \xrightarrow{v}$


v_r
v_a
$\frac{1}{R_a+sL_a}$
i_a
K_m
τ
$\frac{1}{s(Js+(N_1/N_2)^2f_L)}$
θ
$\frac{N_1}{N_2}$
θ_L
v_t
v
$K_b s$
θ
v_t
$K_t s$
θ_L


v
v_a
$\frac{1}{R_a+sL_a}$
i_a
K_m
τ
$\frac{1}{Js+(N_1/N_2)^2f_L}$
$\dot{\theta}$
$\frac{N_1}{N_2}$
$\dot{\theta}_L$
v_t
v
K_b
$\dot{\theta}$
v_t
K_t
$\dot{\theta}_L$

#3 BLOCK DIAGRAM REDUCTION

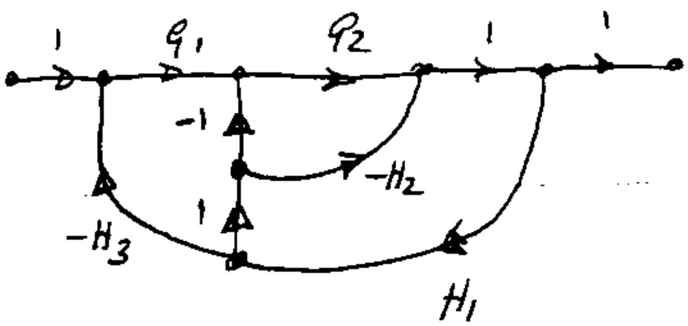




$$\frac{G_1G_2/1+H_1H_2+G_2H_1}{1+\left(G_1G_2/1+H_1H_2+G_2H_1\right)(H_1H_3)}$$

$$\frac{G_1G_2}{1+H_1H_2+G_2H_1+G_1G_2H_1H_3}$$

SIGNAL FLOW DIAGRAM



<u>FORWARD PATHS</u>

$F_1 = G_1G_2$

<u>LOOPS</u>

$L_1 = -G_1G_2H_1H_3$

$L_2 = -G_2H_1$

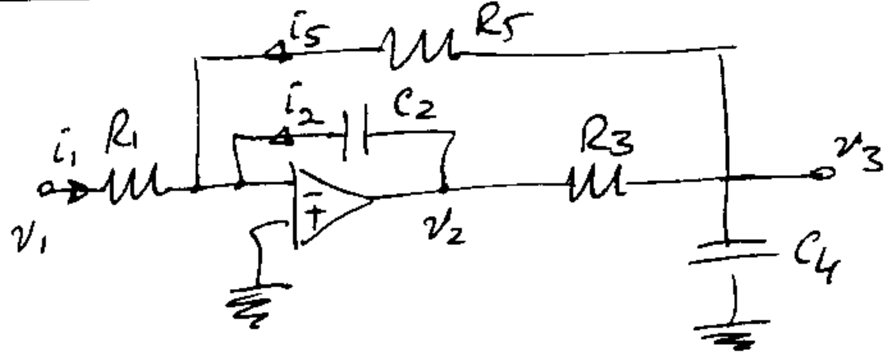
$L_3 = -H_1H_2$

All the loops are touching and they all are on F_1

$$\Delta = 1-(L_1+L_2+L_3) = 1+G_1G_2H_1H_3+G_2H_1+H_1H_2$$

$$\Delta_1 = \Delta\big|_{L_1=L_2=L_3=0} = 1 \implies P = \frac{G_1G_2}{1+G_1G_2H_1H_3+G_2H_1+H_1H_2}$$

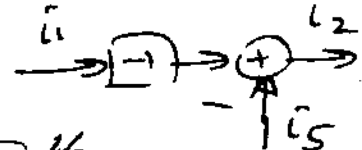
#4



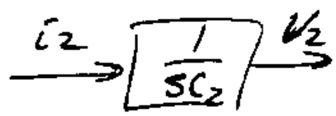
a)

$$i_1 = \frac{1}{R_1} v_1$$

$$i_1 + i_2 + i_5 = 0, \quad i_2 = -i_1 - i_5$$



$$v_2 = \frac{1}{sC_2} i_2$$

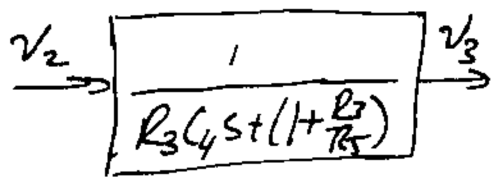


$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{1/sC_4} + \frac{v_3}{R_5} = 0$$

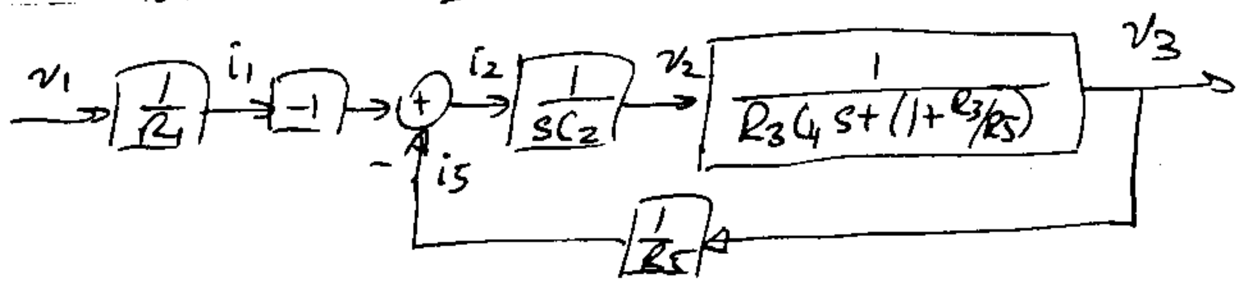
$$\left(\frac{1}{R_3} + sC_4 + \frac{1}{R_5}\right) v_3 = \frac{1}{R_3} v_2$$

$$\left(R_3 C_4 s + \left(1 + \frac{R_3}{R_5}\right)\right) V_3 = V_2$$

$$V_3 = \frac{1}{R_3 C_4 s + \left(1 + \frac{R_3}{R_5}\right)} V_2$$



$$i_5 = \frac{1}{R_5} v_3$$



$$\frac{v_3}{v_1} = \frac{\left(-\frac{1}{R_1}\right)\frac{1}{sC_2}\frac{1}{R_3C_4s+(1+R_3/R_5)}}{1+\frac{1}{sC_2}\frac{1}{R_3C_4s+(1+R_3/R_5)}\cdot\frac{1}{R_5}}$$

$$= -\frac{1/R_1 \cdot R_5}{sC_2\left(R_3C_4s+(1+R_3/R_5)\right)R_5+1}$$

$$= -\frac{R_5/R_1}{R_5C_2R_3C_4s^2+R_5C_2(1+R_3/R_5)s+1}$$

$$= -\frac{R_5/R_1\cdot 1/R_5C_2R_3C_4}{s^2+\frac{(1+R_3/R_5)}{R_3C_4}s+\frac{1}{R_5C_2R_3C_4}}$$

b₁₁ $M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} \Longrightarrow \zeta = \frac{|\ln M_p|}{\sqrt{\pi^2+\ln^2 M_p}}$

for a second order system with no zeros

so for $M_p \approx 9.5\%$ or $M_p \approx 0.095$, $\zeta = \frac{|\ln 0.095|}{\sqrt{\pi^2+\ln^2 0.095}} \approx 0.6$

$t_{2\%s} \approx \frac{4}{\zeta\omega_n}$, so for $t_{2\%s} \approx \frac{2}{3}$, $\zeta\omega_n = 6$

or $\omega_n = \frac{6}{0.6} = 10$

In a second-order system with no zeroes, we have

$$\frac{v_3}{v_1} = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

comparing this expression with the transfer funct from part a, we conclude that

$$2\zeta\omega_n = \frac{(1 + R_3/R_5)}{R_3 C_4} \Rightarrow 2(0.6)(10) = \frac{1 + R_3/R_5}{R_3 C_4}$$

and $$\omega_n^2 = \frac{1}{R_5 C_2 R_3 C_4} \Rightarrow (10)^2 = \frac{1}{R_5 C_2 R_3 C_4}$$

For $R_1 = 100k\Omega$, $R_3 = 10k\Omega$, $C_4 = 50\mu F$

$$\frac{1 + \frac{10k}{R_5}}{10k \times 50\mu} = 12 \Rightarrow \boxed{R_5 = 2k\Omega}$$

$$\frac{1}{2k \times C_2 \times 10k \times 50\mu} = 100 \Rightarrow C_2 = 10\mu F$$