Problems from Math 5222 Lecture 2

Problem

✓ If $\mathbf{x}$: $(x_1, x_2, \ldots, x_n)$ is a unit vector and $Q = a_{ij}x_ix_j$ is a real symmetric quadratic form with nonsingular matrix A, then the extreme values of Q are the characteristic values of A. Prove it. *Hint:* Maximize Q subject to the constraining condition $x_ix_i = 1$ and deduce the system of equations $(a_{ij} - \delta_{ij}\lambda)x_i = 0$, where λ is the Lagrange multiplier. p. 39

Problems

✓1. Reduce the matrix

$$A = (a_{ij}) = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

to the diagonal form S by the similitude transformation $C^{-1}AC$. Show that

$$C = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad C^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad \text{and } S = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}.$$ p. 49

Discuss the meaning of A when it is viewed as an operator characterizing the deformation of space.

✓2. Diagonalize the matrices:

$$\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{pmatrix}.$$

Problem

✓ Discuss the transformations in which the coordinates y^i are rectangular cartesian:

(a)
$$\begin{aligned} y^1 &= \frac{1}{\sqrt{6}}x^1 + \frac{2}{\sqrt{6}}x^2 + \frac{1}{\sqrt{6}}x^3, \\ y^2 &= \frac{1}{\sqrt{2}}x^1 - \frac{1}{\sqrt{3}}x^2 + \frac{1}{\sqrt{3}}x^3, \\ y^3 &= \frac{1}{\sqrt{2}}x^1 \qquad\qquad - \frac{1}{\sqrt{2}}x^3. \end{aligned}$$ p. 52

(b)
$$\begin{aligned} y^1 &= x^1 \cos x^2, \\ y^2 &= x^1 \sin x^2, \\ y^3 &= x^3. \end{aligned}$$