1. Determine the block diagram of the following system, such that the variables \( v_1, i_1, v_2, i_2, i_3, v_3 \), and \( v_4 \) are clearly shown. (25pts)

![Block Diagram 1]

2. For the block diagram given below, determine the transfer function either by block diagram reduction or by Mason’s formula. Show your work clearly. (25pts)

![Block Diagram 2]

3. The block diagram of a control system is given below.

![Block Diagram 3]

Obtain a state-space representation of the system without any block-diagram reduction. (25pts)
4. Consider the following control system.

Design the controller gains $K_1$ and $K_2$, such that the approximate maximum percent overshoot is $M_o \approx 9.5\%$, and the approximate rise time is $t_r \approx 0.7\ s$. (25pts)
1. Determine the block diagram of the following system, such that the variables $v_1$, $i_1$, $v_2$, $i_2$, $i_3$, $v_3$, and $v_4$ are clearly shown.

**Solution:** To determine the block diagram of the system, we first need to separate it into simpler components.

Since the input variable is $v_1$, we find another variable in terms of $v_1$, i.e.

$$i_1 = \frac{1}{R_1} v_1,$$

for an ideal operational amplifier.

Similarly,

$$i_2 = \frac{1}{R_2} v_2.$$
From the node equation, we get
\[ i_1 + i_2 + i_3 = 0, \]
or
\[ i_3 = (-1)(i_1 + i_2). \]

Again for an ideal operational amplifier,
\[ v_3 = R_3 i_3. \]

Finally, for the non-inverting operational amplifier,
\[ v_2 = 1 + \frac{R_4}{1/(sC_4)} = 1 + R_4 C_4 s \]
When we connect all the individual blocks together, we get the following block diagram.

2. For the block diagram given below, determine the transfer function either by block diagram reduction, or by Mason's formula. Show your work clearly.

Solution: If we choose to use the block diagram reduction, best approach is to reduce the block diagram step by step, until we obtain the transfer function.
If we choose to use Mason's formula, we need to draw the signal flow graph of the block diagram.

In drawing the signal flow graph, the unity gains are subscribed for easy tracking of the gain expressions. The forward path gains are

\[ F_1 = l_1 l_2 G_1 l_3 l_4 l_5 G_2 l_6 l_7 l_8 = G_1 G_2, \]
\[ F_2 = l_1 l_2 l_9 l_4 l_5 G_2 l_6 l_7 l_8 = G_2, \]

and

\[ F_3 = l_1 l_2 G_1 l_4 l_7 l_8 = G_1. \]
The loop gains are

\[ L_1 = l_2G_1l_3l_4l_5l_2(-1) = -G_1, \]
\[ L_2 = l_2l_1l_4l_5l_1(-1) = -1, \]
\[ L_3 = l_2G_1l_3l_1l_5G_2l_1l_1l_3(-1) = -G_1G_2, \]
\[ L_4 = l_2l_1l_4l_5G_2l_1l_1l_3(-1) = -G_2, \]
\[ L_5 = l_2G_1l_3l_1l_5(-1) = -G_1, \]

and

\[ L_6 = l_5G_2l_1 = G_2. \]

From the forward path and loop gains, we determine the touching loops and the forward paths.

<table>
<thead>
<tr>
<th>Touching Loops</th>
<th>Loops on Forward Paths</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_1 )</td>
<td>( L_2 )</td>
</tr>
<tr>
<td>( \checkmark )</td>
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</tbody>
</table>

Therefore,

\[ \Delta = 1 - (L_1 + \ldots + L_6) + L_5L_6 \]
\[ = 1 - (-G_1 - 1 - G_1G_2 - G_2 - G_1 + G_2) + (-G_1)G_2 \]
\[ = 2(1 + G_1), \]

and

\[ \Delta_1 = \Delta|_{L_1=\ldots=L_6=0} = 1, \]
\[ \Delta_2 = \Delta|_{L_1=\ldots=L_6=0} = 1, \]
\[ \Delta_3 = \Delta|_{L_1=\ldots=L_6=0} = 1 - L_6 = 1 - G_1. \]

So,

\[ \frac{Y(s)}{U(s)} = \frac{1}{\Delta} \sum_{i=1}^{3} F_i \Delta_i = \frac{G_1G_2(1) + G_2(1) + G_1(1 - G_2)}{2(1 + G_1)}. \]
or

\[
\frac{Y(s)}{U(s)} = \frac{G_1 + G_2}{2(1 + G_1)}.
\]

3. The block diagram of a control system is given below.

Obtain a state-space representation of the system without any block-diagram reduction.

**Solution:** In order to obtain a state-space representation without any block-diagram reduction or without determining the closed-loop transfer function, we need to realize the individual blocks and use the complete block diagram to generate the state-space equations.

(a) The feedforward gain block.

(b) Controller realization form.

(c) The feedback gain block.

(d) Controller realization form.

The connected and “expanded” block diagram is shown below.
After assigning the state variables as shown in the figure, we obtain

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -2x_1 - x_2 + (r - y_2), \\
\dot{x}_3 &= -x_3 + y_1,
\end{align*}
\]

and

\[ y = y_1, \]

where

\[
\begin{align*}
y_1 &= 2x_1, \\
y_2 &= x_3 + y_1.
\end{align*}
\]

After eliminating the intermediate variables: \(y_1\) and \(y_2\), we obtain the state-space representation

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t) \\
\dot{x}_3(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-4 & -1 & 1 \\
2 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} r(t),
\]

\[
y(t) = \begin{bmatrix}
2 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix}.
\]

If we use the observability realization form for each of the blocks, then we obtain a different state-space representation.

(a) The feedforward gain block.

(b) Observer realization form.
(c) The feedback gain block.

(d) Observer realization form.

The connected and “expanded” block diagram for this case is shown below.

Similarly, we obtain

\[ \dot{x}_1 = -y_1 + x_2, \]
\[ \dot{x}_2 = -2y_1 + 2(r - y_2), \]
\[ \dot{x}_3 = -y_2, \]

and

\[ y = y_1, \]

where

\[ y_1 = x_1, \]
\[ y_2 = x_3 + y_1. \]
And,
\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t) \\
\dot{x}_3(t)
\end{bmatrix} =
\begin{bmatrix}
-1 & 1 & 0 \\
-4 & 0 & -2 \\
-1 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
2 \\
0
\end{bmatrix} r(t),
\]
\[
y(t) =
\begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix}.
\]

1. Consider the following control system.

Design the controller gains $K_1$ and $K_2$, such that the approximate maximum percent overshoot is $M_p \approx 9.5\%$, and the approximate rise time is $t_r \approx 0.7s$.

**Solution:** The closed-loop transfer function of the system is
\[
\frac{Y(s)}{R(s)} = \frac{1/(s+1)}{1 + (1/(s+1))(K_1 + K_2/s)}
\]
\[
= \frac{s}{s^2 + (1 + K_1)s + K_2}.
\]

From the general expression of the second-order system poles $s^2 + 2\zeta\omega_n s + \omega^2$, we get
\[
1 + K_1 = 2\zeta\omega_n, \quad (1)
\]
and
\[
K_2 = \omega_n^2, \quad (2)
\]
by comparison. We determine the system parameters: $\zeta$ and $\omega$ from the system requirements.

<table>
<thead>
<tr>
<th>Given Requirements</th>
<th>General System Restrictions</th>
<th>Specific System Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum percent overshoot for a unit step input</td>
<td>$M_p \approx 0.095$.</td>
<td>For a second-order system with no zero, $\frac{\sqrt{1 - \zeta^2}}{\zeta} \approx 0.095$.</td>
</tr>
<tr>
<td>Rise time for a unit step input</td>
<td>$t_r \approx 0.7s$.</td>
<td>For a second-order system with no zero, $\frac{\pi - \cos^{-1}(\zeta)}{\omega_d} \approx 0.7$.</td>
</tr>
</tbody>
</table>
From the maximum overshoot requirement, we obtain
\[ \zeta = \frac{|\ln(M_p)|}{\sqrt{(\ln(M_p))^2 + \pi^2}} \approx 0.6. \]

And, from the rise time requirement, we obtain
\[ \omega_d = \frac{\pi - \cos^{-1}(\zeta)}{t_r} \approx 3.1633. \]

or
\[ \omega_n = \frac{\omega_d}{\sqrt{1 + \zeta^2}} \approx 3.95. \]

Therefore, from Equations (1) and (2), we determine
\[ K_1 \approx 3.74, \]
\[ K_2 \approx 15.63. \]

Note here that these are first approximations, since the closed-loop system has also a zero that alters the expressions for the specifications.