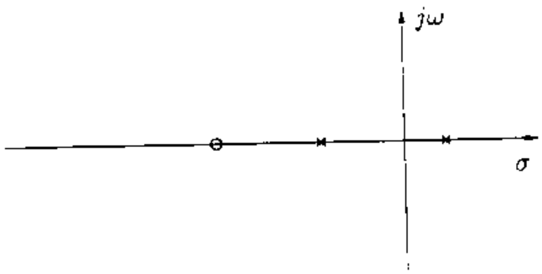
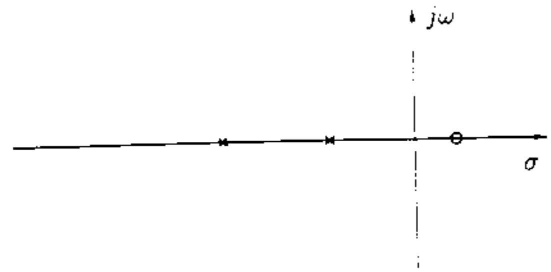
EE 231 **Exam#2** **Dec. 08, 1993**

50 minutes

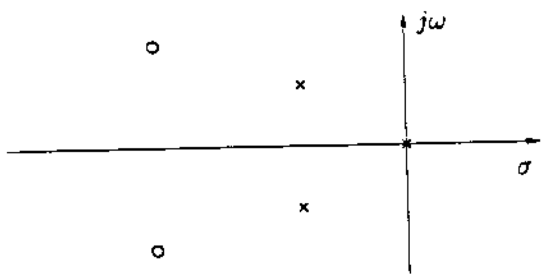
1. For the following open-loop pole/zero locations, sketch *expected* root-locus diagrams. *Do not* determine any features of the diagram, simply show the expected shapes of all the root-locus branches. (20pts)



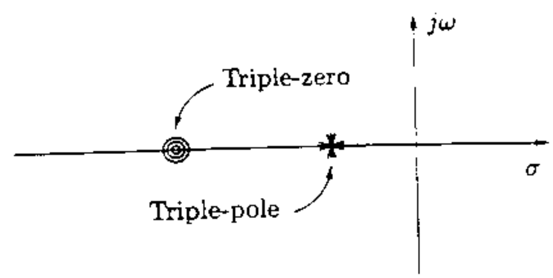
(a)



(b)



(c)



(d)

2. Consider a unity-feedback control system with the open-loop transfer function

$$G(s) = K\frac{12s}{(s+1)^3(s+2)(s+5)} = K\frac{12s}{s^5+10s^4+34s^3+52s^2+37s+10}.$$

   (a) Construct the root-locus diagram. Determine all the important features like asymptotes, imaginary-axis crossings, angle of arrivals and departures; however *do not* determine the break-away and/or break-in points explicitly. Obtain only the equation whose solutions would give those points i.e., *do not solve that equation.* (35pts)

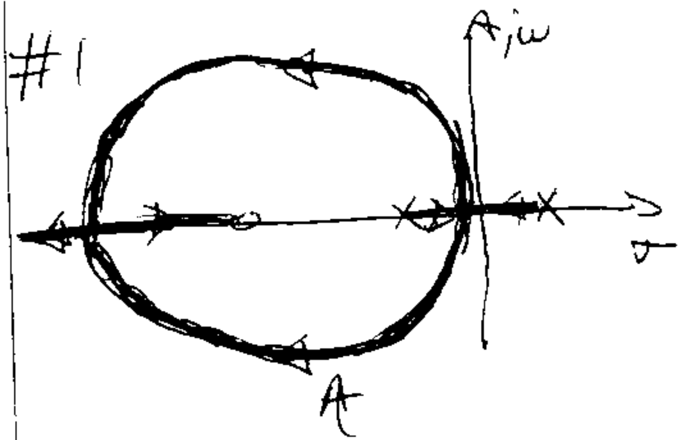
   (b) Determine all the values of K such that the closed-loop system is asymptotically stable. (5pts)

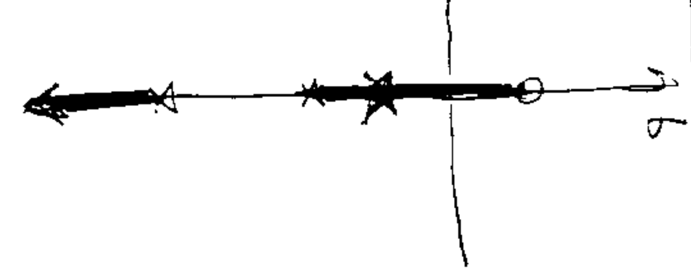
3. Consider a unity-feedback system with the open-loop transfer function

$$D(s)G(s) = D(s)\frac{(s+6)}{(s+2)(s+5)}.$$

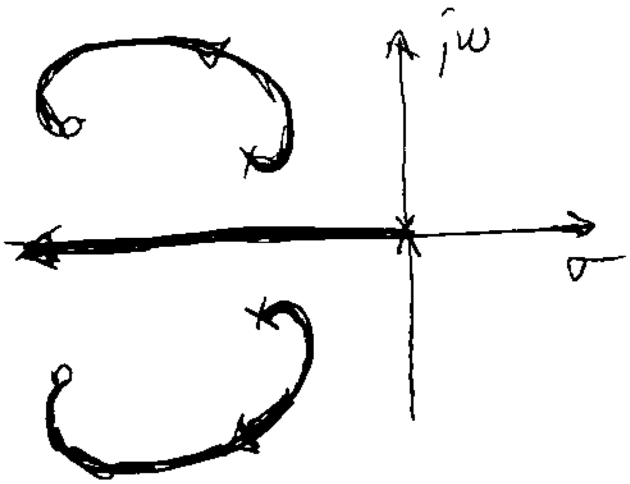
Using graphical root-locus methods, design the simplest controller $D(s)$, such that the steady state error for a step input is zero, and the maximum percent overshoot based on the dominant poles is approximately 16.3%. *Note*: Due to the hand construction of the root-locus diagram, inaccuracies in the closed-loop pole locations will be acceptable. (40pts)

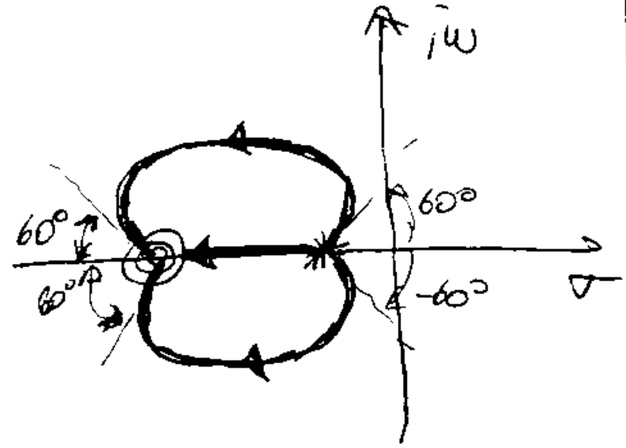
#1





a circle with center at the zero

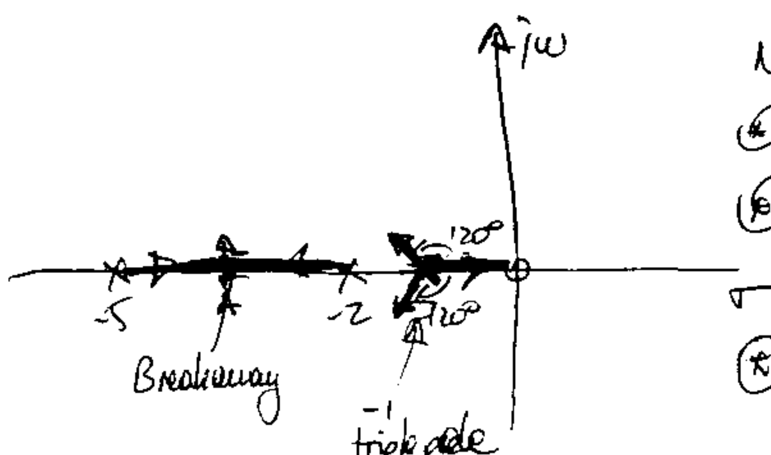




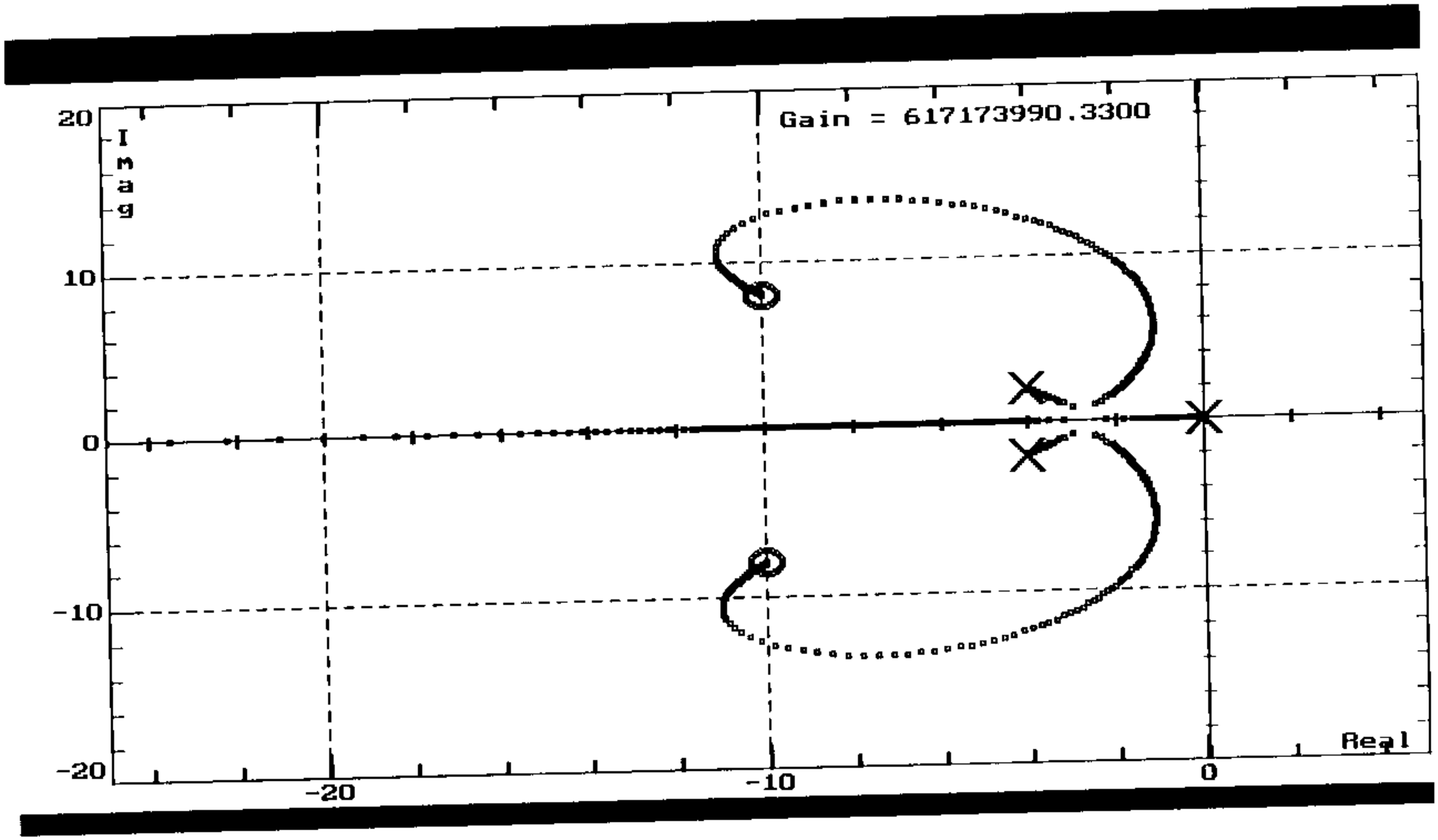
#2 $$G(s) = K\frac{12s}{(s+1)^3(s+2)(s+5)}$$

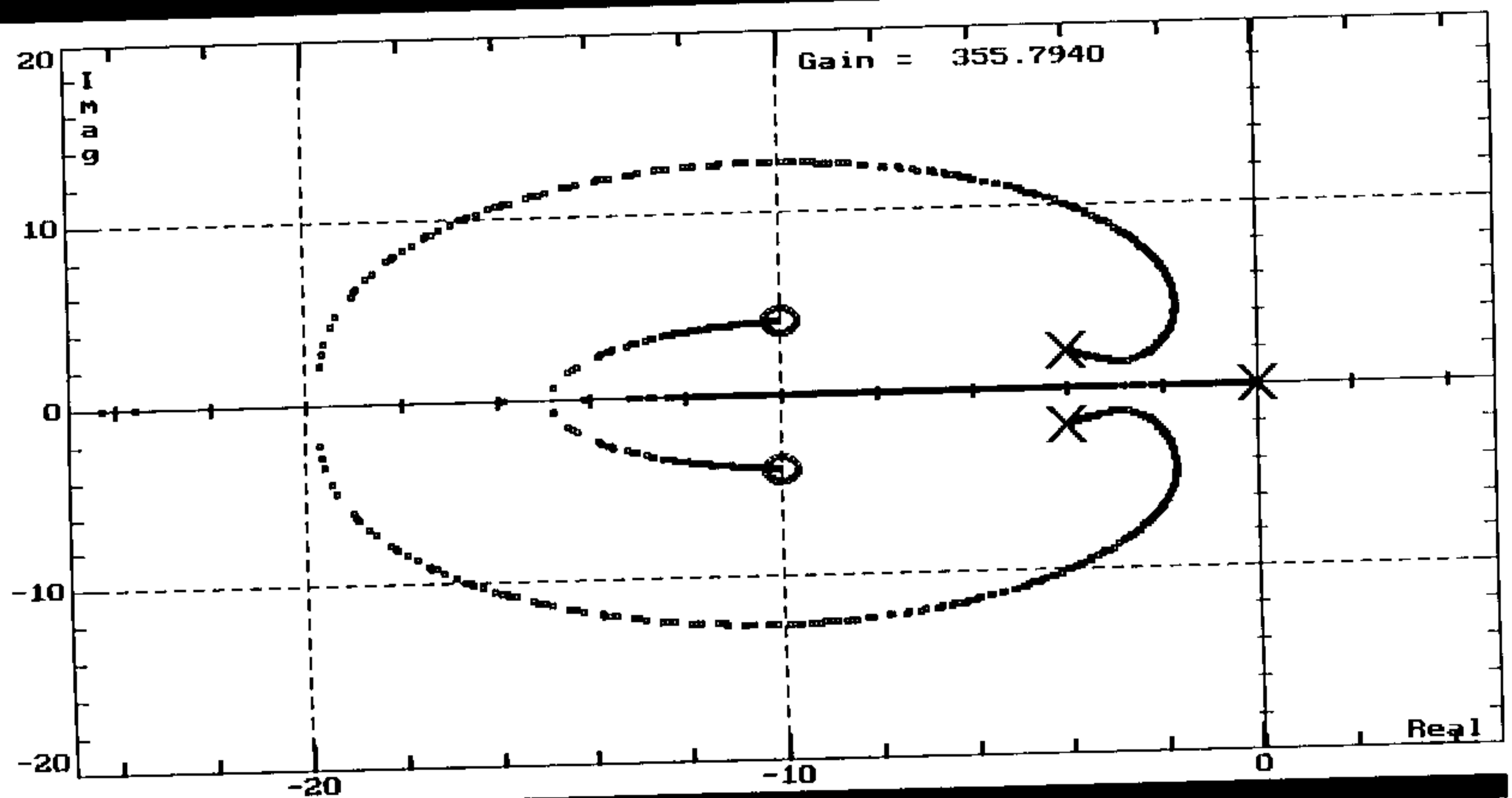
$$= K\frac{12s}{s^5+10s^4+34s^3+52s^2+37s+10}$$

all



NEED TO FIND
- Breakaway
- Asymptotes
- Imag-axis crossings


Gain = 617173990.3300
Imag
Real
20
10
0
-10
-20
-20
-10
0


Gain = 355.7940
Imag
Real
20
10
0
-10
-20
-20
-10
0

(*) Breakaway from $\frac{dK}{ds} = 0$

$$1 + G(s) = 0 \Rightarrow 1 + K \frac{12s}{s^5 + 10s^4 + 34s^3 + 52s^2 + 37s + 10} = 0$$

$$\text{or } -12K = \frac{s^5 + 10s^4 + 34s^3 + 52s^2 + 37s + 10}{s}$$

$$-12\frac{dK}{ds} = \frac{(5s^4 + 40s^3 + 102s^3 + 104s + 37)s - (s^5 + 10s^4 + 34s^3 + 52s^2 + 37s + 10)}{s^2}$$

So breakaway from the solution of

$$4s^5 + 30s^4 + 68s^3 + 52s^2 - 10 = 0$$

↳ (Soln: $s = -4.15003, -1.70358, -1, -1, 0.313612$)

for $K \geq 0$

(*) Asymptotes

$$\sigma_a = \frac{-1-1-1-2-5+0}{5-1} = -2.5$$

$$\theta_a = \frac{\pm 180^\circ + k360^\circ}{5-1} = \pm 45, \pm 135^\circ$$

(*) Imag-axis crossings from the Routh table

$$1 + G(s) = 0 \Rightarrow s^5 + 10s^4 + 34s^3 + 52s^2 + 37s + 10 + 12Ks = 0$$

s^5	1	34	37+12K
~~s^4~~	~~10~~	~~52~~	~~10~~
s^4	5	26	5
~~s^3~~	$\cancel{\frac{144}{5}}$	$\cancel{\frac{180+60K}{5}}$	
s^3	12	15+5K	
~~s^2~~	$\cancel{\frac{237-25K}{12}}$	$\cancel{\frac{60}{12}}$	
s^2	237−25K	60	
s^1	X		
s^0	60		

$$X = (237-25K)(15+5K) - 720$$

Imag-axis crossings when $X = 0$

or $(237-25K)(15+5K) - 720 = 0$

$$(237-25K)(3+K) - 144 = 0$$

$$711 + 162K - 25K^2 - 144 = 0$$

$$25K^2 - 162K - 567 = 0$$

$$K_{1,2} = \frac{162 \pm \sqrt{162^2 - 4(25)(-567)}}{2(25)} = \frac{162 \pm 288}{50}$$

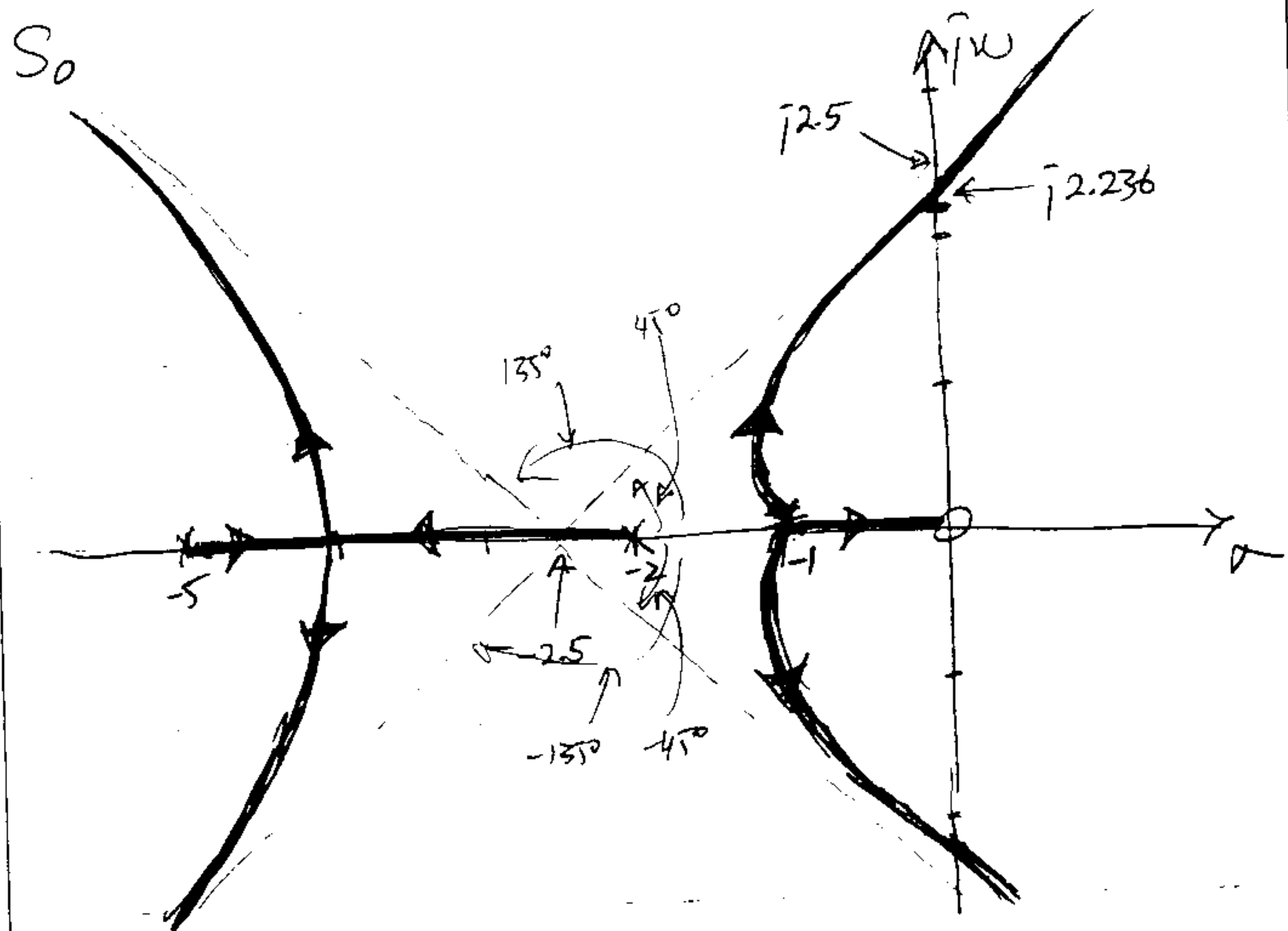
$$= -2.52,\ 9$$

We want $K>0$

From the s^2-term & $K=9$

$$(237-25K)s^2+60=0$$

$$12s^2+60=0 \Rightarrow s_{1,2}=j\sqrt{5}=\pm j2.236$$



b) Closed-loop stability can be obtained from the Routh table. For stability

i) $237-25K>0$, $-25K>-237$, $K<+9.48$

ii) $X>0$, $711+162K-25K^2-144>0$

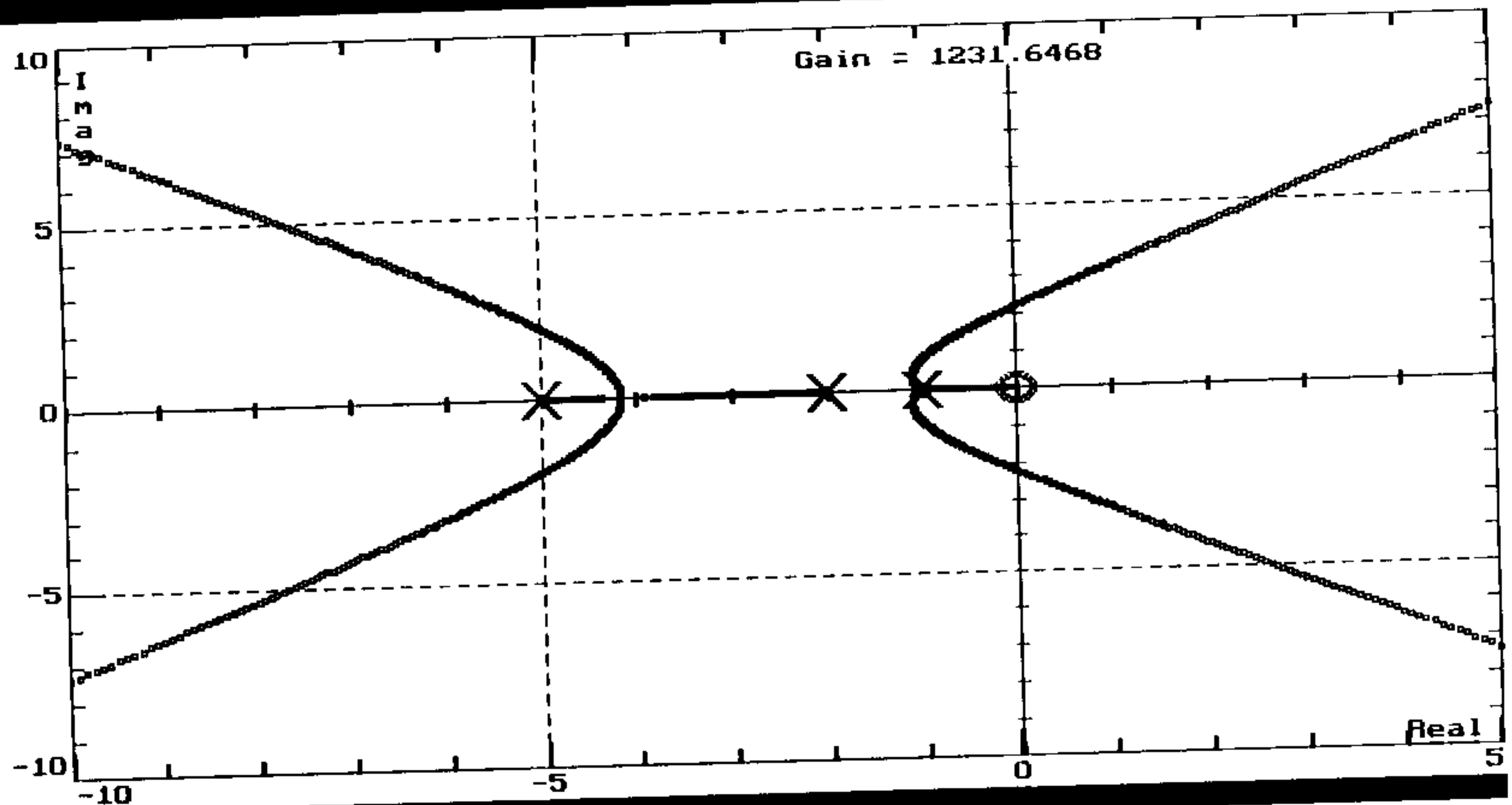
$$567+162K-25K^2>0$$

$$-25(K-9)(K+2.52)>0 \Rightarrow -2.52<K<9$$

$-$ $+$ $-$

-2.52 0 9 K

Conditions i & ii imply $-2.52<K<9$.


Gain = 1231.6468
Imag
Real
10
5
0
-5
-10
-10
-5
0
5

#3 $D(s)\,G_1(s) = D(s)\,\dfrac{s+6}{(s+2)(s+T)}$

Requirements: Steady-state error for step input $=0$ $\Rightarrow$ Open-loop system TYPE 1

$$\Rightarrow D(s) = D^1(s)\,\frac{1}{s}$$

: Max. % overshoot is $\approx 16.3\%$ $\quad M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} \approx 0.163$

$$-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi \approx \ln 0.163$$

$$-\zeta\pi \approx \sqrt{1-\zeta^2}\,\ln 0.163$$

$$\zeta^2\pi^2 \approx (1-\zeta^2)\ln^2 0.163$$

$$(\ln^2 0.163 + \pi^2)\,\zeta^2 \approx \ln^2 0.163$$

$$\zeta^2 \approx \frac{\ln^2 0.163}{\ln^2 0.163 + \pi^2}$$

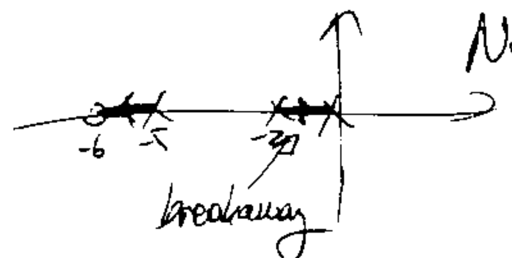
$$\zeta \approx \frac{|\ln 0.163|}{\sqrt{\ln^2 0.163 + \pi^2}} \approx 0.5$$

i.e. $\Rightarrow \zeta \approx 0.5$

Simplest Control $D'(s) = K$ or $D(s) = \frac{K}{s}$

$$\Rightarrow D(s)G(s) = K\frac{s+6}{s(s+2)(s+5)}$$

Root-locus



Need to find
(*) Breakaway
(*) Asymptotes

(*) Break-away from $\frac{dK}{ds} = 0$

$$1 + D(s)G(s) = 0 \Rightarrow -K = \frac{s(s+2)(s+5)}{s+6} = \frac{s^3+7s^2+10s}{s+6}$$

$$-\frac{dK}{ds} = \frac{(3s^2+14s+10)(s+6) - (s^3+7s^2+10s)}{(s+6)^2}$$

$$\frac{dK}{ds} = 0 \Rightarrow 2s^3 + 25s^2 + 84s + 60 = 0$$ for $K>0$

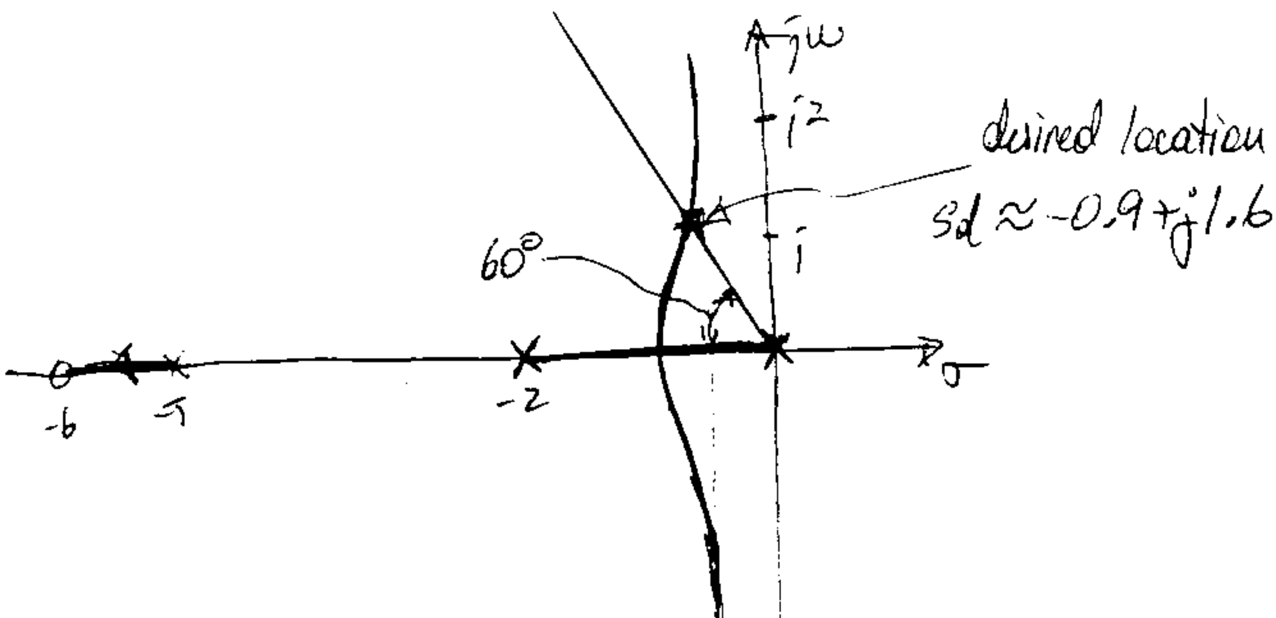
↳ SOLN: $s = -7.3259, -4.19881, -0.97529$

(BUT YOU DIDN'T HAVE TO FIND THE SOLUTION, YOU COULD ~~HAND~~ SKETCH THE ROOT-LOCUS.)

(*) Asymptotes

$$\sigma_a = \frac{0-2-5-(-6)}{3-1} = -0.5$$

$$\theta_a = \frac{\pm 180^\circ + k360^\circ}{3-1} = \pm 90^\circ$$



$\zeta = 0.5$ is a line with angle $= \cos^{-1}\zeta = \cos^{-1} 0.5 = 60°$

$\Longrightarrow s_d \approx -0.9 \pm j1.6$ (YOU ARE NOT EXPECTED TO FIND s_d EVEN THIS ACCURATELY. A GENERAL NEIGHBORHOOD IS ACCEPTABLE)

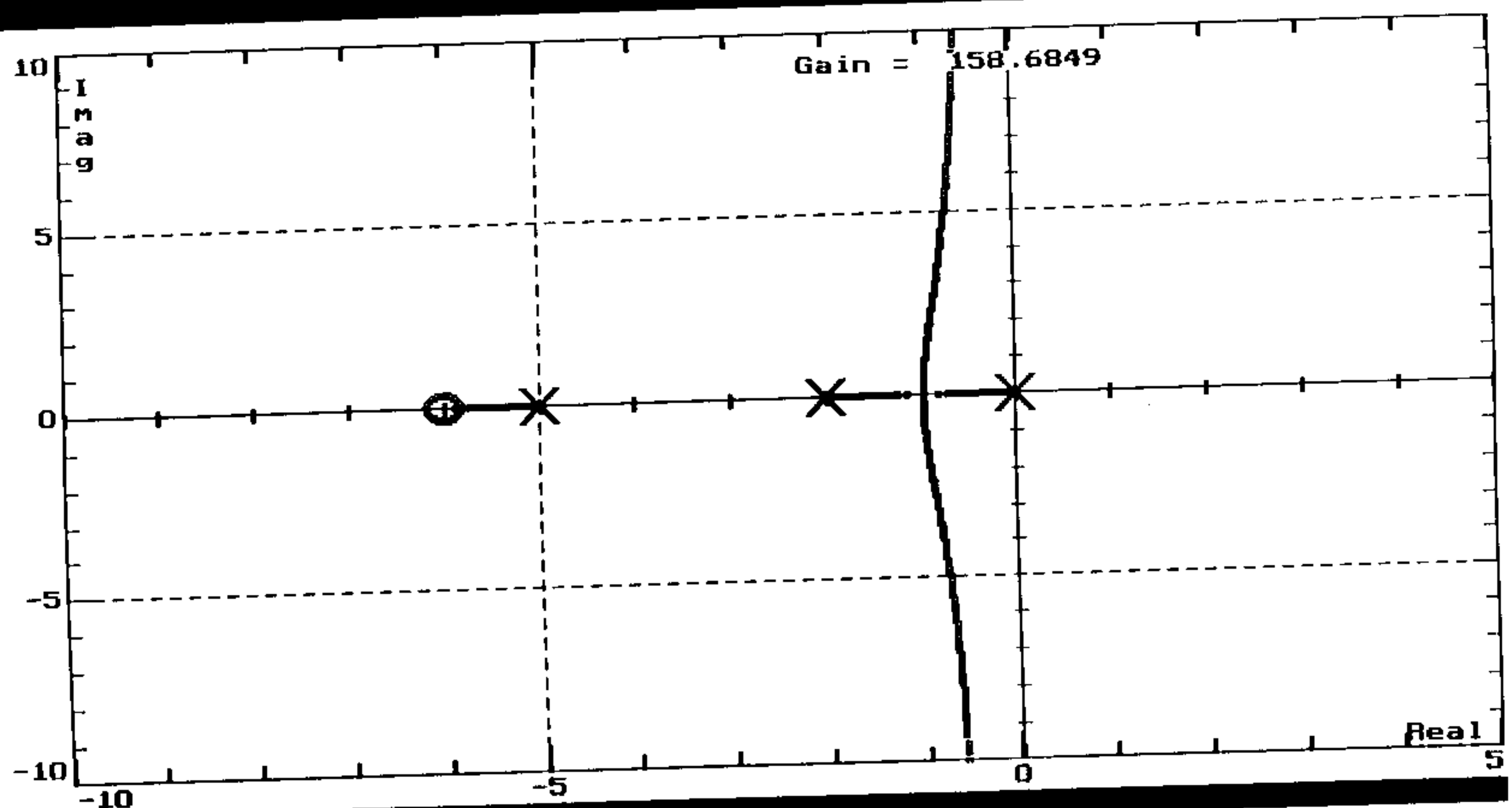
Actually $s_d = -0.923 \pm j1.603$

To find K in $D(s) = \frac{K}{s}$ use mag. cond.

$$\left| K \frac{s+6}{s(s+2)(s+5)} \right|_{s=-0.9+j1.6} = 1 \Longrightarrow K = 2.935$$

Actually $K = 2.94$

So $D(s) = \frac{2.935}{s}$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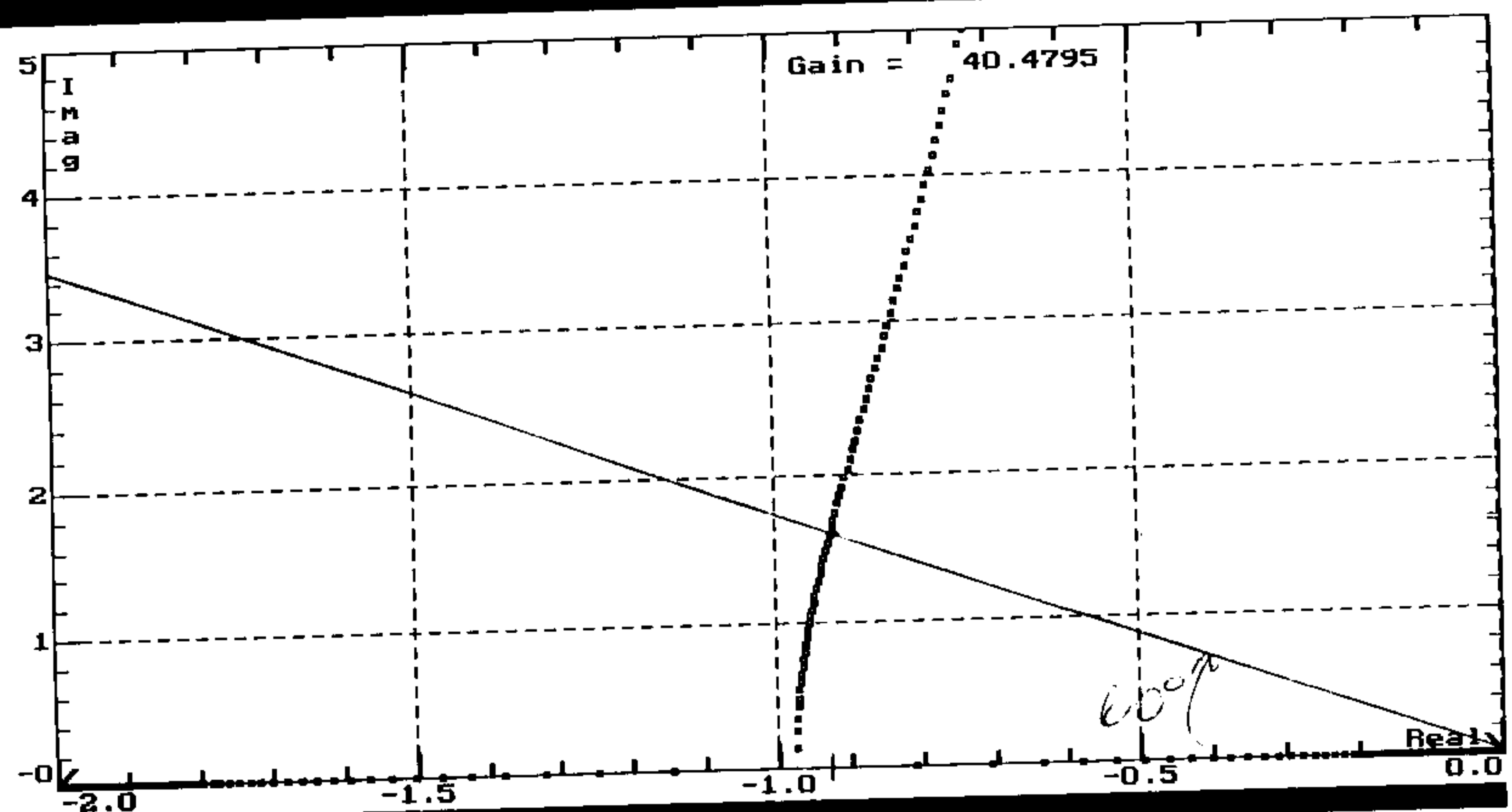
Imag
Real
10
5
0
-5
-10
-10
-5
0
5



Gain = 40.4795
Imag
Real
5
4
3
2
1
-0
-2.0
-1.5
-1.0
-0.5
0.0
60°