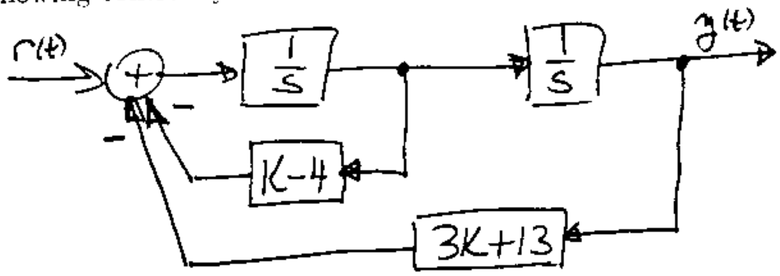
EE 231

Exam#2
50 minutes

Dec. 08, 1995

1. Consider the following control system.

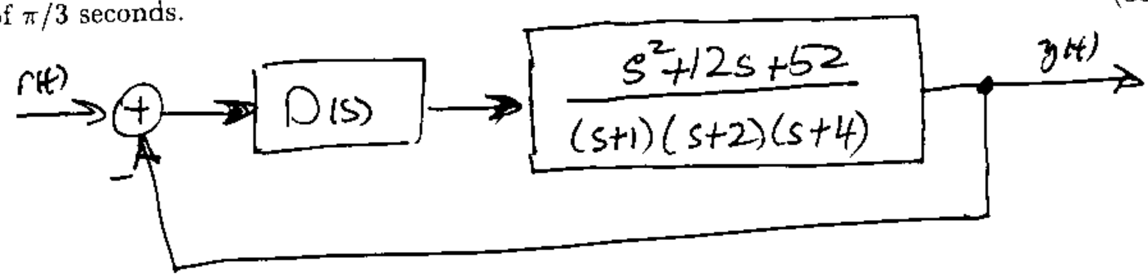


   (a) Construct the root-locus diagram as K goes from 0 to ∞. (15pts)

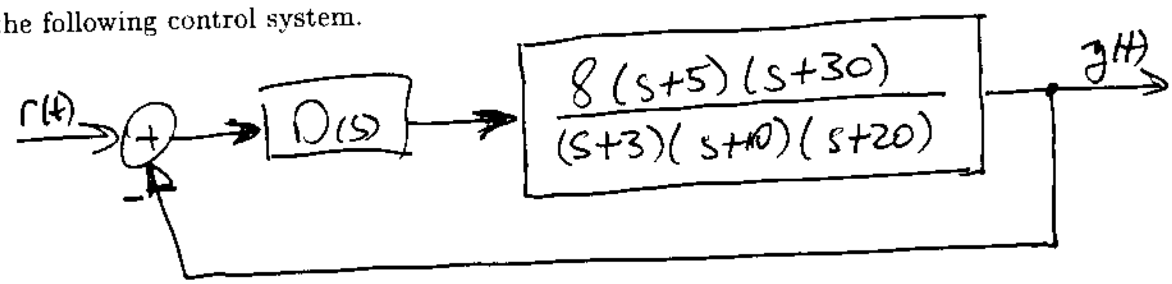
   (b) Construct the root-locus diagram as K goes from 0 to $-\infty$. (15pts)

   (c) Determine the range of K for the asymptotical stability of the closed-loop system. (15pts)

2. For the following system, design a first order compensator $D(s)$, without increasing the order of the system, such that the dominant complex poles would produce a 5% settling time of 1.5 seconds and a peak time of $\pi/3$ seconds. (35pts)



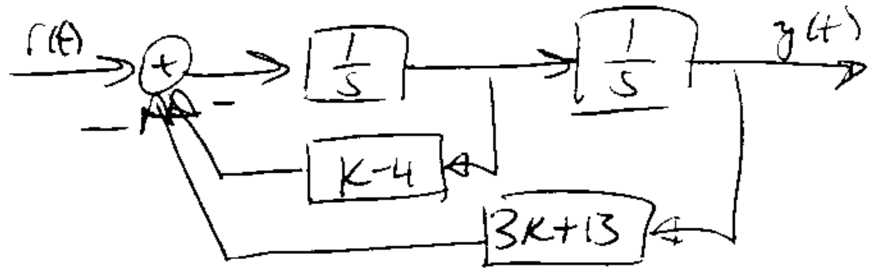
3. Consider the following control system.

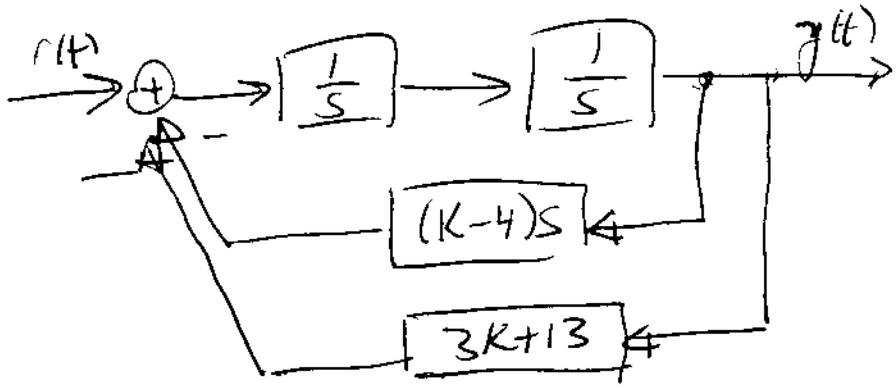


   (a) Design a compensator $D(s)$, such that the steady state error, $e(\infty)$ is zero for a step input, and it is 0.1 for the unit ramp input. (20pts)

   (b) Design a compensator $D(s)$, such that the steady state error, $e(\infty)$ is zero for a ramp input. (+20pts)

#1





$$\Rightarrow \quad G(s) = \frac{1}{s^2}, \quad H(s) = (K-4)s + (3K+13)$$

$$1 + GH = 1 + \frac{(K-4)s + (3K+13)}{s^2} = 0$$

$$s^2 + (K-4)s + (3K+13) = 0$$

$$s^2 + Ks - 4s + 3K + 13 = 0$$

$$s^2 - 4s + 13 + Ks + 3K = 0$$

$$(s^2 - 4s + 13) + K(s+3) = 0$$

$$1 + K\frac{s+3}{s^2 - 4s + 13} = 0$$

So the new open-loop gain is $G'H' = K\dfrac{s+3}{s^2-4s+13}$

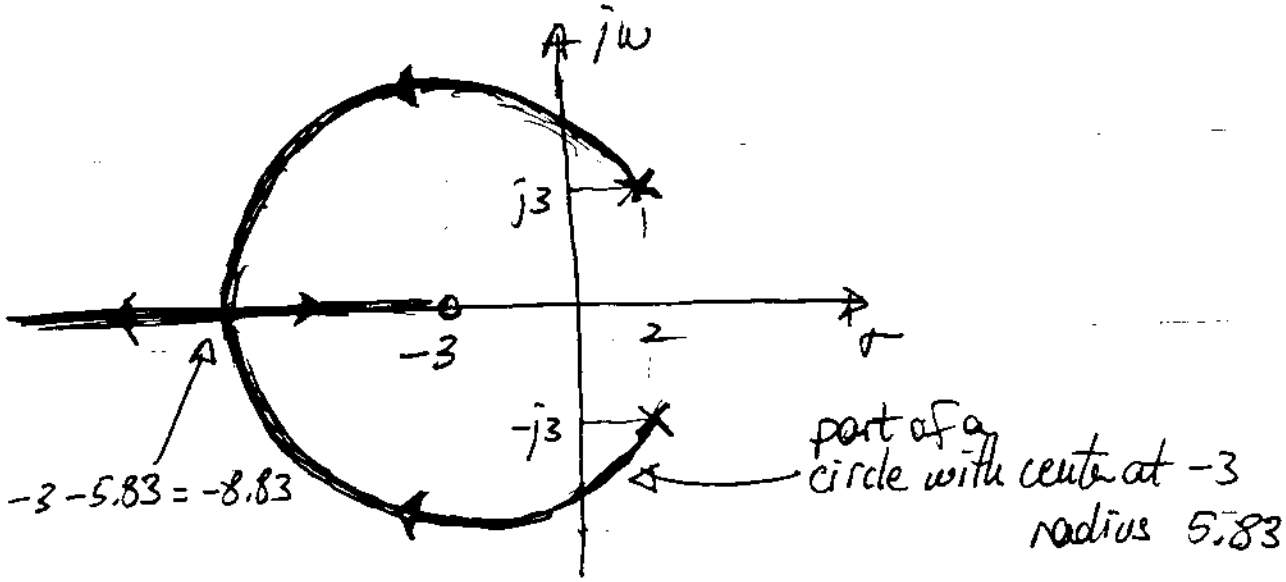
s.t. the original gain GH and the new gain $G'H'$ give the same closed-loop poles. So let's use the new gain for root-locus.

q_{11} $K>0$ $G'H' = K\dfrac{s+3}{s^2-4s+13}$ two poles & one zero

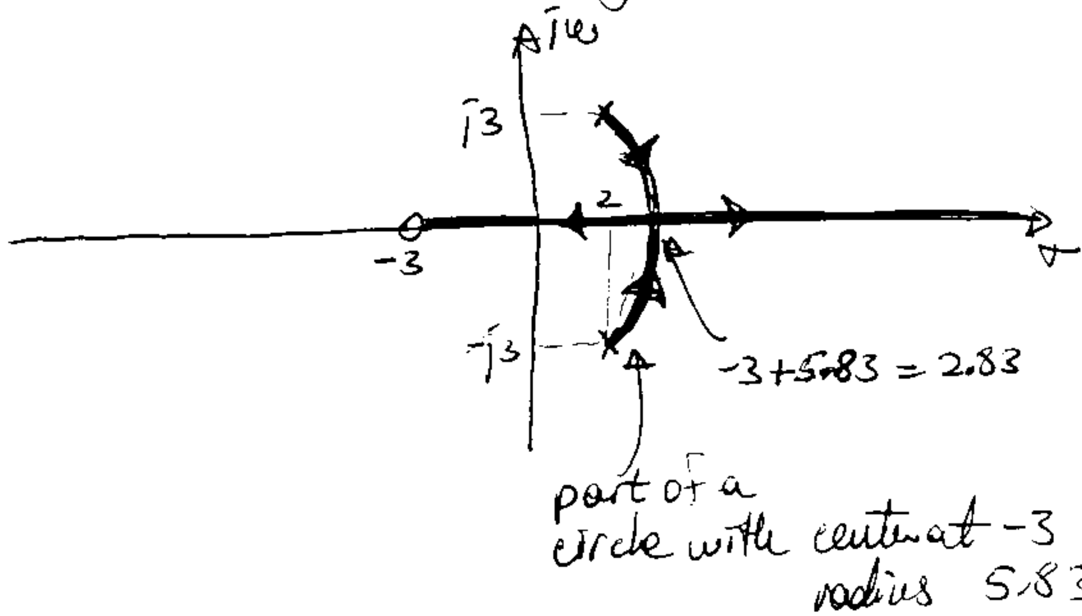
$s_{1,2} = +2 \pm j3$

$\Rightarrow$ root-locus is part of a circle with center at the zero or $s=-3$ and radius $r = \sqrt{(p_1-z)(p_2-z)}$

$$= \sqrt{(+2+j3-(-3))(+2-j3-(-3))}$$
$$= \sqrt{(5+j3)(5-j3)}$$
$$= \sqrt{5^2+3^2} = \sqrt{34} \approx 5.83$$



b// $K<0$ The root-locus in this case will be the remaining portion of the circle



c// The best way to determine the range of K for asymptotical stability is to use Routh table

The closed-loop poles satisfy

$1+GH=0$ or from previous analysis

$$s^2+(K-4)s+(3K+13)=0$$

s^2	1	$3K+13$
s	$K-4$ $\Longrightarrow$ $K-4>0$, $K>4$	
1	$3K+13$ $\Longrightarrow$ $3K+13>0$ $3K>-13$, $K>-\frac{13}{3}$	

so $K>4$ and $K>-\frac{13}{3}$ $\Longrightarrow$ $\boxed{K>4}$

#2 $D(s)$ s.t. $t_{5\%s} = 1.5$

$t_p = \frac{\pi}{3}$

For dominant complex poles $t_{5\%s} = \frac{3}{\sigma_0}$

$\Rightarrow \quad 1.5 = \frac{3}{\sigma_0} \Rightarrow \sigma_0 = 2$

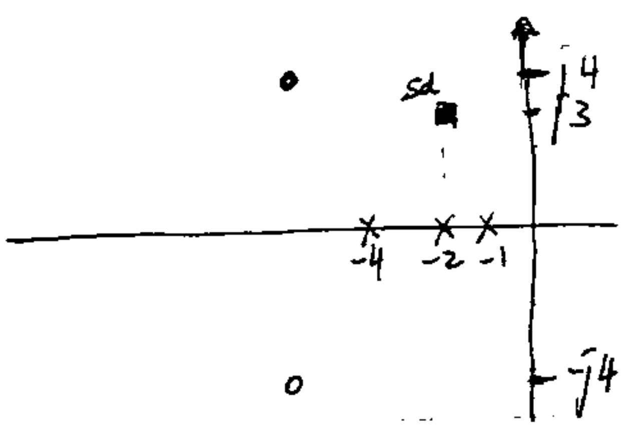
$t_p = \frac{\pi}{\omega_d}$

$\Rightarrow \quad \frac{\pi}{3} = \frac{\pi}{\omega_d} \Rightarrow \omega_d = 3$

$\Rightarrow$ desired closed-loop poles are at
$s_d = -2 \pm j3$

$s_{1,2} = -6 \pm j4$

$$GH = D(s)\,\frac{s^2+12s+52}{(s+1)(s+2)(s+4)}$$



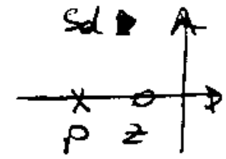
So the angle needed for the root-locus go through s_d is

$$\phi + \tan^{-1}\frac{3-4}{-2-(-6)} + \tan^{-1}\frac{3-(-4)}{-2-(-6)} - \tan^{-1}\frac{3-0}{-2-(-1)}$$
$$-\tan^{-1}\frac{3-0}{-2-(-2)} - \tan^{-1}\frac{3-0}{-2-(-4)} = (2k+1)\,180^\circ$$

$$\phi + (-14.04^\circ) + 60.26^\circ - 108.43^\circ - 90^\circ - 56.31^\circ = (2k+1)180^\circ$$

$\phi = 28.53^\circ \Rightarrow$ Design a lead compensator

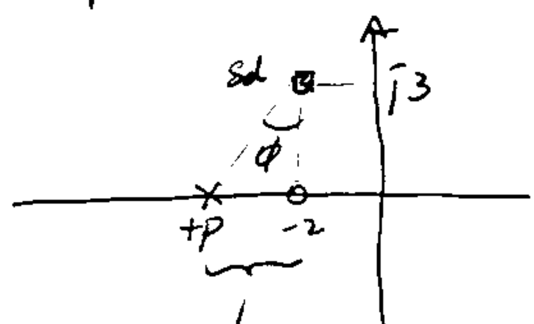
$\hookrightarrow D(s) = K \dfrac{s - z}{s - p}$



In order not to increase the system order, the lead compensator should cancel one of the poles and place another pole.

One possible pole to cancel is at $s = -2$, i.e. $z = -2$

so



$\hookrightarrow 3 \tan\phi = 3 \tan 28.53^\circ = 1.63$

$\Rightarrow +p = -2 - 1.63 = -3.63$

or $D(s) = K \dfrac{s+2}{s+3.63}$

To find K, we need to use the magnitude condition

$$\left| GH \right|_{s=s_d} = 1 \quad \text{or} \quad \left| K \frac{\cancel{s+2}}{s+3.63} \cdot \frac{s^2+12s+52}{(s+1)\cancel{(s+2)}(s+4)} \right|_{s=-2+j3} = 1$$

$\Rightarrow K = 1.17$ or $D(s) = 1.17 \dfrac{s+2}{s+3.63}$

#3

$$GH = D(s) \frac{8(s+5)(s+30)}{(s+3)(s+10)(s+20)}$$

a) For $e(\infty) = 0$ for a step input, the system has to be TYPE 1 or $D(s) = D'(s) \frac{1}{s}$

For $e(\infty) = 0.1$ for unit ramp input, we first need to find K_v, assuming $D'(s) = K$

$$K_v = \lim_{s \to 0} s\, GH = \lim_{s \to 0} \cancel{s}\, K \frac{8(s+5)(s+30)}{\cancel{s}(s+3)(s+10)(s+20)}$$

$$= 2K \Rightarrow e(\infty) = \frac{1}{K_v} = \frac{1}{2K}$$

Requirement $\quad e_{desired}(\infty) = 0.1$

$$\Rightarrow \frac{1}{2K} = 0.1 \text{ or } K = 5$$

So if $D(s) = \frac{5}{s}$ keeps the system stable, we are successful. We can check the stability using Routh criterion

Closed-loop poles $1+GH=0$

$$1+\frac{40(s+5)(s+30)}{s(s+3)(s+10)(s+20)}=0$$

$$s(s+3)(s+10)(s+20)+40(s+5)(s+30)=0$$

$$s^4+33s^3+330s^2+2000s+6000=0$$

$$\begin{array}{c|ccc}
s^4 & 1 & 330 & 6{,}000 \\
s^3 & 33 & 2{,}000 & \\
s^2 & 330-\frac{2000}{33}=269.39 & 6{,}000 & \\
s & 2000-\frac{33\times 6000}{269.39}=1{,}265.02 & & \\
1 & 6{,}000 & &
\end{array}$$

$\checkmark$ $\checkmark$ $\checkmark$ $\checkmark$

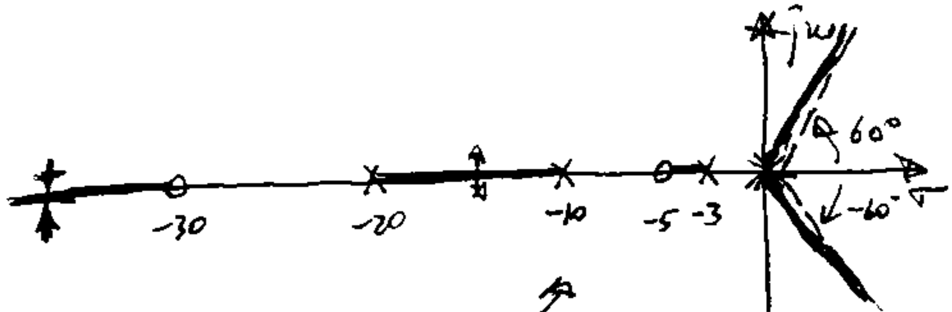
So the system is stable and $D(s)=\frac{5}{s}$ is acceptable.

b) For $e(\infty)=0$ for unit ramp, we need

$$D(s)=D'(s)\frac{1}{s^2}$$

so $$GH=D'(s)\frac{8(s+5)(s+30)}{s^2(s+3)(s+10)(s+20)}$$

Unfortunately, in this case $D(s)=K$ does not give a stable system since from the rough sketch of the root locus plot, we get



asymptotes $\sigma_a = \dfrac{\sum p_i - \sum z_i}{n-m} = \dfrac{0+0+(-3)+(-10)+(-20)-((-5)+(-30))}{5-2}$

$= \dfrac{2}{3}$

$\theta_a = \dfrac{(2k+1)180^\circ}{n-m} = \dfrac{(2k+1)180^\circ}{5-2} = \pm 60^\circ, 180^\circ$

The two poles at $s=0$, immediately become unstable for any gain (even for the open-loop). As a result, we conclude that we need more dynamics in the compensator. At this point, we have two choices

① add more poles → a bad idea, since it would decrease some of θ_a (but it would probably move σ_a s.t. $\sigma_a < 0$)

② add zeros → a better idea, since we can easily add two zeros to the compensator, and it would still be proper.

→ add one zero → bad idea since it would increase σ_a

→ add two zeros → a good idea

One possibility is to add complex zeros so that the poles leaving $s=0$ would go towards these zeros

so let the zeros be at $s=-4\pm j4$

i.e. $\boxed{D(s)=K\,\dfrac{s^2+8s+32}{s^2}}$

Here any $K>0$ would work.