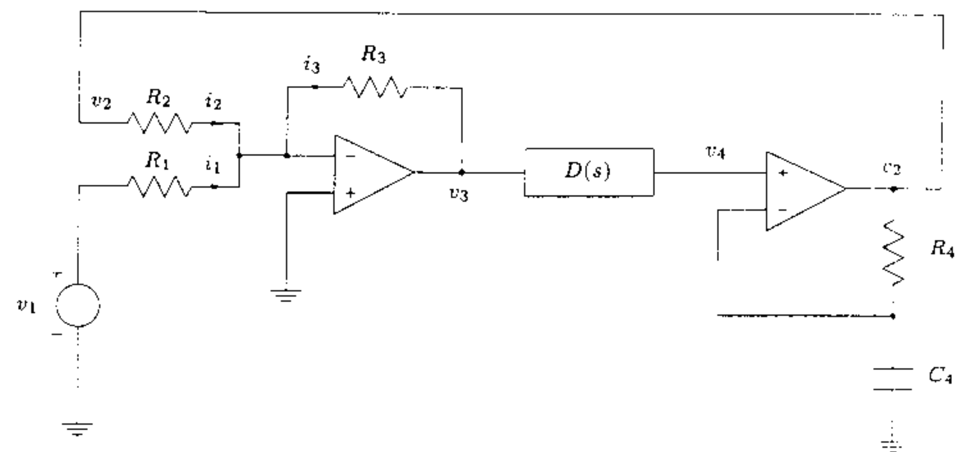
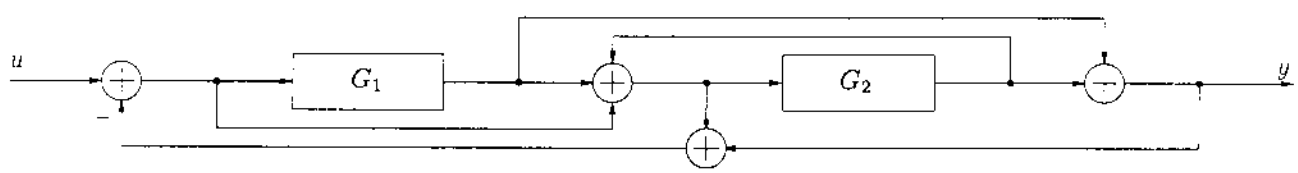
EE 231 Exam#1 Mar. 08, 1999

50 minutes

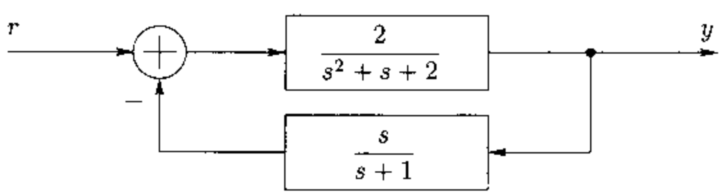
1. Determine the block diagram of the following system, such that the variables v_1, i_1, v_2, i_2, i_3, v_3, and v_4 are clearly shown. (25pts)



2. For the block diagram given below, determine the transfer function *either* by block diagram reduction, *or* by Mason's formula. Show your work clearly. (25pts)

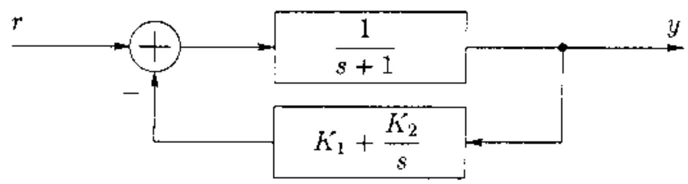


3. The block diagram of a control system is given below.



Obtain a state-space representation of the system without any block-diagram reduction. (25pts)

4. Consider the following control system.

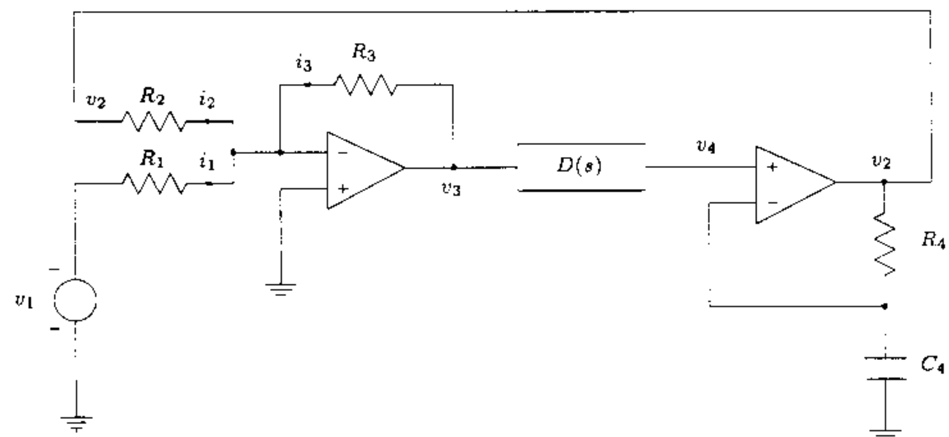


Design the controller gains K_1 and K_2, such that the approximate maximum percent overshoot is $M_p \approx 9.5\%$, and the approximate rise time is $t_r \approx 0.7\,\text{s}$. (25pts)

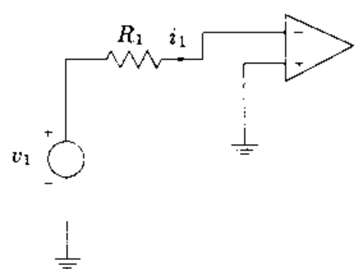
EE 231 **Exam#1** **Mar. 08, 1999**

Solutions

1. Determine the block diagram of the following system, such that the variables v_1, i_1, v_2, i_2, i_3, v_3, and v_4 are clearly shown.



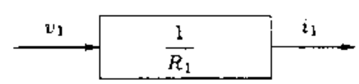
Solution: To determine the block diagram of the system, we first need to separate it into simpler components.

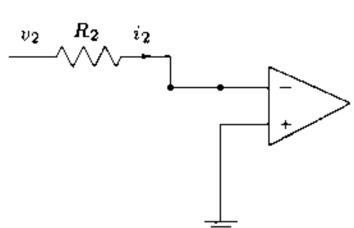


Since the input variable is v_1, we find another variable in terms of v_1, i.e.

$$i_1 = \frac{1}{R_1} v_1,$$

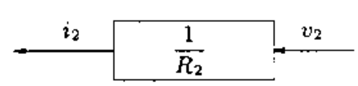
for an ideal operational amplifier.

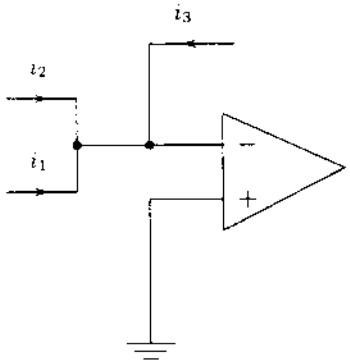




Similarly,

$$i_2 = \frac{1}{R_2} v_2.$$



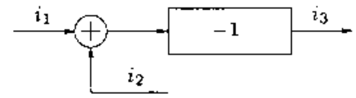


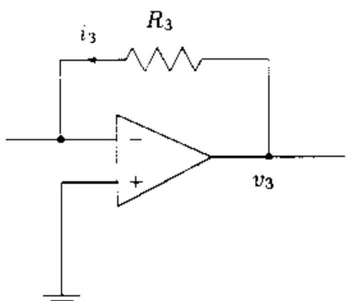
From the node equation, we get

$$i_1 + i_2 + i_3 = 0,$$

or

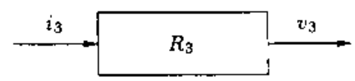
$$i_3 = (-1)(i_1 + i_2).$$

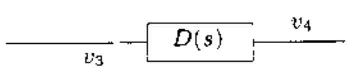




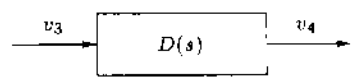
Again for an ideal operational amplifier,

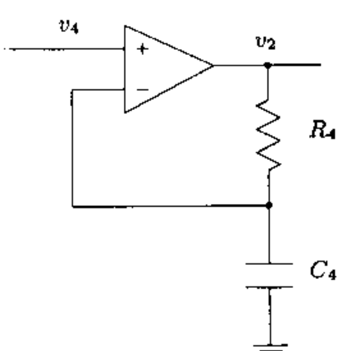
$$v_3 = R_3 i_3.$$





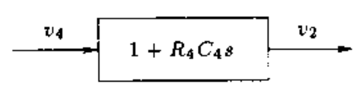
$$v_4 = D(s)v_3.$$



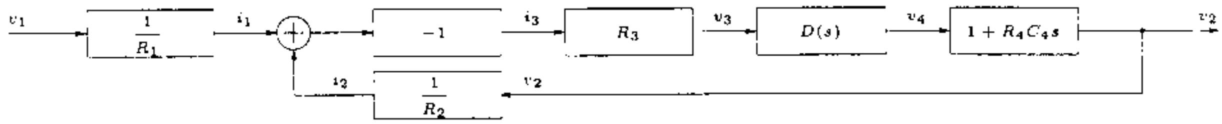


Finally, for the non-inverting operational amplifier,

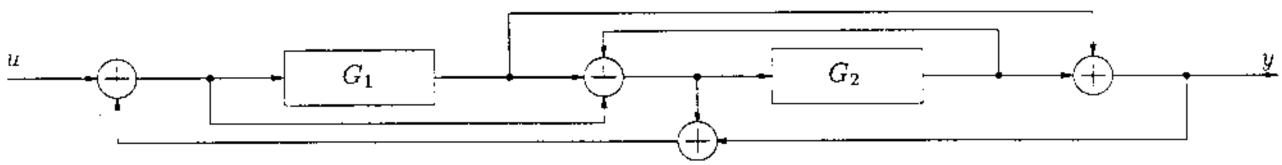
$$v_2 = 1 + \frac{R_4}{1/(sC_4)}$$
$$= 1 + R_4 C_4 s$$



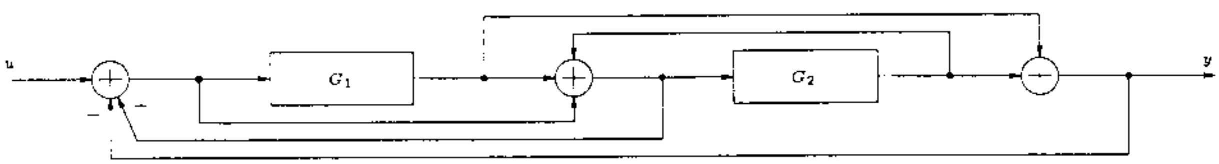
When we connect all the individual blocks together, we get the following block diagram.

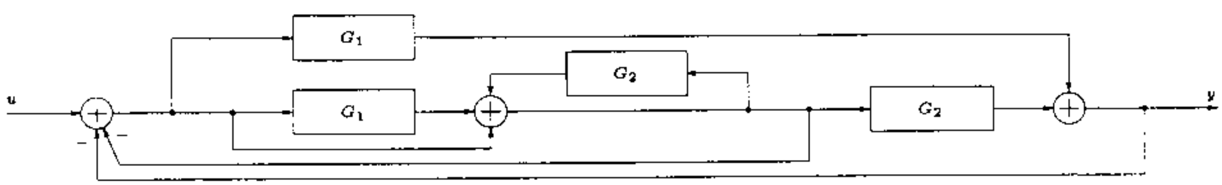


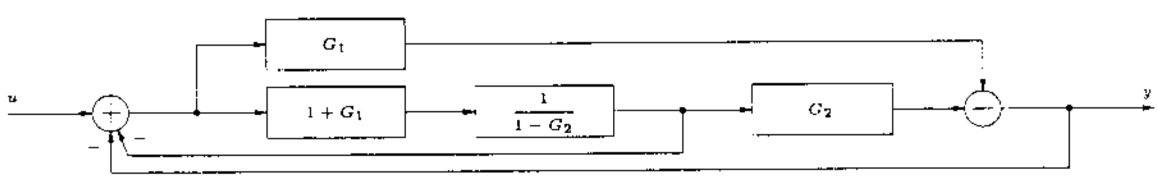
2. For the block diagram given below, determine the transfer function *either* by block diagram reduction, *or* by Mason's formula. Show your work clearly.

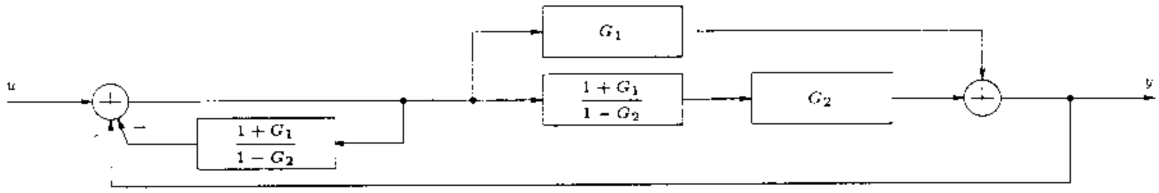


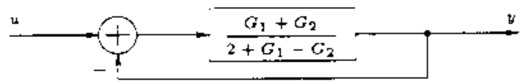
Solution: If we choose to use the block diagram reduction, best approach is to reduce the block diagram step by step, until we obtain the transfer function.

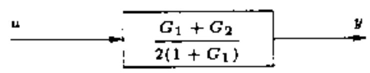




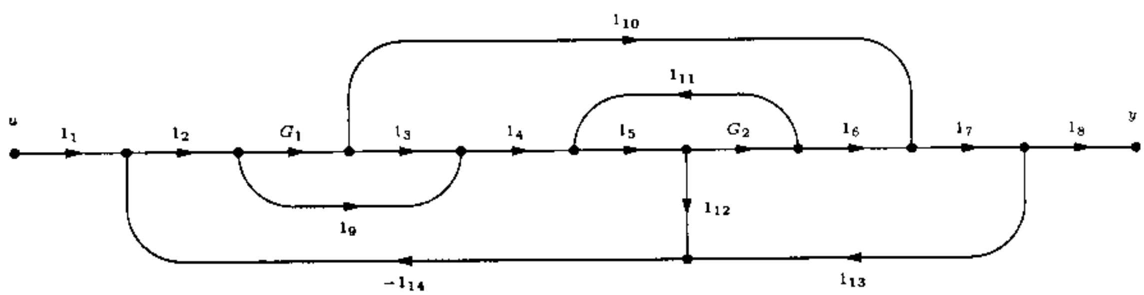








If we choose to use Mason's formula, we need to draw the signal flow graph of the block diagram.



In drawing the signal flow graph, the unity gains are subscribed for easy tracking of the gain expressions. The forward path gains are

$$F_1 = 1_1 1_2 G_1 1_3 1_4 1_5 G_2 1_6 1_7 1_8 = G_1 G_2,$$

$$F_2 = 1_1 1_2 1_9 1_4 1_5 G_2 1_6 1_7 1_8 = G_2,$$

and

$$F_3 = 1_1 1_2 G_1 1_{10} 1_7 1_8 = G_1.$$

The loop gains are

$$
\begin{aligned}
L_1 &= 1_2 G_1 1_3 1_4 1_5 1_{12}(-1_{14}) = -G_1,\\
L_2 &= 1_2 1_9 1_4 1_5 1_{12}(-1_{14}) = -1,\\
L_3 &= 1_2 G_1 1_3 1_4 1_5 G_2 1_6 1_7 1_{13}(-1_{14}) = -G_1 G_2,\\
L_4 &= 1_2 1_9 1_4 1_5 G_2 1_6 1_7 1_{13}(-1_{14}) = -G_2,\\
L_5 &= 1_2 G_1 1_{10} 1_7 1_{13}(-1_{14}) = -G_1,
\end{aligned}
$$

and

$$L_6 = 1_5 G_2 1_{11} = G_2.$$

From the forward path and loop gains, we determine the touching loops and the forward paths.

Touching Loops

	L_1	L_2	L_3	L_4	L_5	L_6
L_1	✔	✔	✔	✔	✔	✔
L_2		✔	✔	✔	✔	✔
L_3			✔	✔	✔	✔
L_4				✔	✔	✔
L_5					✔	✘
L_6						✔

Loops on Forward Paths

	L_1	L_2	L_3	L_4	L_5	L_6
F_1	✔	✔	✔	✔	✔	✔
F_2	✔	✔	✔	✔	✔	✔
F_3	✔	✔	✔	✔	✔	✘

Therefore,

$$
\begin{aligned}
\Delta &= 1 - (L_1 + \ldots + L_6) + L_5 L_6\\
&= 1 - (-G_1 - 1 - G_1 G_2 - G_2 - G_1 + G_2) + (-G_1)G_2\\
&= 2(1 + G_1),
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta_1 &= \Delta|_{L_1=\cdots=L_6=0} = 1,\\
\Delta_2 &= \Delta|_{L_1=\cdots=L_6=0} = 1,\\
\Delta_3 &= \Delta|_{L_1=\cdots=L_5=0} = 1 - L_6 = 1 - G_1.
\end{aligned}
$$

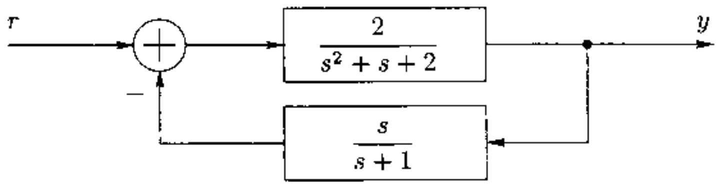
So,

$$\frac{Y(s)}{U(s)} = \frac{1}{\Delta}\sum_{i=1}^{3} F_i \Delta_i = \frac{G_1 G_2(1) + G_2(1) + G_1(1 - G_2)}{2(1 + G_1)},$$

or

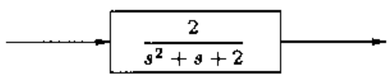
$$\frac{Y(s)}{U(s)} = \frac{G_1 + G_2}{2(1+G_1)}.$$

3. The block diagram of a control system is given below.

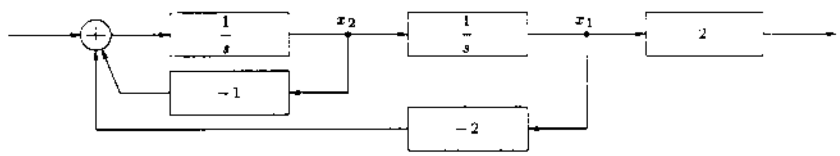


Obtain a state-space representation of the system without any block-diagram reduction.

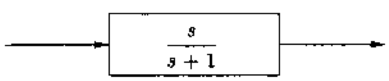
Solution: In order to obtain a state-space representation without any block-diagram reduction or without determining the closed-loop transfer function, we need to realize the individual blocks and use the complete block diagram to generate the state-space equations.



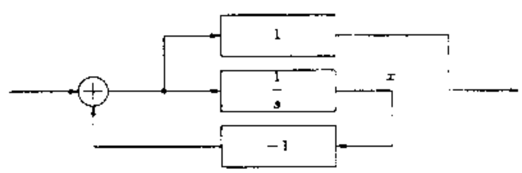
(a) The feedforward gain block.



(b) Controller realization form.

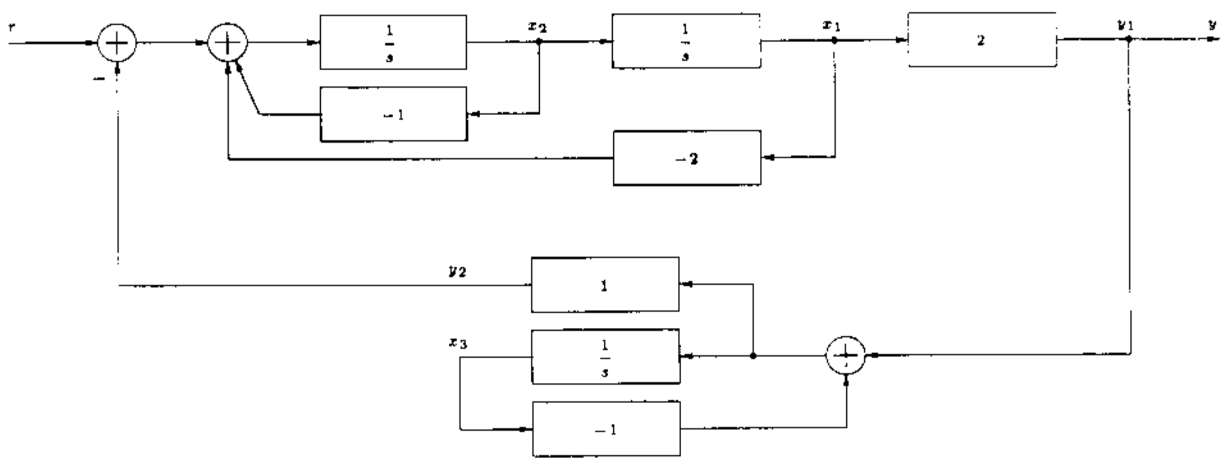


(c) The feedback gain block.



(d) Controller realization form.

The connected and "expanded" block diagram is shown below.



After assigning the state variables as shown in the figure, we obtain

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -2x_1 - x_2 + (r - y_2),$$
$$\dot{x}_3 = -x_3 + y_1,$$

and

$$y = y_1,$$

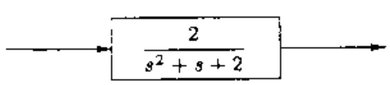
where

$$y_1 = 2x_1,$$
$$y_2 = -x_3 + y_1.$$

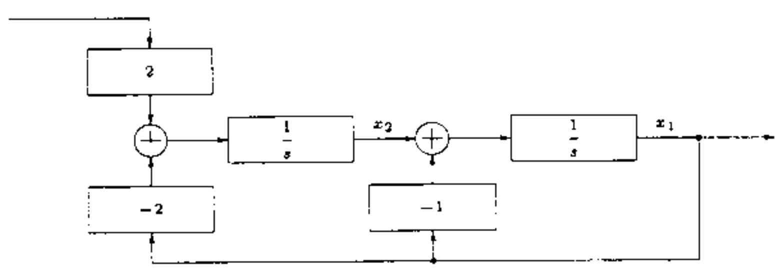
After eliminating the intermediate variables: y_1 and y_2, we obtain the state-space representation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} r(t),$$
$$y(t) = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

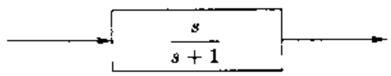
If we use the observability realization form for each of the blocks, then we obtain a different state-space representation.



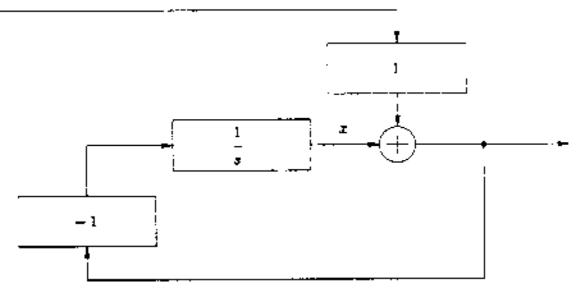
(a) The feedforward gain block.



(b) Observer realization form.

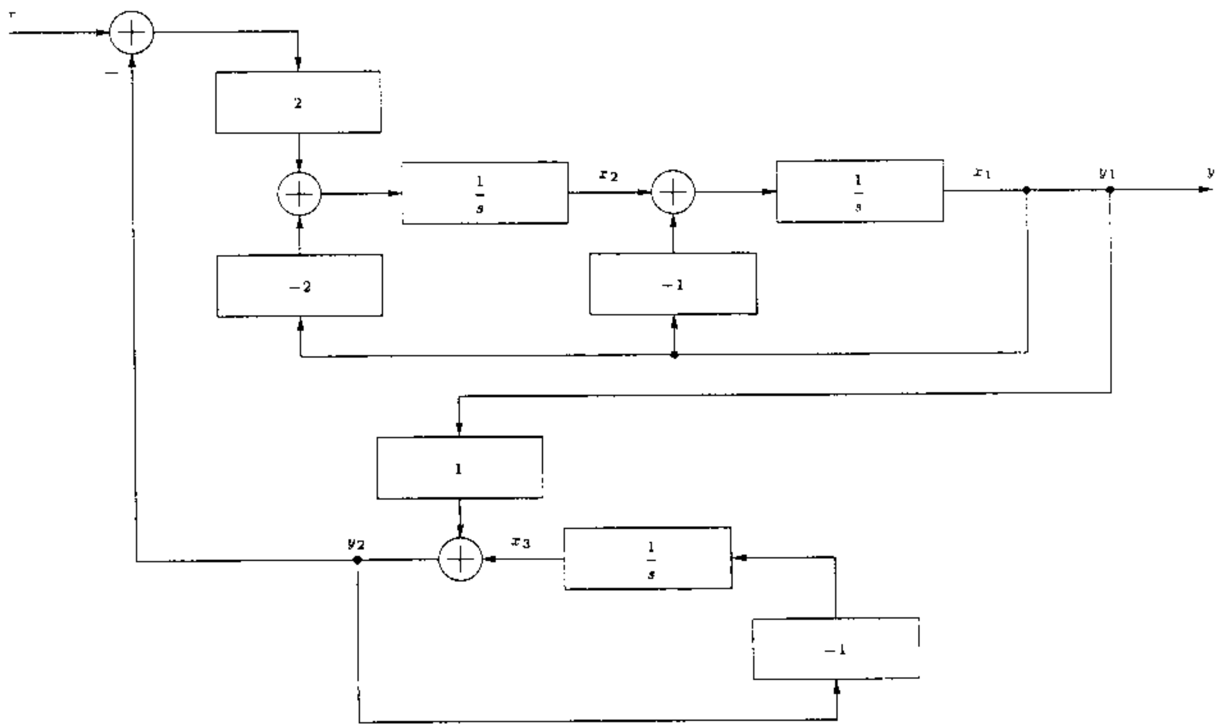


(c) The feedback gain block.



(d) Observer realization form.

The connected and "expanded" block diagram for this case is shown below.



Similarly, we obtain

$$\dot{x}_1 = -y_1 + x_2,$$
$$\dot{x}_2 = -2y_1 + 2(r - y_2),$$
$$\dot{x}_3 = -y_2,$$

and

$$y = y_1,$$

where

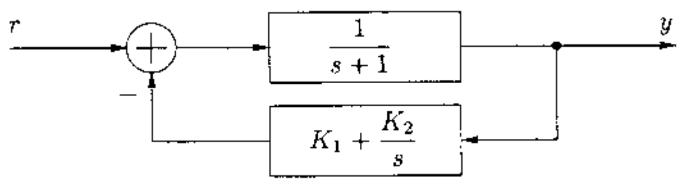
$$y_1 = x_1,$$
$$y_2 = x_3 + y_1.$$

And,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -4 & 0 & -2 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} r(t),$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

4. Consider the following control system.



Design the controller gains K_1 and K_2, such that the approximate maximum percent overshoot is $M_p \approx 9.5\%$, and the approximate rise time is $t_r \approx 0.7\,\mathrm{s}$.

Solution: The closed-loop transfer function of the system is

$$\frac{Y(s)}{R(s)} = \frac{1/(s+1)}{1 + (1/(s+1))(K_1 + K_2/s)} = \frac{s}{s^2 + (1+K_1)s + K_2}.$$

From the general expression of the second-order system poles $s^2 + 2\zeta\omega_n s + \omega^2$, we get

$$1 + K_1 = 2\zeta\omega_n, \tag{1}$$

and

$$K_2 = \omega_n^2, \tag{2}$$

by comparison. We determine the system parameters: ζ and ω from the system requirements.

Given Requirements	General System Restrictions	Specific System Restrictions
Maximum percent overshoot for a unit step input	$M_p \approx 0.095.$	For a second-order system with no zero. $e^{\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} \approx 0.095.$
Rise time for a unit step input	$t_r \approx 0.7\,\mathrm{s}.$	For a second-order system with no zero, $\frac{\pi - \cos^{-1}(\zeta)}{\omega_d} \approx 0.7.$

From the maximum overshoot requirement, we obtain

$$\zeta = \frac{|\ln(M_p)|}{\sqrt{(\ln(M_p))^2 + \pi^2}} \approx 0.6.$$

And, from the rise time requirement, we obtain

$$\omega_d = \frac{\pi - \cos^{-1}(\zeta)}{t_r} \approx 3.1633,$$

or

$$\omega_n = \frac{\omega_d}{\sqrt{1+\zeta^2}} \approx 3.95.$$

Therefore, from Equations (1) and (2), we determine

$$K_1 \approx 3.74,$$

$$K_2 \approx 15.63.$$

Note here that these are first approximations, since the closed-loop system has also a zero that alters the expressions for the specifications.