EE 431 | Exam#1 | Oct. 30, 2001

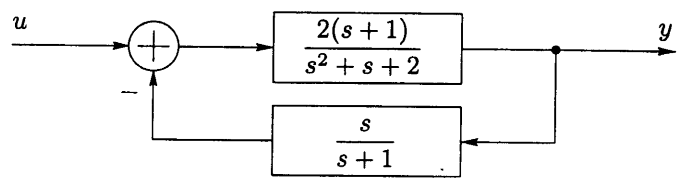
75 minutes

1. Assume

$$\mathbf{a} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \text{ and } \mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Determine a set of vectors that span the subspace that is orthogonal to the subspace spanned by the vectors **a**, **b**, and **c**. (15pts)

2. The block diagram of a control system is given below.



Obtain a state-space representation of the system without any block-diagram reduction. (20pts)

3. A control system is described in state-space representation, such that

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Bu(t),$$

where u and $\mathbf{x}$ are the input and the state variables, respectively.

(a) Determine $\mathbf{x}(1)$, when

$$A = \begin{bmatrix} -3 & -5 \\ 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \text{ and } u(t) = 0 \text{ for } t \geq 0.$$

(25pts)

(b) Determine $\mathbf{x}(1)$, when

$$A = \begin{bmatrix} -3 & -4 \\ 4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } u(t) = 1 \text{ for } t \geq 0.$$

(25pts)

4. A control system is described in state-space representation, such that

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 5 & -2 \\ 8 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 2 \\ 4 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} -5 & 3 \end{bmatrix} \mathbf{x}(t) + u(t),$$

where u, $\mathbf{x}$, and y are the input, the state, and the output variables, respectively.
Determine the transfer function or the transfer matrix of the system. (15pts)

EE 431 — Exam#1 Solutions — Oct. 30, 2001

1. Assume

$$\mathbf{a} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \text{ and } \mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Determine a set of vectors that span the subspace that is orthogonal to the subspace spanned by the vectors **a**, **b**, and **c**.

Solution: Since, the vectors **a**, **b**, and **c** are from a 3-dimensional Euclidean vector space, there can only be three linearly-independent vectors. If **a**, **b**, and **c** are linearly independent, then there is no orthogonal subspace to **a**, **b**, and **c**. However in our case,

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = (-3) \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + (2) \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix},$$

or $\mathbf{c} = (-3)\mathbf{a} + (2)\mathbf{b}$; and **a** and **b** are linearly independent. As a result, there should be one vector **n** that is orthogonal to **a** and **b** (and **c**). To find this vector **n**, we may start with the usual basis vectors

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \text{ and } \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};$$

and remove the components of **a** and **b** from each of the vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$, until we get a non-zero vector. Once we get one non-zero vector, we don't need to proceed any further. But first, we need to orthogonalize the vectors **a** and **b**. From the Gram-Schmidt orthogonalization procedure, we get

$$\mathbf{b}' = \mathbf{b} - \frac{\langle \mathbf{b}, \mathbf{a} \rangle}{\langle \mathbf{a}, \mathbf{a} \rangle}\mathbf{a} = \mathbf{b} - \frac{\mathbf{b}^T\mathbf{a}}{\mathbf{a}^T\mathbf{a}}\mathbf{a}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \frac{(2)(1) + (1)(0) + (1)(1)}{(1)^2 + (0)^2 + (1)^2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix}.$$

Second, we remove the components of **a** and $\mathbf{b}'$ from $\mathbf{e}_1$. (Note that we could have also chosen the vectors $\mathbf{e}_2$ or $\mathbf{e}_3$. Indeed, if we get the zero vector with our choice of $\mathbf{e}_1$, we will try them one by one.)

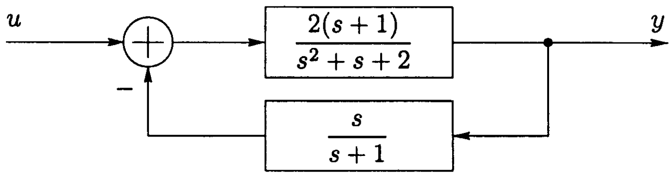
$$\mathbf{n} = \mathbf{e}_1 - \frac{\langle \mathbf{e}_1, \mathbf{a} \rangle}{\langle \mathbf{a}, \mathbf{a} \rangle}\mathbf{a} - \frac{\langle \mathbf{e}_1, \mathbf{b}' \rangle}{\langle \mathbf{b}', \mathbf{b}' \rangle}\mathbf{b}'$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \left(\frac{1}{2}\right) \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \left(\frac{1}{3}\right) \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 1/3 \\ -1/3 \\ -1/3 \end{bmatrix}.$$

Since **n** is non-zero, we have the desired vector. To simplify the appearance of the final result, we may want to scale **n** and describe set of vectors that span the subspace that is orthogonal to the subspace spanned by the vectors **a**, **b**, and **c** as

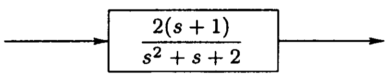
$$\{ [\, 1 \;\; -1 \;\; -1 \,]^T \}.$$

2. The block diagram of a control system is given below.

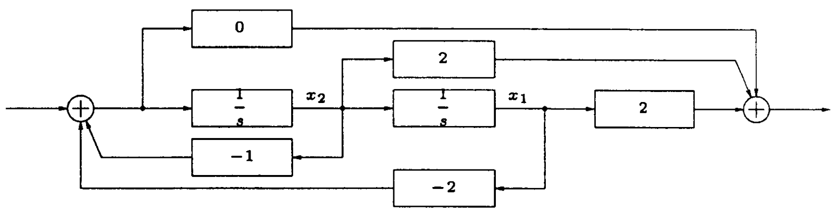


Obtain a state-space representation of the system without any block-diagram reduction.

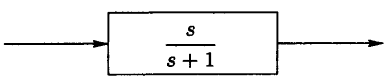
Solution: In order to obtain a state-space representation without any block-diagram reduction or without determining the closed-loop transfer function, we need to realize the individual blocks and use the complete block diagram to generate the state-space equations.



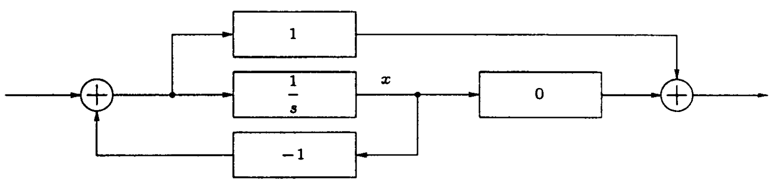
(a) The feedforward gain block.



(b) Controller realization form.

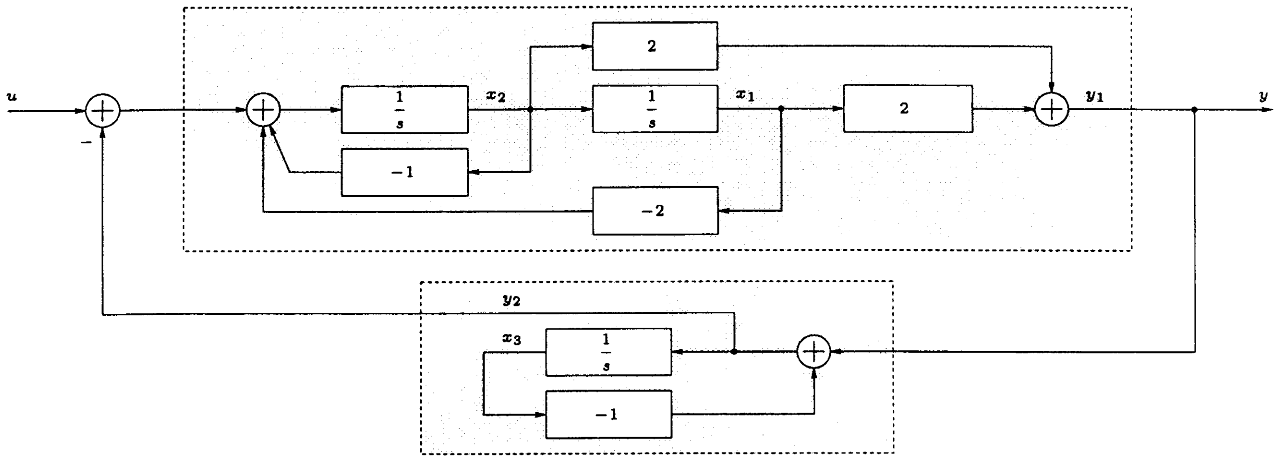


(c) The feedback gain block.



(d) Controller realization form.

The connected and "expanded" block diagram is shown below.



After assigning the state variables as shown in the figure, we obtain

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -2x_1 - x_2 + (u - y_2),$$
$$\dot{x}_3 = -x_3 + y_1,$$

and

$$y = y_1,$$

where

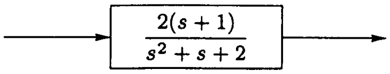
$$y_1 = 2x_1 + 2x_2,$$
$$y_2 = -x_3 + y_1.$$

After eliminating the intermediate variables: y_1 and y_2, we obtain the state-space representation

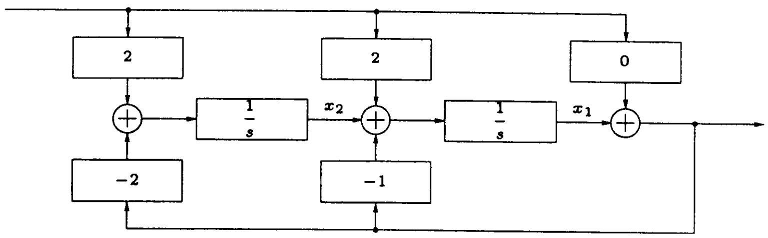
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -3 & 1 \\ 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

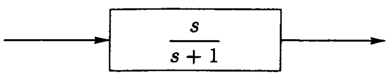
If we use the observer realization form for each of the blocks, then we obtain a different state-space representation.



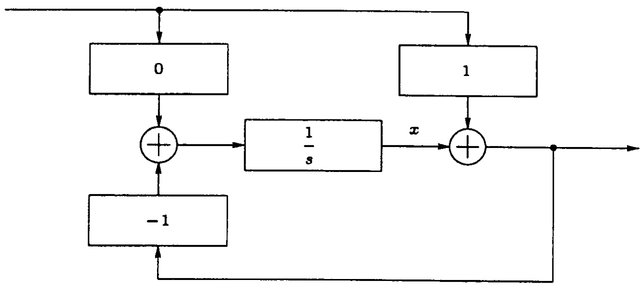
(a) The feedforward gain block.



(b) Observer realization form.

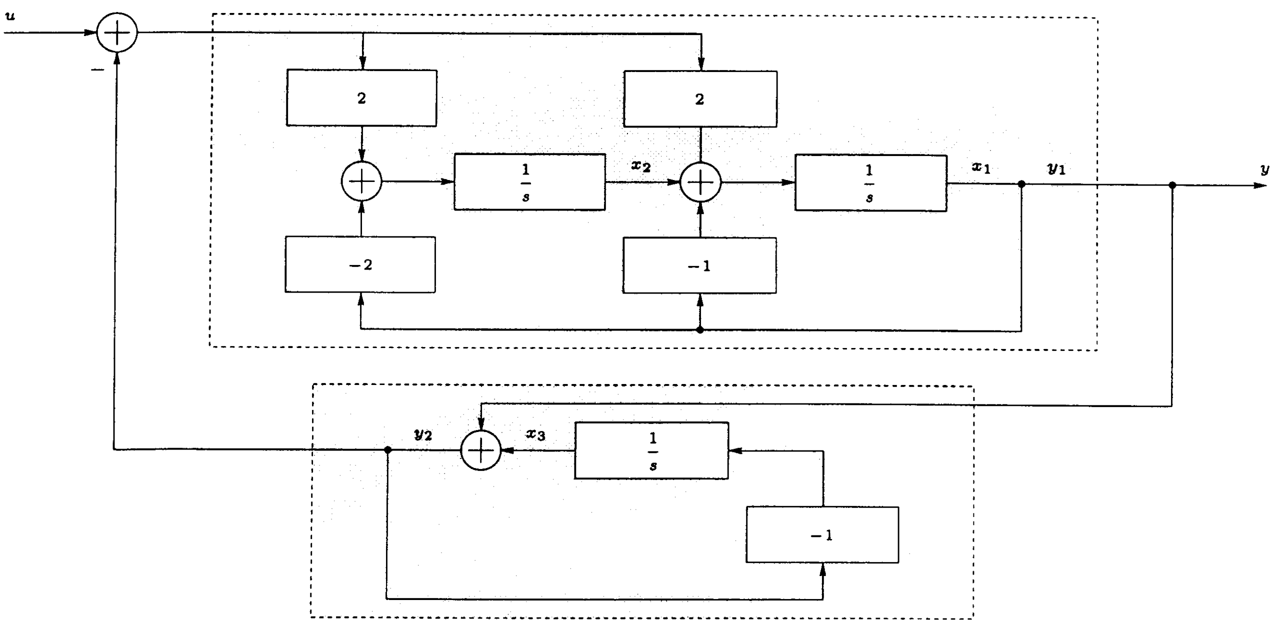


(c) The feedback gain block.



(d) Observer realization form.

The connected and "expanded" block diagram for this case is shown below.



Similarly, we obtain

$$\dot{x}_1 = -y_1 + x_2 + 2(u - y_2),$$
$$\dot{x}_2 = -2y_1 + 2(u - y_2),$$
$$\dot{x}_3 = -y_2,$$

and

$$y = y_1,$$

where

$$y_1 = x_1,$$
$$y_2 = x_3 + y_1.$$

And,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 & -2 \\ -4 & 0 & -2 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

3. A control system is described in state-space representation, such that

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Bu(t),$$

where u and $\mathbf{x}$ are the input and the state variables, respectively.

(a) Determine $\mathbf{x}(1)$, when

$$A = \begin{bmatrix} -3 & -5 \\ 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \text{ and } u(t) = 0 \text{ for } t \geq 0.$$

Solution: The solution to the given control system is given by

$$\begin{aligned} \mathbf{x}(t) &= e^{At}\mathbf{x}(0) + \int_0^t e^{A(t-\tau)} Bu(\tau)\, d\tau \\ &= e^{At}\mathbf{x}(0), \end{aligned}$$

since $u(t) = 0$ for $t \geq 0$. To determine e^{At}, we may use a few different methods. Here, we will only have two of the methods.

Computation of e^{At} using the Cayley-Hamilton theorem:

In this method, we observe that e^{At} may be described by a linear combination of A^k for $k = 0, \ldots, (n-1)$, so that

$$e^{At} = \alpha_0 I + \alpha_1 A + \ldots + \alpha_{n-1} A^{n-1},$$

where I is the appropriately dimensioned identity matrix, n is the dimension of the system, and $\alpha_0, \ldots, \alpha_{n-1}$ are scalars. The scalars are determined by the application of the eigenvectors to the above equation that results in the set of equations

$$\begin{aligned} e^{\lambda_1 t} &= \alpha_0 + \alpha_1 \lambda_1 + \ldots + \alpha_{n-1} \lambda_1^{n-1} \\ &\vdots \\ e^{\lambda_n t} &= \alpha_0 + \alpha_1 \lambda_n + \ldots + \alpha_{n-1} \lambda_n^{n-1}, \end{aligned}$$

where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues, and they are determined from

$$\det(\lambda I - A) = 0.$$

In our case, $n = 2$, so

$$e^{At} = \alpha_0 I + \alpha_1 A,$$

and the eigenvalues are determined from

$$\begin{aligned} \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -5 \\ 4 & 6 \end{bmatrix}\right) = \det \begin{bmatrix} \lambda + 3 & 5 \\ -4 & \lambda - 6 \end{bmatrix} &= \lambda^2 - 3\lambda + 2 \\ &= (\lambda - 1)(\lambda - 2) = 0. \end{aligned}$$

or $\lambda_1 = 1$ and $\lambda_2 = 2$. The set of equations becomes

$$\begin{aligned} e^{(1)t} &= \alpha_0 + \alpha_1(1) \\ e^{(2)t} &= \alpha_0 + \alpha_1(2). \end{aligned}$$

Solving the above set of equations simultaneously gives

$$\begin{aligned} \alpha_0 &= 2e^t - e^{2t} \\ \alpha_1 &= -e^t + e^{2t}. \end{aligned}$$

As a result,

$$\begin{aligned} e^{At} = \alpha_0 I + \alpha_1 A &= (2e^t - e^{2t}) I + (-e^t + e^{2t}) A \\ &= \begin{bmatrix} 5e^t - 4e^{2t} & 5e^t - 5e^{2t} \\ -4e^t + 4e^{2t} & -4e^t + 5e^{2t} \end{bmatrix}. \end{aligned}$$

Computation of e^{At} using the Laplace transform:

In this method, we observe that $e^{At} = \mathcal{L}_s^{-1}\left[(sI - A)^{-1}\right](t)$, where I is the appropriately dimensioned identity matrix.

$$
\begin{aligned}
e^{At} &= \mathcal{L}_s^{-1}\left[(sI-A)^{-1}\right](t) \\
&= \mathcal{L}_s^{-1}\left[\begin{bmatrix} s+3 & 5 \\ -4 & s-6 \end{bmatrix}^{-1}\right](t) \\
&= \mathcal{L}_s^{-1}\begin{bmatrix} \dfrac{s-6}{(s-1)(s-2)} & \dfrac{-5}{(s-1)(s-2)} \\ \dfrac{4}{(s-1)(s-2)} & \dfrac{s+3}{(s-1)(s-2)} \end{bmatrix}(t) \\
&= \mathcal{L}_s^{-1}\begin{bmatrix} \left(\dfrac{5}{s-1}+\dfrac{-4}{s-2}\right) & \left(\dfrac{5}{s-1}+\dfrac{-5}{s-2}\right) \\ \left(\dfrac{-4}{s-1}+\dfrac{4}{s-2}\right) & \left(\dfrac{-4}{s-1}+\dfrac{5}{s-2}\right) \end{bmatrix}(t) \\
&= \begin{bmatrix} 5e^{t}-4e^{2t} & 5e^{t}-5e^{2t} \\ -4e^{t}+4e^{2t} & -4e^{t}+5e^{2t} \end{bmatrix}.
\end{aligned}
$$

Since $\mathbf{x}(t) = e^{At}\mathbf{x}(0)$,

$$
\mathbf{x}(t) = \begin{bmatrix} 5e^{t}-4e^{2t} & 5e^{t}-5e^{2t} \\ -4e^{t}+4e^{2t} & -4e^{t}+5e^{2t} \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} e^{2t} \\ -e^{2t} \end{bmatrix}.
$$

Therefore,

$$
\mathbf{x}(1) = \begin{bmatrix} e^{2} \\ -e^{2} \end{bmatrix} \approx \begin{bmatrix} 7.3891 \\ -7.3891 \end{bmatrix}.
$$

(b) Determine $\mathbf{x}(1)$, when

$$
A = \begin{bmatrix} -3 & -4 \\ 4 & 5 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } u(t) = 1 \text{ for } t \geq 0.
$$

Solution: The solution to the given control system is given by

$$
\begin{aligned}
\mathbf{x}(t) &= e^{At}\mathbf{x}(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)\,\mathrm{d}\tau \\
&= \int_0^t e^{A(t-\tau)}Bu(\tau)\,\mathrm{d}\tau,
\end{aligned}
$$

since $\mathbf{x}(0) = 0$. To determine e^{At}, we may use a few different methods. Here, we will only have two of the methods.

Computation of e^{At} using the Cayley-Hamilton theorem:

In this method, we observe that e^{At} may be described by a linear combination of A^k for $k = 0, \ldots, (n-1)$, so that

$$e^{At} = \alpha_0 I + \alpha_1 A + \ldots + \alpha_{n-1} A^{n-1},$$

where I is the appropriately dimensioned identity matrix, n is the dimension of the system, and $\alpha_0, \ldots, \alpha_{n-1}$ are scalars. The scalars are determined by the application of the eigenvectors to the above equation that results in the set of equations

$$\begin{aligned} e^{\lambda_1 t} &= \alpha_0 + \alpha_1 \lambda_1 + \ldots + \alpha_{n-1}\lambda_1^{n-1} \\ &\vdots \\ e^{\lambda_n t} &= \alpha_0 + \alpha_1 \lambda_n + \ldots + \alpha_{n-1}\lambda_n^{n-1}, \end{aligned}$$

where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues, and they are determined from

$$\det(\lambda I - A) = 0.$$

In our case, $n = 2$, so

$$e^{At} = \alpha_0 I + \alpha_1 A,$$

and the eigenvalues are determined from

$$\begin{aligned} \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -4 \\ 4 & 5 \end{bmatrix}\right) &= \det \begin{bmatrix} \lambda + 3 & 4 \\ -4 & \lambda - 5 \end{bmatrix} \\ &= \lambda^2 - 2\lambda + 1 \\ &= (\lambda - 1)^2 = 0. \end{aligned}$$

or $\lambda_1 = \lambda_2 = 1$. In this case, the set of equations is modified to generate a linearly independent equation for the repeated eigenvalue, such that

$$\begin{aligned} e^{\lambda_1 t} &= \alpha_0 + \alpha_1 \lambda_1 \\ \frac{\mathrm{d}}{\mathrm{d}\lambda_1}\left(e^{\lambda_1 t}\right. &= \left.\alpha_0 + \alpha_1 \lambda_1\right), \end{aligned}$$

or

$$\begin{aligned} e^{\lambda_1 t} &= \alpha_0 + \alpha_1 \lambda_1 \\ t e^{\lambda_1 t} &= \alpha_1. \end{aligned}$$

For $\lambda_1 = 1$, we get

$$\begin{aligned} e^{(1)t} &= \alpha_0 + \alpha_1 (1) \\ t e^{(1)t} &= \alpha_1. \end{aligned}$$

Solving the above set of equations simultaneously gives

$$\begin{aligned} \alpha_0 &= (1 - t)e^t \\ \alpha_1 &= t e^t. \end{aligned}$$

As a result,

$$e^{At} = \alpha_0 I + \alpha_1 A = \big((1-t)e^t\big)I + \big(te^t\big)A$$

$$= \begin{bmatrix} (1-4t)e^t & -4te^t \\ 4te^t & (1+4t)e^t \end{bmatrix}.$$

Computation of e^{At} using the Laplace transform:

In this method, we observe that $e^{At} = \mathcal{L}_s^{-1}\left[(sI-A)^{-1}\right](t)$, where I is the appropriately dimensioned identity matrix.

$$e^{At} = \mathcal{L}_s^{-1}\left[(sI-A)^{-1}\right](t)$$

$$= \mathcal{L}_s^{-1}\left[\begin{bmatrix} s+3 & 4 \\ -4 & s-5 \end{bmatrix}^{-1}\right](t)$$

$$= \mathcal{L}_s^{-1}\begin{bmatrix} \dfrac{s-5}{(s-1)^2} & \dfrac{-4}{(s-1)^2} \\ \dfrac{4}{(s-1)^2} & \dfrac{s+3}{(s-1)^2} \end{bmatrix}(t)$$

$$= \mathcal{L}_s^{-1}\begin{bmatrix} \left(\dfrac{1}{s-1} + \dfrac{-4}{(s-1)^2}\right) & \dfrac{-4}{(s-1)^2} \\ \dfrac{4}{(s-1)^2} & \left(\dfrac{1}{s-1} + \dfrac{4}{(s-1)^2}\right) \end{bmatrix}(t)$$

$$= \begin{bmatrix} e^t - 4te^t & -4te^t \\ 4te^t & e^t + 4te^t \end{bmatrix}.$$

Since $\mathbf{x}(t) = \int_0^t e^{A(t-\tau)}Bu(\tau)\,\mathrm{d}\tau$, and $u(t) = 1$ for $t \geq 0$;

$$\mathbf{x}(t) = \int_0^t \begin{bmatrix} (1-4(t-\tau))e^{(t-\tau)} & -4(t-\tau)e^{(t-\tau)} \\ 4(t-\tau)e^{(t-\tau)} & (1+4(t-\tau))e^{(t-\tau)} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}(1)\,\mathrm{d}\tau$$

$$= \int_0^t \begin{bmatrix} (1-4(t-\tau))e^{(t-\tau)} \\ 4(t-\tau)e^{(t-\tau)} \end{bmatrix}\mathrm{d}\tau$$

$$= \begin{bmatrix} \int_0^t (1-4\xi)e^{\xi}\,\mathrm{d}\xi \\ 4\int_0^t \xi e^{\xi}\,\mathrm{d}\xi \end{bmatrix}$$

$$= \begin{bmatrix} \left[(1-4(\xi-1))e^{\xi}\right]_{\xi=0}^{t} \\ 4\left[(\xi-1)e^{\xi}\right]_{\xi=0}^{t} \end{bmatrix}.$$

Therefore, for $t = 1$ we get

$$\mathbf{x}(1) = \begin{bmatrix} e - 5 \\ 4 \end{bmatrix} \approx \begin{bmatrix} -2.2817 \\ 4 \end{bmatrix}.$$

4. A control system is described in state-space representation, such that

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 5 & -2 \\ 8 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 2 \\ 4 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} -5 & 3 \end{bmatrix} \mathbf{x}(t) + u(t),$$

where u, $\mathbf{x}$, and y are the input, the state, and the output variables, respectively. Determine the transfer function or the transfer matrix of the system.

Solution: The transfer matrix of a control system described in the state-state representation

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t),$$

$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t),$$

is

$$F(s) = C(sI - A)^{-1}B + D,$$

where

$$A = \begin{bmatrix} 5 & -2 \\ 8 & -3 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

$$C = \begin{bmatrix} -5 & 3 \end{bmatrix}, \qquad D = 1,$$

and I is the appropriately dimensioned identity matrix. So,

$$F(s) = \begin{bmatrix} -5 & 3 \end{bmatrix} \left(s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 5 & -2 \\ 8 & -3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 2 \\ 4 \end{bmatrix} + 1$$

$$= \begin{bmatrix} -5 & 3 \end{bmatrix} \left(\begin{bmatrix} s-5 & 2 \\ -8 & s+3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 2 \\ 4 \end{bmatrix} + 1$$

$$= \frac{1}{(s-5)(s+3) - (-8)(2)} \begin{bmatrix} -5 & 3 \end{bmatrix} \begin{bmatrix} s+3 & -2 \\ 8 & s-5 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} + 1$$

$$= \frac{1}{(s-1)^2} (2s - 2) + 1$$

$$= \frac{2}{s-1} + 1.$$

Therefore, the transfer matrix is

$$F(s) = \frac{s+1}{s-1}.$$