Exam#2
50 minutes

Apr. 29, 1996

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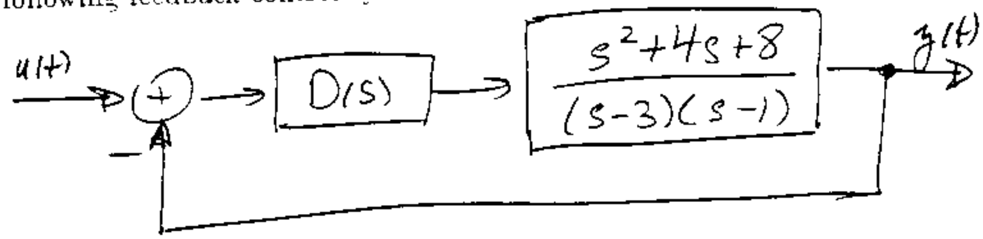
1. Consider the following feedback control systems.



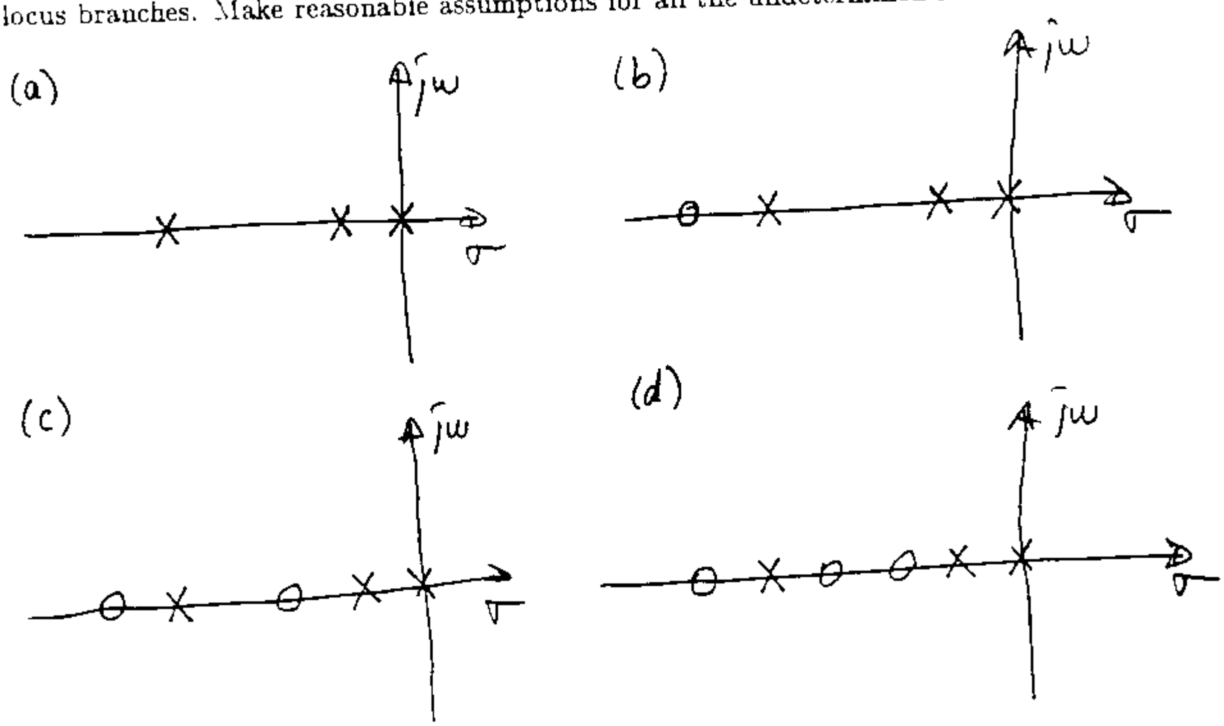
Note: This problem might require hand construction of some diagrams. In this case, the inaccuracies in the closed-loop pole locations are acceptable.

(a) Design a proportional (P) controller, such that the normalized maximum overshoot is $M_p \approx 0.25$. (10pts)

(b) Design an integral (I) controller, such that the steady-state error for the unit ramp input is at most 0.1. (5pts)

(c) Design the simplest controller, such that the 2% settling time is at most 4 seconds. After the design, determine the normalized maximum overshoot. (10pts)

2. For the following open-loop pole/zero locations, sketch *expected* root-locus diagrams. *Do not* determine any features of the diagram, except the asymptote angles. Simply show the expected shapes of all the root-locus branches. Make reasonable assumptions for all the undetermined features. (20pts)



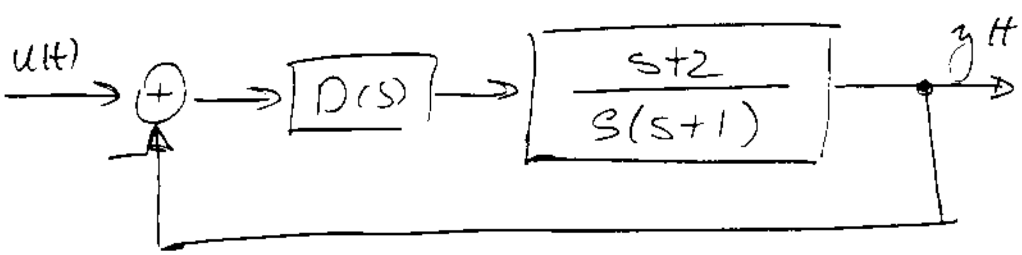
3. Consider a unity-feedback control system with the open-loop transfer function

$$G(s) = K\frac{1}{s(s+2)(s^2+6s+13)} = K\frac{\cancel{s^2+8s+17}\ 1}{s^4+8s^3+25s^2+26s}$$

   (a) Construct the root-locus diagram. Determine all the important features like asymptotes, imaginary-axis crossings, angle of arrivals and departures; however *do not* determine the break-away and/or break-in points explicitly. In other words, obtain the equation whose solutions would give those points, but *do not* solve that equation. **(30pts)**

   (b) Determine all the values of K such that the closed-loop system is asymptotically stable. **(5pts)**

4. For the following system, design a first order compensator $D(s)$, without increasing the order of the system, such that the steady-state error is zero for a step input, and the 5% settling time is approximately 0.5 second. **(20pts)**



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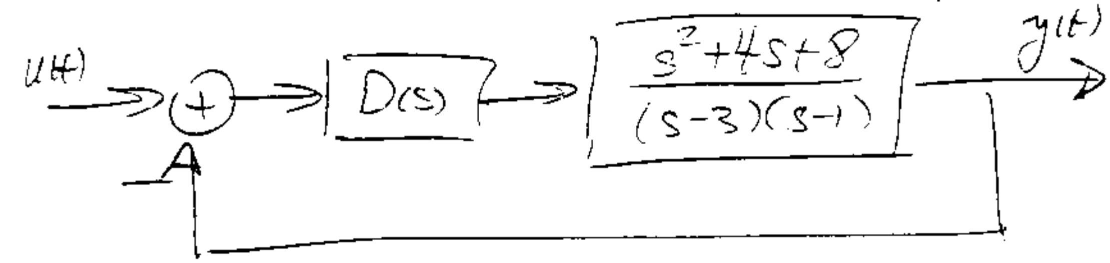
#1

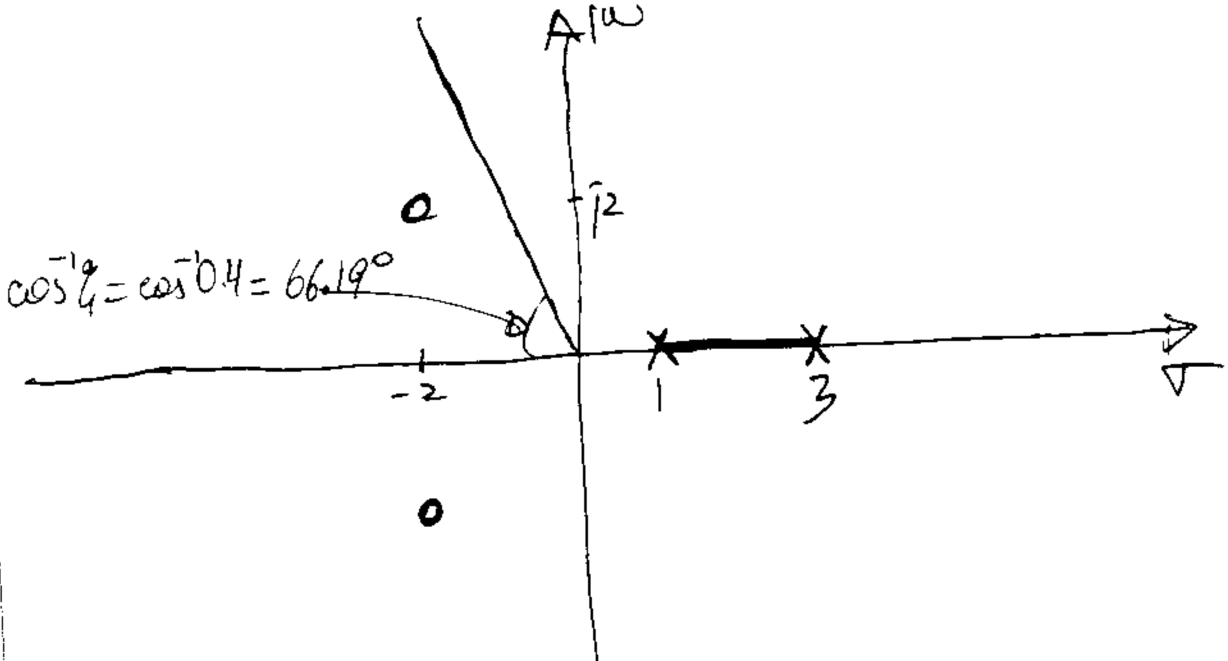


a// Proportional Control $\Rightarrow$ $D(s)=K$

$$GH = K\,\frac{s^2+4s+8}{(s-3)(s-1)} \leftarrow s=-2\pm j2$$

$$M_p \approx 0.25 \Rightarrow e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} \approx 0.25$$

$$\zeta \approx \frac{|\ln 0.25|}{\sqrt{\ln^2 0.25+\pi^2}} = 0.4$$



Need to find out the imaginary axis crossing
and the angle of arrival
and the break away point
(For the purposes of the exam, it was O.K. to sketch the root-locus without finding the above pt.)

Imaginary axis crossing $\longrightarrow$ Routh table

$1+GH=0$

$1+K\frac{s^2+4s+8}{(s-3)(s-1)}=0$

$(s^2-4s+3)+K(s^2+4s+8)=0$

$(K+1)s^2+(4K-4)s+(8K+3)=0$

s^2	$K+1$	$8K+3$
s	$4K-4$	
1	$8K+3$	

$8K+3 \longrightarrow (K+1)s^2+(8K+3)=0$

$4K-4 \longrightarrow 4K-4=0,\ K=1$

$2s^2+11=0$

$s=\pm j2.3452$

Angle of arrival $\longrightarrow$ Angular condition at $s_0=-2+j2$

$-\angle(s_0-1)-\angle(s_0-3)+\angle(s_0+2+j2)+\theta_{in}=180^\circ+k360^\circ$

$-\tan^{-1}\frac{2-0}{-2-1}-\tan^{-1}\frac{2-0}{-2-3}+90^\circ+\theta_{in}=180^\circ+k360^\circ$

$-146.31^\circ-158.20^\circ+90^\circ+\theta_{in}=180^\circ+k360^\circ$

$\theta_{in}=34.51^\circ$

Break-away point $\rightarrow \frac{dK}{ds} = 0$

$$1 + GH = 0$$

$$1 + K\frac{s^2+4s+8}{(s-3)(s-1)} = 0 \qquad -K = \frac{s^2-4s+3}{s^2+4s+8}$$

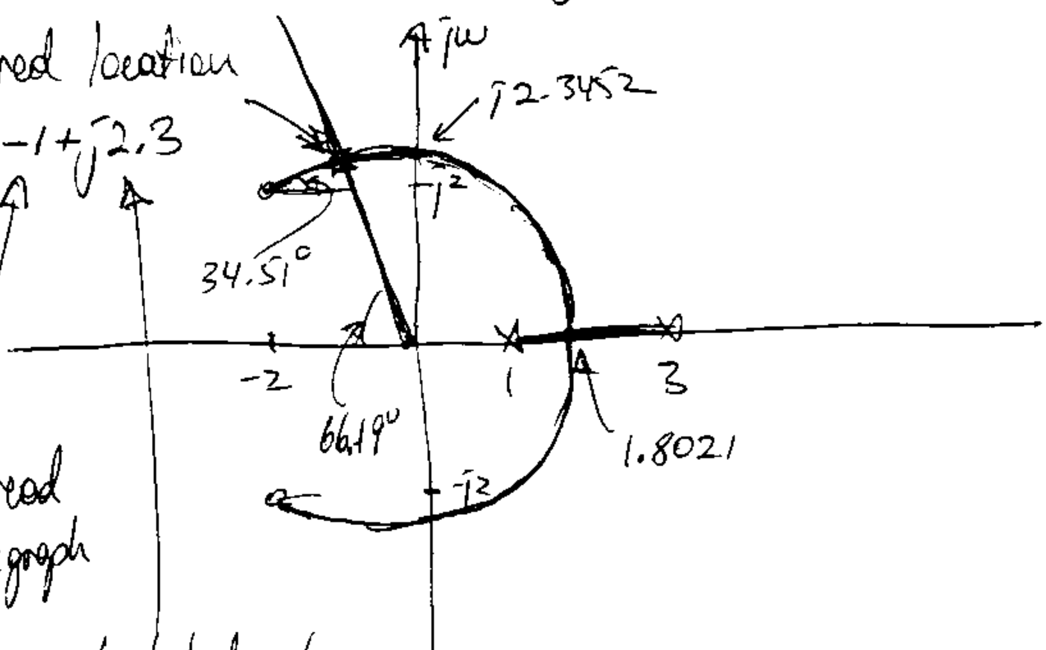
$$\frac{dK}{ds} = \frac{(2s-4)(s^2+4s+8) - (s^2-4s+3)(2s+4)}{(s^2+4s+8)^2}$$

$$\frac{dK}{ds} = 0, \qquad 8s^2+10s-44 = 0$$

$$s = \frac{-5 \pm \sqrt{377}}{8} = 1.8021, -3.0521$$

Desired location

$s_d \approx -1 + j2.3$



approx. read from the graph

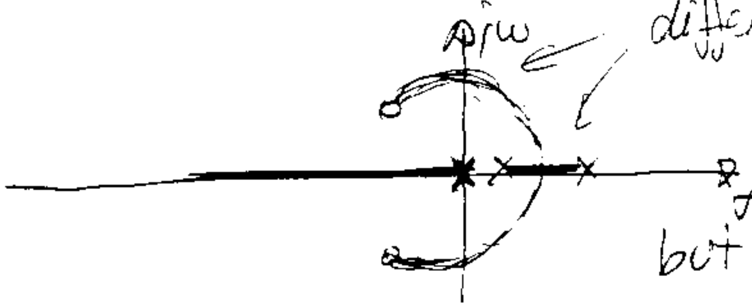
calculated s.t. s_d has $\zeta = 0.4$ from σ_0.

To find K from s_d, use the magnitude condition

or $\left|GH\right|_{s=s_d} = 1, \quad \left|K\frac{s^2+4s+8}{(s-3)(s-1)}\right|_{s=-1+j2.3} = 1 \Rightarrow K = 3.05$

b)) Integral Control $\Rightarrow D(s) = \frac{K}{s}$

$$GH = K \frac{s^2+4s+8}{s(s-3)(s-1)}$$



different imag axis crossing, break-away pt, angle of arrival

but the shape is similar

$$e_{ss} = \frac{1}{K_v} \quad \text{where } K_v = \lim_{s \to 0} sGH \quad \text{for unit ramp}$$

$$K_v = \lim_{s \to 0} sK \frac{s^2+4s+8}{s(s-3)(s-1)} = K\frac{8}{(-3)(-1)} = \frac{8}{3}K$$

$$e_{ss} = \frac{3}{8K}$$

$$e_{ss} \le 0.1 \Rightarrow \frac{3}{8K} \le 0.1 \Rightarrow K \ge 3.75$$

Need to check stability → use Routh-Hurwitz criterion

$$1 + GH = 0$$

$$1 + K\frac{s^2+4s+8}{s(s-3)(s-1)} = 0$$

$$s^3 + (K-4)s^2 + (4K+3)s + 8K = 0$$

s^3	1	$4K+3$
s^2	$K-4$	$8K$
s	$(4K+3)-\frac{8K}{K-4}$	
1	$8K$	

Stability $\Rightarrow$ $K-4>0$, $K>4$

$4K+3-\frac{8K}{K-4}>0$, $(K-4)(4K+3)-8K>0$

$4K^2-21K-12>0$

$\hookrightarrow K=-0.52$

$K=5.77$

eqn>0 , eqn<0 , eqn>0

-0.52 $\quad$ 5.77

so $K<-0.52$

or $K>5.77$

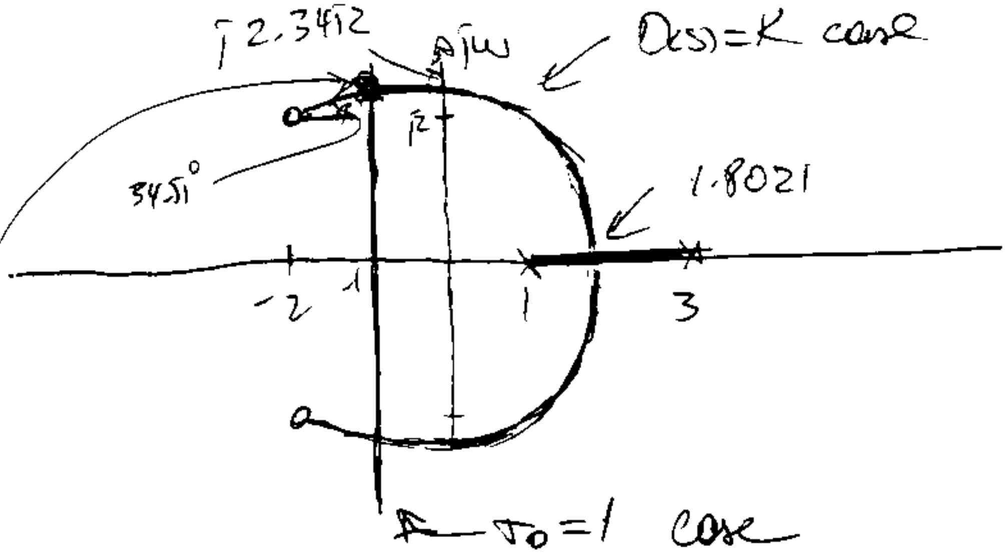
$8K>0$, $K>0$

$\Rightarrow \boxed{K>5.77}$

So let $K=6$ or $D(s)=\frac{6}{s}$

c.ii $t_{2\%s} \le 4 \Rightarrow \frac{4}{\sigma_0} \le 4$, $\sigma_0 \ge 1$ for a 2nd order system

$$GH = D(s)\,\frac{s^2+4s+8}{(s-3)(s-1)}$$



Desired location
$s_d \approx -1 + j2.4$ approx. $\longrightarrow$ $M_p = e^{-\frac{\sigma_0}{\omega_d}\pi} = e^{-\frac{1}{2.4}\pi}$
≈ 0.27 or 27%

K from the magnitude condition

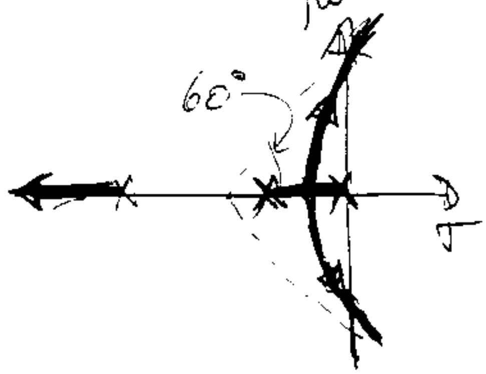
$$\left| GH \right|_{s=s_d} = 1$$

$$\left| K\,\frac{s^2+4s+8}{(s-3)(s-1)} \right|_{s=-1+j2.4} = 1 \Rightarrow K = 2.99$$

i.e. $t_{2\%s} \le 4 \Rightarrow K > 2.99$, let $K = 3.5$

#2

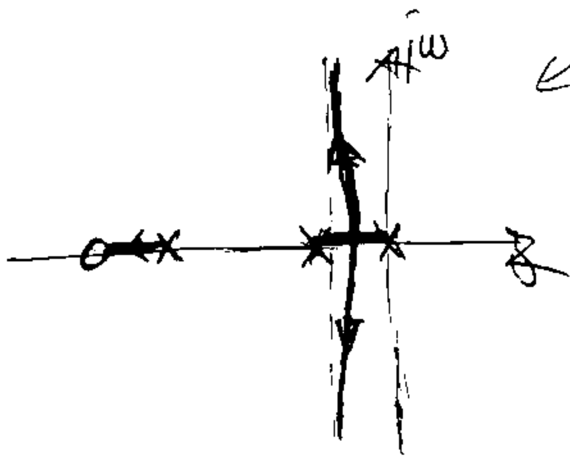
a)



\# of poles = 3
\# of zeros = 0

$$\sigma_a = \frac{180° + k360°}{3-0} = \pm 60°, 180°$$

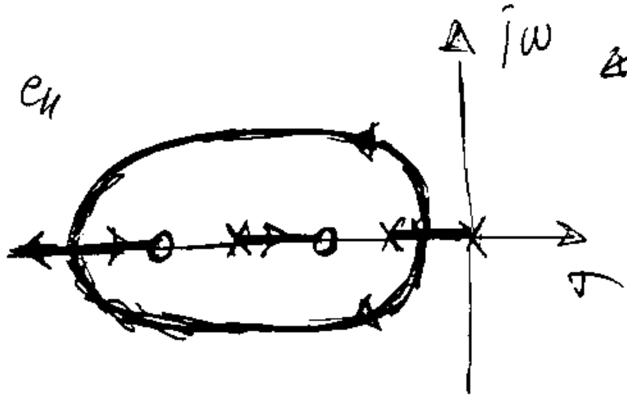
b)



\# of poles = 3
\# of zeros = 1

$$\sigma_a = \frac{180° + k360°}{3-1} = \pm 90°$$

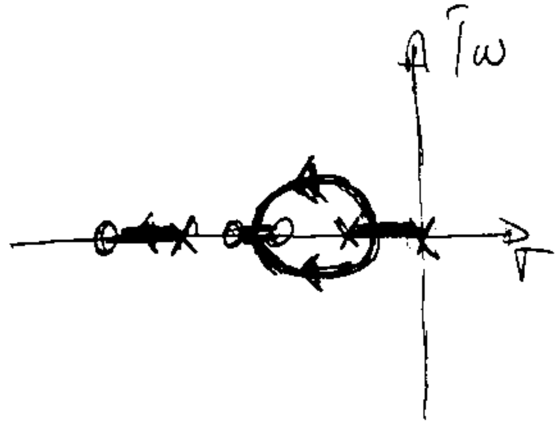
c)



\# of poles = 3
\# of zeros = 2

$$\sigma_a = \frac{180° + k360°}{3-2} = 180°$$

d)



\# of poles = 3
\# of zeros = 3

No asymptotes

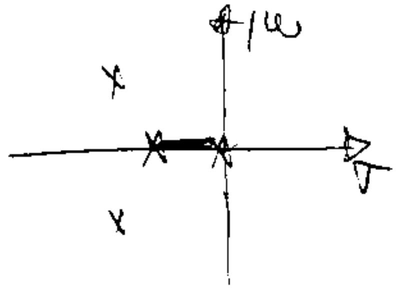
National Brand

#3

$$G(s) = K \frac{1}{s(s+2)(s^2+6s+13)} = K \frac{1}{s^4+8s^3+25s^2+26s}$$

$s = -3 \pm j2$

a. Root-locus plot



Need to determine
→ Breakaway pt
→ Asymptotes
→ Imag. axis crossing
→ Angle of departure

→ Breakaway pt → $\frac{dK}{ds} = 0$

$1 + GH = 0$

$$1 + K \frac{1}{s^4+8s^3+25s^2+26s} = 0$$

$$-K = s^4 + 8s^3 + 25s^2 + 26s$$

$$-\frac{dK}{ds} = 4s^3 + 24s^2 + 50s + 26$$

$$\frac{dK}{ds} = 0 \Rightarrow 4s^3 + 24s^2 + 50s + 26 = 0$$

Soln. $s = -0.7652$
$s = -2.6174 \pm j1.2820$

Not required

→ Asymptotes →
$$\sigma_a = \frac{\sum p_i - \sum z_i}{n-m}$$
$$\theta_a = \frac{180^\circ + k360^\circ}{n-m}$$

$$\sigma_a = \frac{(0+(-2)+(-3+j2)+(-3-j2))}{4-0} = -2$$

$$\theta_a = \frac{180^\circ + k360^\circ}{4-0} = \pm 45^\circ, \pm 135^\circ$$

→ Imag. axis crossing → Routh-Hurwitz criterion

$$1 + GH = 0$$
$$1 + K\frac{1}{s^4+8s^3+25s^2+26s} = 0$$
$$s^4+8s^3+25s^2+26s+K=0$$

s^4	1	25	K
s^3	8	26	
s^2	$25-\frac{26}{8}=21.75$	K	
s	$26-\frac{8K}{21.75}$		
1	K		

s^2 row: $21.75s^2+K=0$

s row: $26-\frac{8K}{21.75}=0 \Rightarrow K=70.69$

$$21.75s^2+70.69=0$$
$$s = \pm j1.8$$

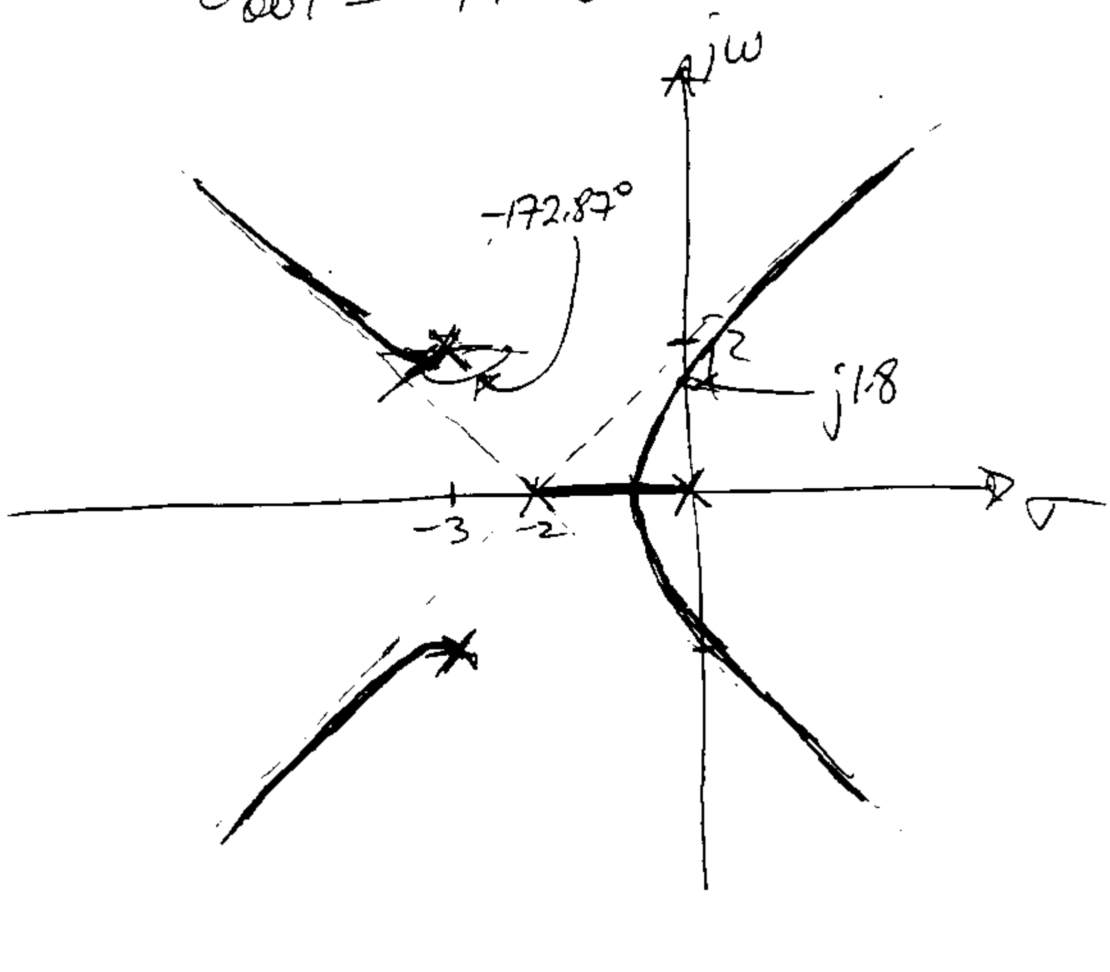
→ Angle of departure → Ang-cond about $s_p = -3+j2$

$$-\angle s_p - \angle(s_p+2) - \angle(s_p+3+j2) - \theta_{out} = 180^\circ + k360^\circ$$

$$-\tan^{-1}\frac{2-0}{-3-0} - \tan^{-1}\frac{2-0}{-3-(-2)} - 90^\circ - \theta_{out} = 180^\circ + k360^\circ$$

$$-146.31^\circ - 116.57^\circ - 90^\circ - \theta_{out} = 180^\circ + k360^\circ$$

$$\theta_{out} = -172.87^\circ$$



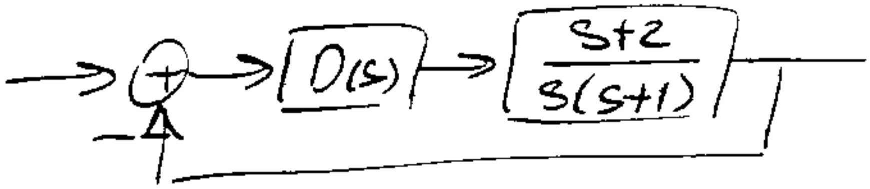
b)ii For stability Routh-Hurwitz table should have positive first column

$s \Rightarrow 26 - \frac{8K}{21.75} > 0 \Rightarrow K < 70.69$

$1 \Rightarrow K \Rightarrow K > 0$

so $0 < K < 70.69$

#4

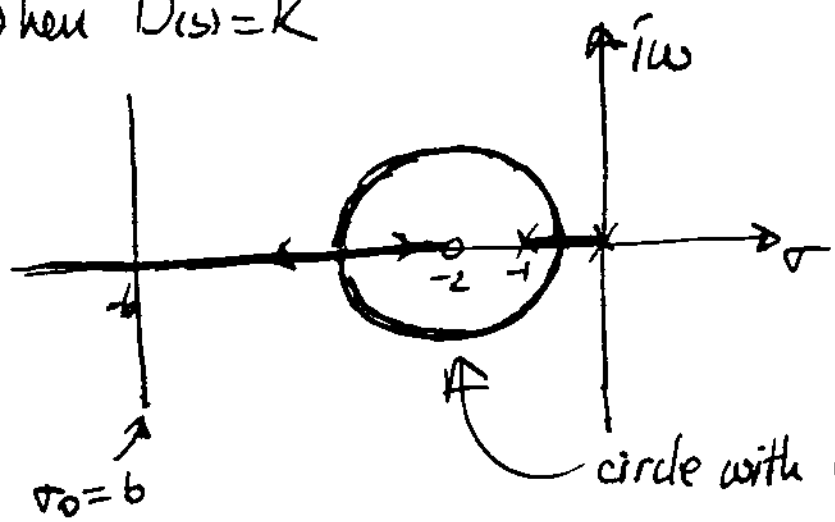


$e_{ss}=0$ for step input $\Rightarrow GH = \frac{1}{s} G_1(s)$

we have already have this term in GH, so there is no need for D(s) to supply it.

$t_{5\%s} \approx 0.5 \Rightarrow \frac{3}{\sigma_0} \approx 0.5 \Rightarrow \sigma_0 = 6$
for a 2nd order system

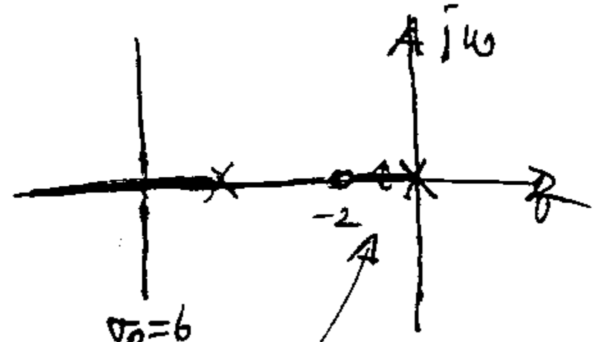
When $D(s)=K$



radius $= \sqrt{(-2-0)(-2-(-1))}$
$= 1.4142$

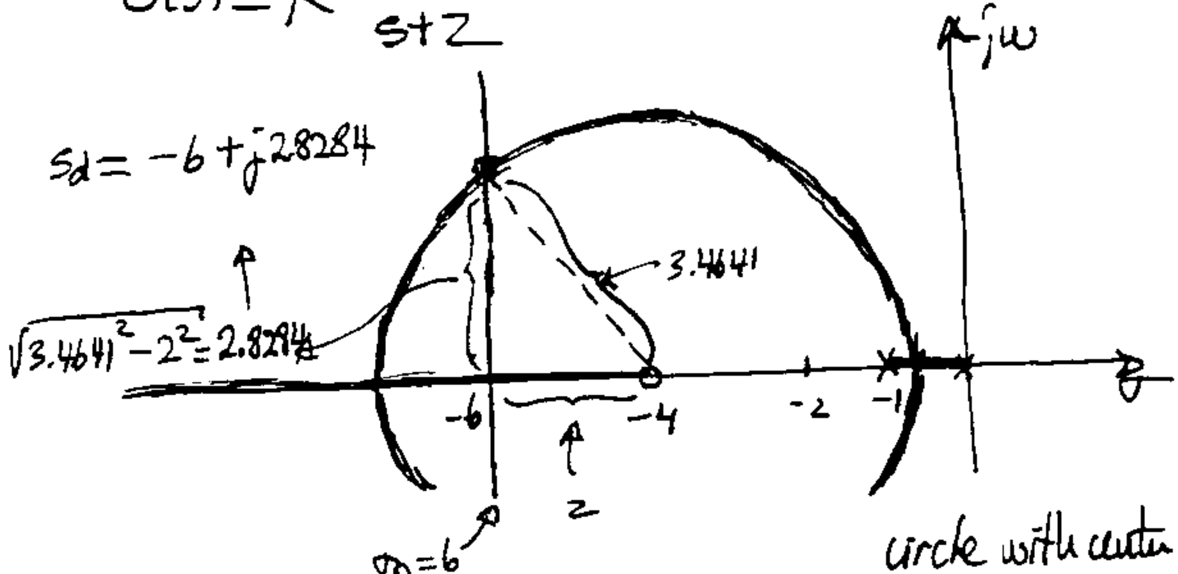
Since for $t_{5\%s} \approx 0.5$, one pole should have $\sigma_0 \approx 6$ and the other one $\sigma_0 \geq 6$. This condition cannot be satisfied from the above root-locus diagram, since one poles approaches to -2.

Canceling one pole and placing it any stable location does not help since the root locus becomes



So cancel the zero, and place it such that the circle intersect with the $\sigma_0=6$. One such choice is a zero at -4, or

$$D(s)=K\,\frac{s+4}{s+2}$$



K from the magnitude condition

$$|GH|_{s=s_d}=1 \Rightarrow \left|K\,\frac{s+4}{s(s+2)}\right|_{s=-6+j2.8284}=1$$

$$\Rightarrow K=9.38 \quad \text{or} \quad D(s)=9.38\,\frac{s+4}{s+2}$$