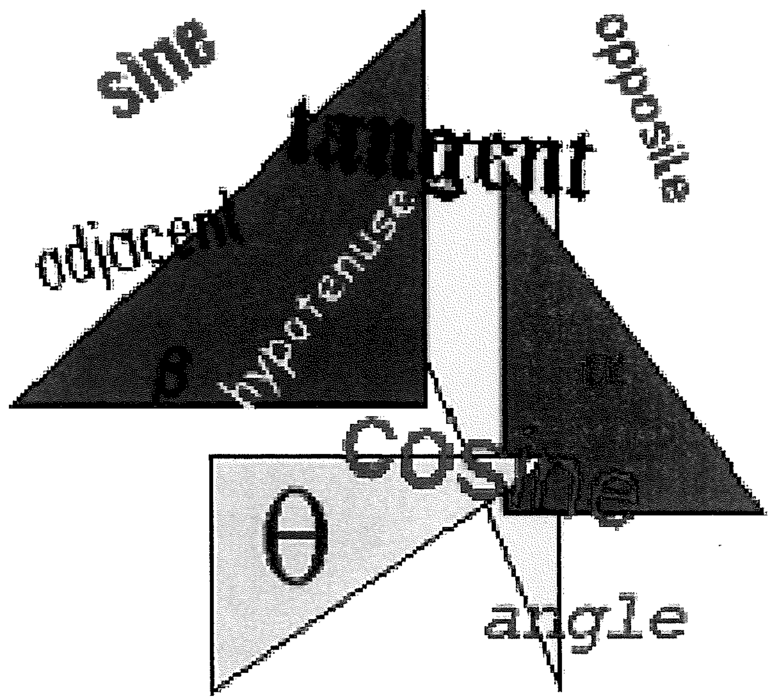
TRIG REVIEW 2014



Missouri University of Science & Technology

August 18-22, 2014

Section 1: Angles and Trigonometric Functions

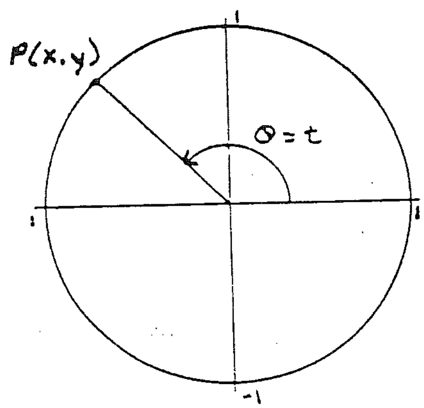
An angle in standard position is one which has its vertex at the origin, and initial side the positive x-axis. An angle has positive (negative) measure if the terminal side is obtained by rotating a copy of the initial side in the counterclockwise (clockwise) direction.

Angles are measured in degrees or radians. One radian is the measure of the central angle of a circle subtended by an arc equal in length to the radius of the circle.

Length of a Circular Arc: If an arc of length s on a circle of radius r subtends a central angle of radian measure θ, then $s = r\theta$.

$180° = \pi$ radians.

Consider the unit circle. For each real number t there corresponds an angle in standard position of radian measure t. The terminal side of this angle intersects the unit circle at a point say $P(x, y)$. Thus, for any real number t there corresponds a point $P(x, y)$ on the unit circle.



Trigonometric Functions in Terms of a Unit Circle: If t is a real number and $P(x, y)$ is the point on the unit circle that corresponds to t, then

$\sin t = y$ $\qquad$ $\csc t = \frac{1}{y}$ (if $y \neq 0$)

$\cos t = x$ $\qquad$ $\sec t = \frac{1}{x}$ (if $x \neq 0$)

$\tan t = \frac{y}{x}$ (if $x \neq 0$) $\qquad$ $\cot t = \frac{x}{y}$ (if $y \neq 0$)

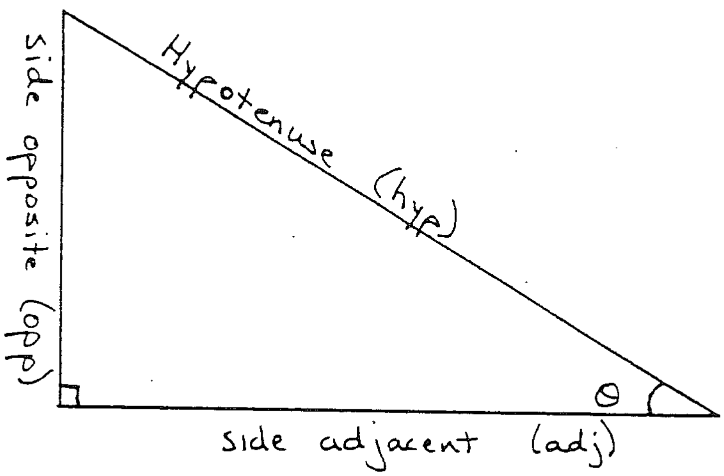
Section 2: Right Triangle Trigonometry

Let θ be an acute angle of a right triangle. The six trigonometric functions of the angle θ are

$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}.$$



Two positive angles are complementary if their sum is $\frac{\pi}{2}$. Two positive angles are supplementary if their sum is π.

Cofunctions of complementary angles are equal.

Let θ be an angle in standard position. Its reference angle is the acute angle formed by the terminal side of θ and the x-axis.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$

Section 3: Trigonometric Identities

A trigonometric identity is an equation which is true for all angles in the domain of the trigonometric functions involved in the equation.

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$
$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$
$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$
$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$
$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$
$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$$2\sin A\cos B=\sin(A+B)+\sin(A-B)$$
$$2\cos A\sin B=\sin(A+B)-\sin(A-B)$$
$$2\cos A\cos B=\cos(A+B)+\cos(A-B)$$
$$2\sin A\sin B=-\cos(A+B)+\cos(A-B)$$
$$\sin A+\sin B=2\sin\left[\tfrac{1}{2}(A+B)\right]\cos\left[\tfrac{1}{2}(A-B)\right]$$
$$\sin A-\sin B=2\cos\left[\tfrac{1}{2}(A+B)\right]\sin\left[\tfrac{1}{2}(A-B)\right]$$
$$\cos A+\cos B=2\cos\left[\tfrac{1}{2}(A+B)\right]\cos\left[\tfrac{1}{2}(A-B)\right]$$
$$\cos A-\cos B=-2\sin\left[\tfrac{1}{2}(A+B)\right]\sin\left[\tfrac{1}{2}(A-B)\right]$$

$$\sin 2A=2\sin A\cos A$$
$$\begin{aligned}\cos 2A&=\cos^2 A-\sin^2 A\\&=1-2\sin^2 A\\&=2\cos^2 A-1\end{aligned}$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

$$\sin\tfrac{1}{2}A=\pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\tfrac{1}{2}A=\pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\tfrac{1}{2}A=\pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\tan\tfrac{1}{2}A=\frac{1-\cos A}{\sin A}$$

$$\tan\tfrac{1}{2}A=\frac{\sin A}{1+\cos A}$$

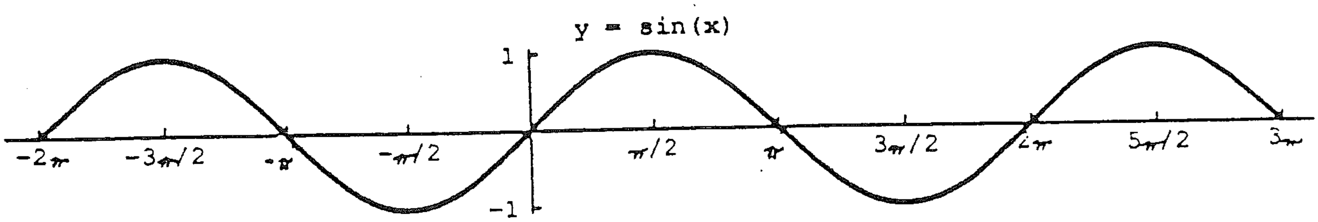
Section 4: Graphs of the Trigonometric Functions

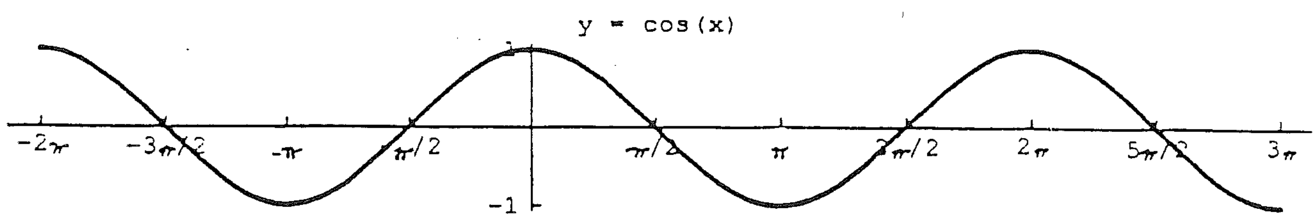
A function f is periodic if there exists a positive real number c such that $f(t+c)=f(t)$ for all t in the domain of f. The least number c for which f is periodic is the period of f.

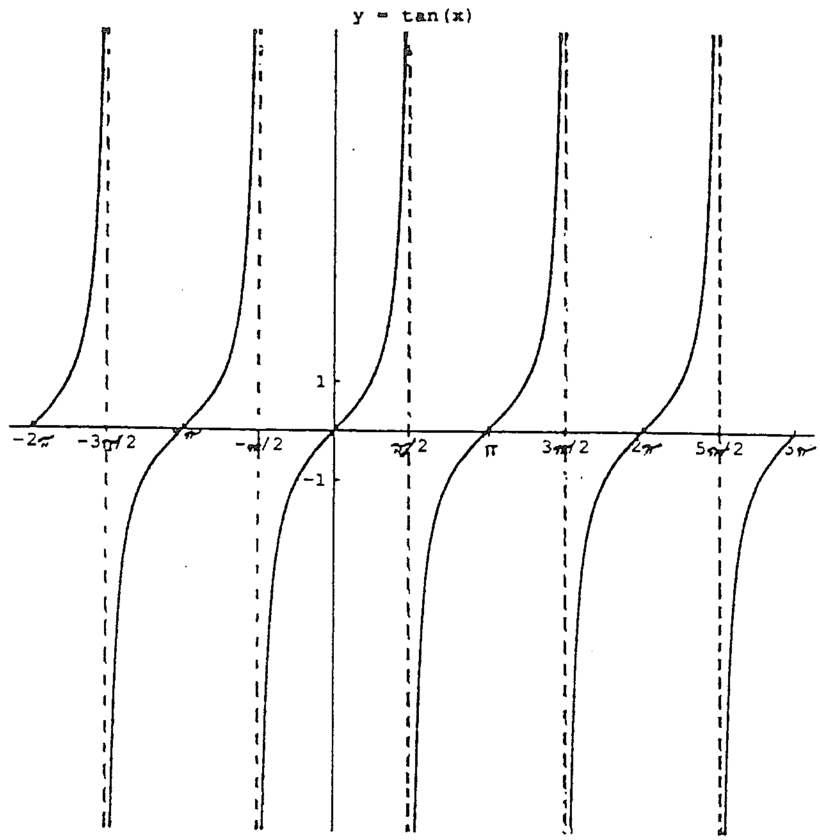
The sine, cosine, secant, and cosecant functions are periodic with period 2π.
The tangent and cotangent functions are periodic with period π.

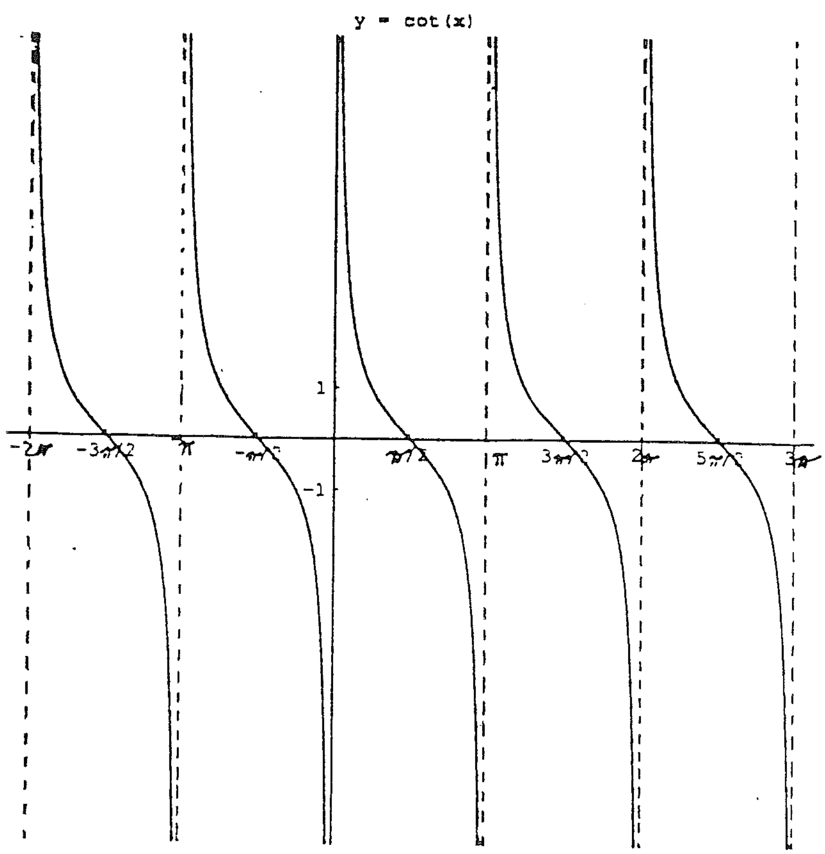
The graphs of the six trigonometric functions are given below.

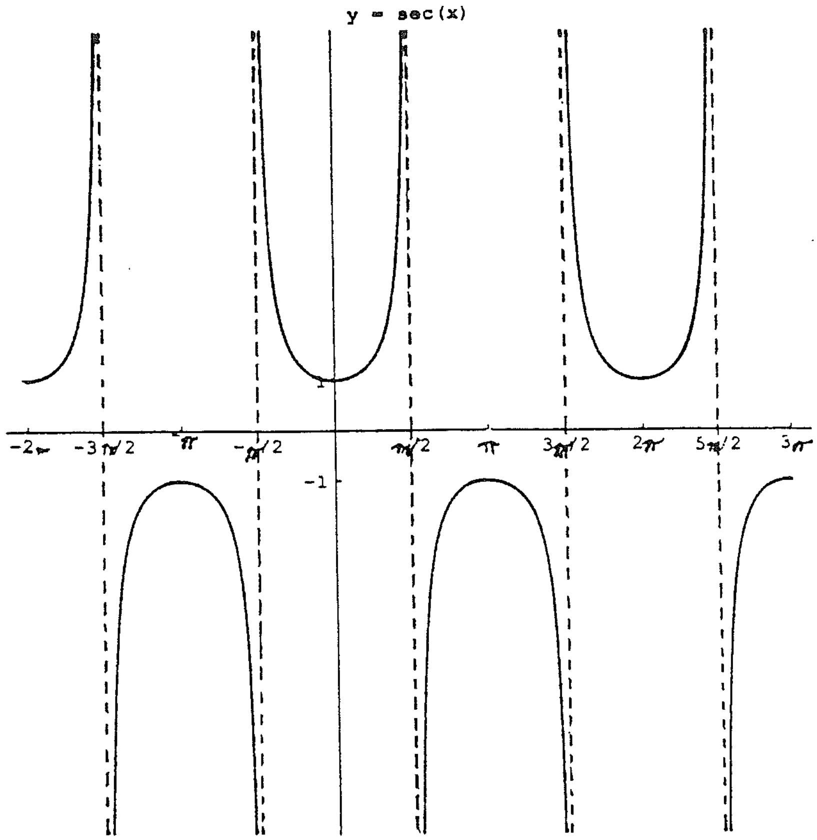
The graphs of $y=a\sin(bx-c)$ and $y=a\cos(bx-c)$ $(b>0)$ have amplitude $|a|$ and period $\frac{2\pi}{b}$.
The "beginning" of a cycle can be found by solving the equation $bx-c=0$. Dividing the period by 4 gives the distance between intercepts and maximums or minimums.

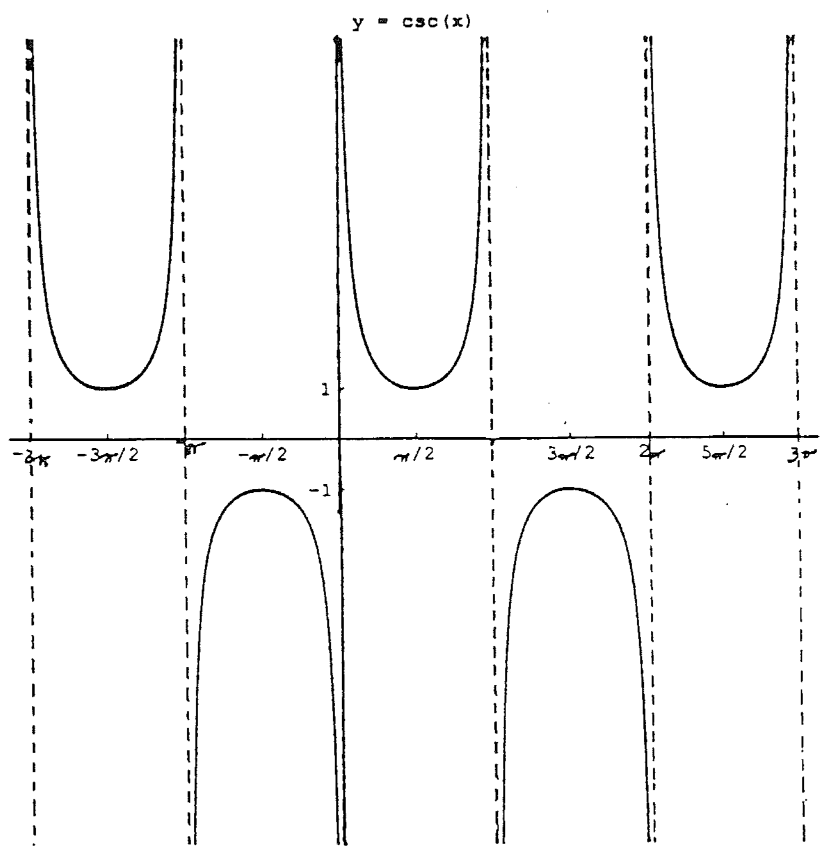





y = tan(x)
1
-1
-2π
-3π/2
-π
-π/2
π/2
π
3π/2
2π
5π/2
3π


y = cot(x)
1
-1
-2π
-3π/2
-π
-π/2
π/2
π
3π/2
2π
5π/2
3π


y = sec(x)
1
-1
-2π
-3π/2
π
-π/2
π/2
π
3π/2
2π
5π/2
3π


y = csc(x)
1
-1
-2π
-3π/2
π
-π/2
π/2
3π/2
2π
5π/2
3π

Section 5: Trigonometric Equations and Inverse Functions

A trigonometric equation is an equation which is true for some angles. To solve a trigonometric equation is to find the angles for which the equation holds.

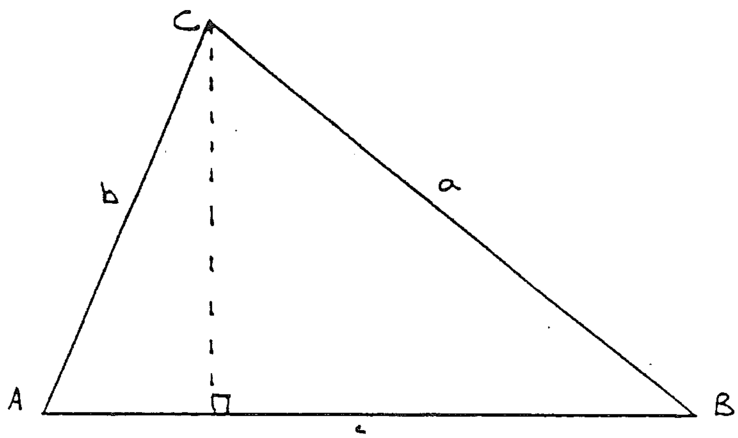
The domain of the inverse sine function is [-1, 1] and its range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. $y = \mathrm{Sin}^{-1}x$ if and only if $\sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

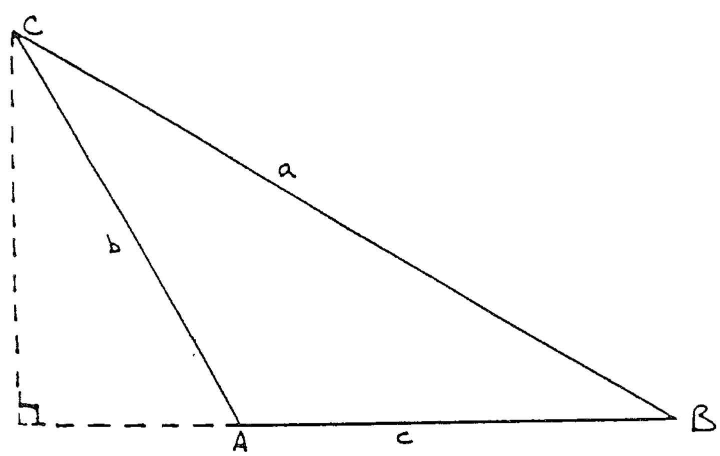
The domain of the inverse cosine function is [-1, 1] and its range is $[0, \pi]$. $y = \mathrm{Cos}^{-1}x$ if and only if $\cos y = x$ and $0 \le y \le \pi$.

The domain of the inverse tangent function is the entire real line and its range is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. $y = \mathrm{Tan}^{-1}x$ if and only if $\tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Section 6: Oblique Triangles

An oblique triangle is a triangle which does not contain a right angle. The laws given refer to triangles labeled in this manner.





Law of Sines: If ABC is a triangle with sides a, b, and c, then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Law of Cosines: If ABC is a triangle with sides a, b, and c, then $a^2 = b^2 + c^2 - 2bc\cos A$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C.$$

The area of a triangle equals one-half the product of the lengths of any two sides and the sine of the angle between them.

The area A of a triangle with sides a, b, and c is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$. This is know as Heron's Formula.

Section 7: Complex Numbers

The imaginary unit is i and has the property that $i^2 = -1$.

A number of the form $a + bi$, with a and b real constants, is called a complex number. The number a is called the real component and the number b is called the imaginary component.

Two complex numbers are equal if and only if the real components are equal and the imaginary components are equal.

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$
$$(a+bi)(c+di)=(ac-bd)+(bc+ad)i$$
$$(a+bi)-(c+di)=(a-c)+(b-d)i$$
$$\frac{a+bi}{c+di}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

The complex number $a+bi$ is given in trigonometric form by $a+bi = r(\cos\theta + i\sin\theta)$ where $r=\sqrt{a^2+b^2}$ and $\tan\theta = \frac{b}{a}$. We call r the modulus, or absolute value, of the complex number and θ the argument, or amplitude, of the complex number.

The modulus of the product of two complex numbers is equal to the product of their modulus. The argument of the product of two complex number is equal to the sum of their arguments.

The modulus of the quotient of two complex numbers is the quotient of their modulus. The argument of the quotient of two complex numbers is the argument of the dividend minus the argument of the divisor.

A nonzero number $r(\cos\theta + i\sin\theta)$ has n nth roots which are given by the formula

$$\sqrt[n]{r}\left(\cos\frac{\theta+2\pi k}{n}+i\sin\frac{\theta+2\pi k}{n}\right)$$

where $k = 0,1,2,\ldots,n-1$.

Exercises: Section 1

1) Express each of the following angles in terms of π radians.

a) 20°	d) 80°	g) -21°	j) 450°
b) 35°	e) 210°	h) 118°	k) -75°
c) -135°	f) -330°	i) 27°	l) 72°

2) Change each angle from radian measure to degrees.

a) $\frac{7\pi}{6}$	d) $-\frac{\pi}{4}$	g) $\frac{2\pi}{15}$	j) $\frac{5\pi}{12}$
b) $-\frac{7\pi}{18}$	e) $\frac{\pi}{30}$	h) $\frac{5\pi}{24}$	k) $\frac{4\pi}{3}$
c) $\frac{3\pi}{10}$	f) $\frac{2\pi}{9}$	i) $-\frac{7\pi}{36}$	l) $\frac{13\pi}{6}$

3) On a circle of radius r find the length of the arc intercepted by the central angle θ.

a) $r = 15$ inches $\quad \theta = 180°$

b) $r = 40$ cm $\quad \theta = \frac{3\pi}{4}$ radians

c) $r = 9$ ft $\quad \theta = 60°$

4) The pendulum of a clock swings through an angle of 10°. Find the distance which the tip travels in one swing if the length of the pendulum is 30 inches.

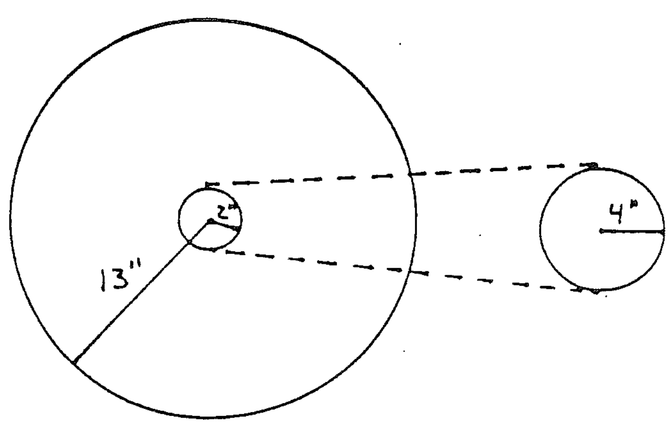
5) Approximate the radian measure of the central angle of a circle subtended by an arc of length 7 cm on a circle of radius 4 cm.

6) A typical tire for a compact car is 22 inches in diameter. If the car is traveling at a rate of 60 mi/hr, find the number of revolutions the tire makes per minute.

7) The radii of the sprocket assemblies and the wheel of the bicycle are 4 inches, 2 inches, and 13 inches, respectively (see diagram). If the cyclist is pedaling at the rate of 1 revolution per second, find the speed of the bicycle in

a) feet per second and

b) miles per hour.



8) A railroad curve is to be laid out on a circle. What radius should be used if the track is to change its direction 6° in a distance of 25 m?

9) Assuming the earth to be a sphere of radius 3960 miles, find the distance of a point in latitude 36°N from the equator.

10) Determine the speed of the earth (in miles per second) in its course around the sun. Assume the earth's orbit to be a circle of radius 93,000,000 miles and 1 yr = 365 days.

11) A train is traveling at the rate 12 mi/hr on a curve of radius 3000 ft. Through what angle has it turned in one minute?

12) Let each of the following angles be in standard position and name the quadrant in which the terminal side lies.

a) $-72°$	d) $-\frac{11\pi}{3}$	g) $-\frac{17\pi}{4}$	j) $75°$
b) $\frac{7\pi}{6}$	e) $230°$	h) $165°$	k) $\frac{5\pi}{4}$
c) $-\frac{3\pi}{5}$	f) $-154°$	i) $350°$	l) 5π

13) Place each of the following angles in standard position; draw a curved arrow to indicate the rotation. Draw and find the size of two other angles, one positive and one negative, that are coterminal with the given angle.

a) $70°$	d) $700°$	g) $\frac{11\pi}{9}$
b) $550°$	e) $\frac{3\pi}{2}$	h) $\frac{37\pi}{18}$
c) $110°$	f) 4π	

14) Name the quadrant or quadrants in which the terminal side of an angle in standard position may be and satisfy the given condition or conditions.

a) The sine is negative	e) The tangent is positive
b) The cosine is positive	f) The secant is negative
c) $\sin\theta < 0,\ \cos\theta < 0$	g) $\sin\theta > 0,\ \tan\theta < 0$
d) $\cos\theta < 0,\ \cot\theta > 0$	h) $\cot\theta > 0,\ \sec\theta > 0$

15) For each of the following find the values of the other trigonometric functions.

a) $\sin\theta = \frac{\sqrt{5}}{5}, \cos\theta < 0$	f) $\tan\theta = \frac{3}{2}, \sin\theta = \frac{3}{\sqrt{13}}$
b) $\sec\theta = -\frac{\sqrt{7}}{2}, \tan\theta = \frac{\sqrt{3}}{2}$	g) $\sin\theta = \frac{2}{5}, \cos\theta > 0$
c) $\cos\theta = \frac{3}{5}$, θ in quadrant IV	h) $\cos\theta = -\frac{1}{3}, \sin\theta < 0$
d) $\tan\theta = -\frac{24}{7}$, θ in quadrant II	i) $\cot\theta = \frac{3}{4}, \cos\theta < 0$
e) $\cos\theta = -\frac{1}{2}, \cot\theta = \frac{\sqrt{3}}{3}$	j) $\cos\theta = -\frac{12}{13}$, θ in quadrant II

16) Construct two angles in standard position which have the given function value and different terminal sides, and write the values of the other trigonometric function of each angle.

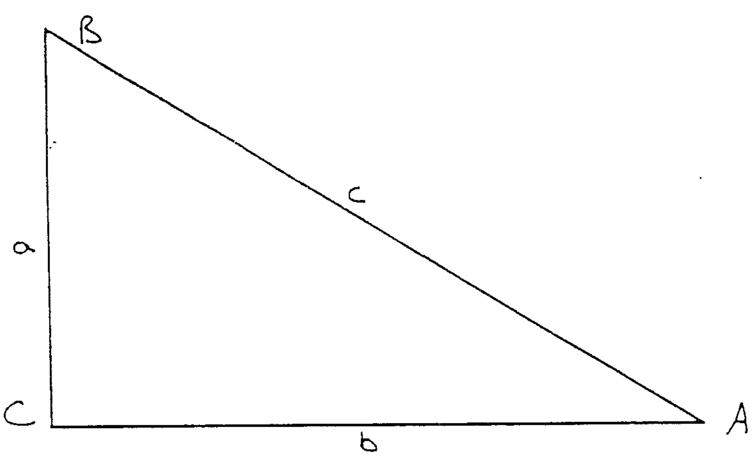
a) $\cos\theta = \frac{9}{41}$	d) $\cos\theta = \frac{24}{25}$	g) $\cot\theta = \frac{3}{5}$
b) $\csc\theta = \frac{29}{21}$	e) $\cot\theta = -\frac{1}{4}$	h) $\sin\theta = \frac{1}{2}$
c) $\sin\theta = -\frac{2}{3}$	f) $\sec\theta = -2$	i) $\tan\theta = -\frac{7}{24}$

Exercises: Section 2

1) Draw each angle in standard position and find the reference angle.

a) 150°	d) 244°	g) -320°	j) 440°
b) -318°	e) -234°	h) 134°	k) -68°
c) $\frac{4\pi}{5}$	f) $\frac{7\pi}{6}$	i) $\frac{7\pi}{5}$	l) $-\frac{9\pi}{4}$

2) Draw a right triangle in the form given and write the values of the trigonometric functions of angles A and B.



a) a = 5, b = 12
b) a = 3, b = 7
c) a = 24, c = 25
d) b = 1, c = 4
e) a = 35, c = 37
f) a = 7, b = 8
g) A = 30°, b = 20
h) B = 45°, b = 35
i) $b = 5\sqrt{3}$, $c = 10\sqrt{3}$
j) $b = 7\sqrt{2}$, $c = 14$

3) Find θ if functions of positive acute angles are involved in each equation.

a) $\cot\theta = \tan\frac{\pi}{5}$
b) $\cos 3\theta = \sin 2\theta$
c) $\sin\frac{1}{2}\theta = \cos\frac{1}{4}\theta$
d) $\csc\left(\theta + \frac{\pi}{4}\right) = \sec 3\theta$
e) $\tan\left(3\theta + \frac{\pi}{9}\right) = \cot\left(\theta + \frac{\pi}{4}\right)$
f) $\sec\left(2\theta + \frac{2\pi}{7}\right) = \csc\left(\theta + \frac{\pi}{7}\right)$

4) Evaluate each of the following expressions without using a calculator.

a) $\cos\frac{\pi}{6}\sin\frac{\pi}{6}/\cos\frac{\pi}{3}-\sin\frac{\pi}{3}$

b) $\cos\frac{\pi}{3}\sin\frac{\pi}{4}-\cos\frac{\pi}{4}\sin\frac{\pi}{3}$

c) $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}}$

d) $\dfrac{1+\cot\frac{\pi}{6}\cot\frac{\pi}{4}}{\cot\frac{\pi}{4}-\cot\frac{\pi}{6}}$

5) Find the length of a side of an equilateral triangle which is inscribed in a circle of radius 10.4 feet.

6) Find the perimeter of a regular octagon which is

a) inscribed in a circle of radius 21 inches.
b) circumscribed about the circle.

7) A 20 ft ladder leaning against the side of a house makes a 75° angle with the ground. How far up the side of the house does the ladder reach?

8) A 6 ft person standing 12 ft from a streetlight casts an 8 ft shadow. What is the height of the streetlight?

9) A ramp $17\frac{1}{2}$ feet in length rises to a loading platform that is $3\frac{1}{3}$ feet off the ground. Find the angle θ that the ramp makes with the ground.

10) The bearing from A to C is S 52°E. The bearing from A to B is N 84°E. The bearing from B to C is S 38°W. A plane flying 250 km/h takes 2.4 hours to go from A to B. Find the distance from A to C.

11) If A is acute and $\sin A = \frac{2x}{3}$, determine the values of the remaining trigonometric functions.

12) If A is acute and $\tan A = x$, determine the values of the remaining trigonometric functions.

13) Find the height of a tree if the angle of elevation of its top changes from 20° and 40° as the observer advances 75 ft toward its base.

14) To find the width of a river, a surveyor set up his transit at C on one bank and sighted across to a point B on the opposite bank; then turning through an angle of 90°, he laid off a distance CA = 225 ft. Finally, setting the transit at A, he measured $\angle CAB$ at 48°20′. Find the width of the river.

15) A wall is 15 ft high and 10 ft from a house. Find the length of the shortest ladder which will just touch the top of the wall and reach a window 20.5 ft above the ground.

16) A vertical stake 20.0 cm high casts a horizontal shadow 12.5 cm long. What time is it if the sun rose at 6:00 a.m. and will be directly overhead at noon?

17) A balloon hovers 307 ft above one end of the Rip Van Winkle Bridge, which spans the Hudson River at Catskill, NY. The angle of depression of the other end of the bridge from the balloon is 21°. How long is the bridge?

18) As a hot-air balloon rises vertically, its angle of elevation from a point P on level ground 110 km from the point Q directly underneath the balloon changes from 19°20′ to 31°50′. Approximately how far does the balloon rise during this period?

19) A rectangular box has dimensions 8 in. × 6 in. × 4 in. Approximate, to the nearest minute, the angle θ formed by a diagonal of the base and the diagonal of the box.

20) A ship leaves port at 1:00 p.m. and sails in the direction N 34° W at a rate of 24 mi/hr. Another ship leaves port at 1:30 p.m. and sails in the direction N 56° E at a rate of 18 mi/hr.

a) Approximately how far apart are the ships at 3:00 p.m.?
b) What is the bearing to the nearest degree, from the first ship to the second?

21) An airplane flying at an altitude of 10,000 feet passes directly over a fixed object on the ground. One minute later the angle of depression of the object is 42°. Approximate the speed of the airplane to the nearest mile per hour.

22) From a tower 124 feet high, the angles of depression of two rocks which are on a horizontal line through the base of the tower are 16° and 12°. Find the distance between the rocks if they are on

a) opposite sides of the tower,
b) the same side of the tower.

23) Find the distance between the rocks in exercise 22 if one rock is directly south, the other directly east, of the tower. The other data are unchanged.

24) A ship sailing in the direction S42° W passes a point A directly east of a lighthouse. If the angle of elevation from A to the top of the lighthouse is 19°, find the angle of elevation when the ship is closest to the lighthouse.

25) A ship sailing in a direction N 54° E passes a point A directly south of a lighthouse. If the angle of elevation of the top of the lighthouse from A is 24°, find the angle of elevation when the ship is closest to the lighthouse.

Exercises: Section 3

1) Express each of the other trigonometric functions of θ in terms of $\sin\theta$.

2) Express each of the other trigonometric functions of θ in terms of $\cot\theta$.

3) Find the values of $\sin\frac{1}{2}\theta$, $\cos\frac{1}{2}\theta$ and $\tan\frac{1}{2}\theta$ given θ is between 0° and 360° and

a) $\sin\theta = \frac{5}{13}$ and θ is in quadrant II

b) $\cos\theta = \frac{3}{7}$ and θ is in quadrant IV

c) $\tan\theta = \frac{1}{3}$ and θ is in quadrant I

d) $\csc\theta = -\frac{25}{7}$ and θ is in quadrant IV

e) $\sec\theta = \frac{6}{5}$ and θ is in quadrant I

f) $\cot\theta = \frac{2}{\sqrt{5}}$ and θ is in quadrant III

4) Find the values of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$ and the quadrant in which 2θ terminates if

a) $\sin\theta = -\frac{1}{3}$ and θ is in quadrant IV

b) $\cos\theta = -\frac{8}{17}$ and θ is in quadrant III

c) $\tan\theta = -\frac{4}{3}$ and θ is in quadrant II

d) $\csc\theta = \frac{13}{12}$ and θ is in quadrant I

e) $\sec\theta = -\frac{5}{4}$ and θ is in quadrant II

f) $\cot\theta = \frac{1}{u}$ and θ is in quadrant I

5) Find the exact values of each of the following

a) $\sin\frac{\pi}{12}$

b) $\cos\frac{5\pi}{12}$

c) $\tan 112.5°$

d) $\sin 15° + \sin 75°$

e) $\cos 15° \sin 75°$

f) $2\cos^2\frac{\pi}{12} - 1$

6) For each of the following find $\sin(A+B)$, $\sin(A-B)$, $\cos(A+B)$, $\cos(A-B)$, $\tan(A+B)$ and $\tan(A-B)$

a) $\cos A = \frac{3}{5}$, $\sin B = \frac{5}{13}$, A & B are in quadrant I

b) $\sin A = \frac{2}{3}$, $\sin B = -\frac{1}{2}$ A is in quadrant II and B is in quadrant IV

c) $\cos A = -\frac{8}{12}$. $\cos B = -\frac{3}{5}$, A & B are in quadrant III

d) $\sin A = -\frac{4}{5}$, $\cos B = \frac{12}{13}$ A is in quadrant III and B is in quadrant IV

Verify each of the following identities

7) $\sec^2\theta\csc^2\theta = \sec^2\theta + \csc^2\theta$

8) $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

9) $2\csc\theta = \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$

10) $\dfrac{1-\sin\theta}{\cos\theta}=\dfrac{\cos\theta}{1+\sin\theta}$

11) $\dfrac{\sec\theta-\csc\theta}{\sec\theta+\csc\theta}=\dfrac{\tan\theta-1}{\tan\theta+1}$

12) $\dfrac{\tan\theta-\sin\theta}{\sin^3\theta}=\dfrac{\sec\theta}{1+\cos\theta}$

13) $\dfrac{\cos\theta\cot\theta-\sin\theta\tan\theta}{\csc\theta-\sec\theta}=1+\cos\theta\sin\theta$

14) $\left(1-\sin^2\theta\right)\left(1+\tan^2\theta\right)=1$

15) $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\dfrac{\sin\theta+1}{\cos\theta}$

16) $\cos\theta+\sin\theta\tan\theta=\sec\theta$

17) $\sec^6\theta(\sec\theta\tan\theta)-\sec^4\theta(\sec\theta\tan\theta)=\sec^5\theta\tan^3\theta$

18) $\dfrac{\sec\theta-1}{1-\cos\theta}=\sec\theta$

19) $\dfrac{\cos\theta\cot\theta}{1-\sin\theta}-1=\csc\theta$

20) $2\sec^2\theta-2\sec^2\theta\sin^2\theta-\sin^2\theta-\cos^2\theta=1$

21) $\dfrac{\cot\theta}{\csc\theta-1}=\dfrac{\csc\theta+1}{\cot\theta}$

22) $\dfrac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}=1-\sin\theta\cos\theta$

23) $\dfrac{\cos(-\theta)}{1+\sin(-\theta)}=\sec\theta+\tan\theta$

24) $\csc^4\theta-\cot^4\theta=2\csc^2\theta-1$

25) $\dfrac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}=\dfrac{\tan x+\tan y}{1-\tan x\tan y}$

26) $\dfrac{\cos x-\cos y}{\sin x+\sin y}+\dfrac{\sin x-\sin y}{\cos x+\cos y}=0$

27) $2\tan x\csc 2x-\tan^2 x=1$

28) $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta}$

29) $\tan\theta \sin 2\theta = 2 - 2\cos^2\theta$

30) $8\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} = 1-\cos 2\theta$

31) $\cos^4\theta = \frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$

32) $2\tan\theta \csc 2\theta - \tan^2\theta = 1$

33) $\sec^2\theta - 1 = \dfrac{\sec 2\theta - 1}{\sec 2\theta + 1}$

34) $\csc\theta \sin 2\theta - \sec\theta = \cos 2\theta \sec\theta$

35) $\tan\alpha = \dfrac{1-\cos 2\alpha}{\sin 2\alpha}$

36) $\dfrac{2\cot x}{\tan 2x} = \csc^2 x - 2$

37) $-\dfrac{\sin(A-B)}{\sin(A+B)} = \dfrac{\cot A - \cot B}{\cot A + \cot B}$

38) $2\cos(A+B)\sin(A+B) = \sin 2A \cos 2B + \sin 2B \cos 2A$

39) $\cos 2x = \cos^4 x - \sin^4 x$

40) $1 - \frac{1}{2}\sin 2x = \dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x}$

41) $2\tan 2\theta = \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} - \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

42) $\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x} = 2$

43) $\tan\theta \sin 2\theta = 2\sin^2\theta$

44) $\cot\theta \sin 2\theta = 1 + \cos 2\theta$

45) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

46) $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

47) $\left(\sin\frac{1}{2}\theta - \cos\frac{1}{2}\theta\right)^2 = 1 - \sin\theta$

48) $\tan\frac{1}{2}\theta = \csc\theta - \cot\theta$

49) $\dfrac{\sin 4A + \sin 2A}{\cos 4A - \sin 2A} = \tan 3A$

50) $\cos^3 x \sin^2 x = \frac{1}{16}(2\cos x - \cos 3x - \cos 5x)$

51) $\dfrac{\sin A + \sin B}{\sin A - \sin B} = \dfrac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$

52) $\cos\theta + \cos 2\theta + \cos 3\theta = \cos 2\theta(1 + 2\cos\theta)$

53) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \dfrac{1+\sin\theta}{|\cos\theta|}$

54) $\dfrac{1}{\cot x + 1} + \dfrac{1}{\tan x + 1} = 1$

55) $\sec^2\left(\frac{\pi}{2} - x\right) - 1 = \cot^2 x$

56) $\sqrt{\dfrac{\csc\theta + \cot\theta}{\csc\theta - \cot\theta}} = \dfrac{1}{\csc\theta - \cot\theta}$

57) $\dfrac{\cos 2\theta}{\sec\theta} - \dfrac{\sin\theta}{\csc 2\theta} = \cos 3\theta$

58) $\dfrac{\tan(A-B) + \tan C}{1 - \tan(A-B)\tan C} = \dfrac{\tan A - \tan(B-C)}{1 + \tan A \tan(B-C)}$

59) $\sec(A-B) = \dfrac{\sec A \sec B}{1 + \tan A \tan B}$

60) $\sec 10\theta = \dfrac{1}{1 - 2\sin^2 5\theta}$

61) $\sec^2\frac{\theta}{2} = \dfrac{2\tan\theta}{\tan\theta + \sin\theta}$

62) $\dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$

63) $\sin 2\theta + \cos 2\theta = \dfrac{(1+\cot\theta)^2 - 2}{1 + \cot^2\theta}$

64) $\sin A + \cos B = 2\sin\left(\frac{A-B}{2} + \frac{\pi}{4}\right)\cos\left(\frac{A+B}{2} - \frac{\pi}{4}\right)$

65) $\cos^2 A - \cos^2 B = \sin(B + A)\sin(B - A)$

Exercises: Section 4

1) Sketch the graphs of the following for one period starting at 0. Identify the amplitude, (when appropriate) and period of each.

a) $y = 4\sin x$

b) $y = \sin 3x$

c) $y = \frac{1}{2}\cos 5x$

d) $y = \tan \frac{4}{3}x$

e) $y = \sec 8x$

f) $y = \cos\left(x + \frac{\pi}{12}\right)$

g) $y = \sin\left(2x + \frac{\pi}{4}\right)$

h) $y = \tan\left(\frac{1}{2}x - \frac{\pi}{6}\right)$

i) $y = 4\cos(3x + \pi)$

j) $y = \csc\left(2x + \frac{\pi}{4}\right)$

2) Sketch one period of the graph of each equation by the method of addition or ordinates.

a) $y = \sin x + \cos 2x$

b) $y = 2\sin x + \cos 3x$

c) $y = 3\sin 2x - \cos x$

d) $y = \cos x - \cos 2x$

e) $y = \sin 3x + \sin x$

f) $y = 2\sin 2x - \cos 3x$

3) Reduce each equation to the form $y = C\sin(kx + \theta)$ and sketch the graph.

a) $y = \sqrt{2}(\sin x - \cos x)$

b) $y = 2\cos 2x - \sin 2x$

c) $y = 3\cos 3x + 4\sin 3x$

d) $y = 5\sin x + 12\cos x$

e) $y = \sin x + \sqrt{3}\cos x$

f) $y = \cos 2x + \sin 2x$

<u>Exercises</u>: <u>Section 5</u>

For exercises 1-22, evaluate each of the following

1) $\mathrm{Sin}^{-1}0$
2) $\mathrm{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
3) $\mathrm{Tan}^{-1}(-1)$
4) $\mathrm{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
5) $\mathrm{Tan}^{-1}\left(-\sqrt{3}\right)$
6) $\mathrm{Sin}^{-1}\left(\frac{1}{3}\right)$
7) $\mathrm{Arctan}\left(\frac{1}{\sqrt{3}}\right)$
8) $\cos\left(\mathrm{Arcsin}\frac{1}{2}\right)$
9) $\tan\left(\mathrm{Arccos}\left(-\frac{1}{2}\right)\right)$
10) $\mathrm{Arcsin}\left(\sin\frac{3\pi}{4}\right)$
11) $\mathrm{Arccos}\left(\cos\left(-\frac{\pi}{4}\right)\right)$
12) $\sin\left(2\,\mathrm{Arc}\sin\frac{2}{3}\right)$
13) $\cos\left(2\,\mathrm{Sin}^{-1}\frac{3}{5}\right)$
14) $\cos\left(\frac{1}{2}\mathrm{Sin}^{-1}\frac{3}{4}\right)$
15) $\tan\left(2\,\mathrm{Arccos}\frac{1}{3}\right)$
16) $\sin\left(2\,\mathrm{Sin}^{-1}\frac{3}{4}\right)$
17) $\sin\left(\mathrm{Sin}^{-1}\frac{5}{13}+\mathrm{Sin}^{-1}\frac{4}{5}\right)$
18) $\cos\left(\mathrm{Tan}^{-1}\left(-\frac{4}{3}\right)+\mathrm{Sin}^{-1}\frac{12}{13}\right)$
19) $\tan\left(2\,\mathrm{Sin}^{-1}\frac{4}{5}+\mathrm{Cos}^{-1}\frac{12}{13}\right)$
20) $\tan\left(\mathrm{Arctan}\frac{3}{4}+\mathrm{Arccot}\frac{15}{8}\right)$
21) $\sin\left(\mathrm{Arcsin}\frac{5}{13}+\mathrm{Arcsin}\frac{4}{5}\right)$
22) $\cos\left(\mathrm{Arcsin}\frac{3}{4}+\mathrm{Arctan}\left(-\frac{4}{3}\right)\right)$

For exercises 23-104, find the solution set of each equation in the interval $0 \le x < 2\pi$

23) $3\tan x+5=2$
24) $2\sin x+1=0$
25) $\cos 2x=0$
26) $\sec x=2$
27) $2\cos x-\sqrt{3}=0$
28) $\cot\frac{1}{2}x-1=0$
29) $4\cos\frac{1}{2}x=1$
30) $4\cos^2 x=3$
31) $4\sin x=3$
32) $3\cot^2 x-1=0$
33) $\sin 3x=0$
34) $\tan x-\cot x=0$
35) $\sin x\cos x-\sin x=0$
36) $\sqrt{3}\csc x=2\sin x\csc x$
37) $2\cos^2 x+\cos x=1$
38) $\sin x\cos x=0$
39) $(\tan x-1)\left(4\sin^2 x-3\right)=0$
40) $3\cos^2 x=\sin^2 x$
41) $\tan^2 x+\tan x+1=0$
42) $3\cos^2 x-7\cos x=6$
43) $2\sec^2 x-\sec x=1$
44) $2\cos^2 x-5\cos x+2=0$
45) $\sin\frac{x}{2}=\sqrt{2}-\sin\frac{x}{2}$
46) $\cos 2x-\cos x=0$
47) $3\tan 2x=\sqrt{3}$
48) $\csc^2\frac{x}{2}=2\sec x$
49) $3\cot^3 x=\cot x$
50) $2\sin^2 x+\cos x+4=0$

51) $\csc x + 4\sin x + 4 = 0$

52) $\cot x + \csc x = -\sin x$

53) $2\cos x + 1 = \sin x$

54) $2\sec x = \tan x + \cot x$

55) $\csc x + \cot x = \sqrt{3}$

56) $\cos x - \sqrt{3}\sin x = 1$

57) $\sin 2x + \cos x = 0$

58) $\tan 2x + 2\sin x = 0$

59) $2\cos^2 \frac{1}{2}x + \sin 2x = 1$

60) $\tan \frac{1}{2}x + 2\sin 2x = \csc x$

61) $\cos^3 x \csc^3 x = 4\cot^2 x$

62) $2\cot x + \csc x = 1$

63) $\cos x - \sin x = \frac{\sqrt{2}}{2}$

64) $\sin 2x = \cos 4x$

65) $\sin 5x - \sin 3x - \sin x = 0$

66) $2\sin\theta - 1 = 0$

67) $\cot\theta + 1 = 0$

68) $4\cos^2\theta - 3 = 0$

69) $3\tan^2\theta - 1 = 0$

70) $\sin 2\theta + \sin\theta = 0$

71) $\cos 2\theta - \cos\theta = 0$

72) $\sin^2\theta - \cos^2\theta + 1 = 0$

73) $\sin\theta - \sin\frac{\theta}{2} = 0$

74) $\cos\theta + \cos\frac{\theta}{2} = 0$

75) $2\cos^2\theta - \sin\theta - 1 = 0$

76) $2\sin^2\theta - 3\cos\theta - 3 = 0$

77) $\cot\theta + 2\sin\theta = \csc\theta$

78) $2\cos^2\theta - 2\cos 2\theta = 1$

79) $\cos 2\theta + 2\cos^2\frac{\theta}{2} = 6$

80) $\sin 2\theta\cos\theta + \cos 2\theta\sin\theta = 1$

81) $\cos 3\theta\cos\theta + \sin 3\theta\sin\theta = 0$

82) $\cot\theta - \tan\frac{\theta}{2} = 0$

83) $\tan 2\theta = \cot\theta$

84) $2\sin 2\theta - 2\cos\theta + 2\sin\theta = 1$

85) $2\cos^2 2\theta - 2\sin^2 2\theta = 1$

86) $\cot\frac{\theta}{2} + \sin 2\theta = \csc\theta$

87) $\tan\frac{\theta}{2} = \cos\theta - 1$

88) $\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = \cos 2\theta$

89) $\sin^2\frac{\theta}{2} = \cos^2\theta$

90) $2\sin^2\frac{\theta}{2} = 1 - \sin\theta$

91) $2\cos^2 2\theta + \cos 2\theta - 1 = 0$

92) $\sin 6\theta - \sin 3\theta = 0$

93) $\cos^2 5\theta - \sin^2 5\theta = 1$

94) $2\sin^3 3\theta - \sin 3\theta = 1 - 2\sin^2 3\theta$

95) $\tan 4\theta + \cot 4\theta = 2$

96) $\dfrac{1-\cos\theta}{\sin\theta} = \sin\theta$

97) $\dfrac{\sin\theta}{1+\cos\theta} = 1 - \cos\theta$

98) $\dfrac{2\tan\theta}{1-\tan^2\theta} = \cos\theta$

99) $4\tan^2\theta = 3\sec^2\theta$

100) $\sin^2\frac{\theta}{2} - \cos\theta + 1 = 0$

101) $1 - 2\cos^2\theta = 2(1 - 2\sin^2\theta)$

102) $\cot\theta + 2\sin\theta = \csc\theta$

103) $2\cos^2 2\theta - 3\cos 2\theta + 1 = 0$

104) $2\sin^2\theta - \sin\theta = 1$

For exercises 105-115, solve each equation for x.

105) $\text{Arccos } 2x = \text{Arcsin } x$

106) $\text{Arccos}\left(2x^2-1\right) = 2\text{Arccos } \frac{1}{2}$

107) $\text{Tan}^{-1}x - \text{Cot}^{-1}x = 0$

108) $\text{Sin}^{-1}2\sqrt{x} = \text{Cos}^{-1}x$

109) $\text{Cos}^{-1}x + \text{Sin}^{-1}(1-x) = \frac{\pi}{2}$

110) $\text{Sin}^{-1}\frac{1}{2}x = \text{Cos}^{-1}x$

111) $\text{Cos}^{-1}x = \text{Cos}^{-1}\frac{1}{2x}$

112) $\text{Arccos } 2x - \text{Arccos } x = \frac{\pi}{3}$

113) $\text{Arctan } x + \text{Arctan }(1-x) = \text{Arctan } \frac{4}{3}$

114) $\text{Arcsin } 2x = \frac{\pi}{4} - \text{Arcsin } x$

115) $\text{Arcsin } x + \text{Arctan } x = \frac{\pi}{2}$

For exercises 116-125 find the solution set of each equation.

116) $\cos\left(\text{Sin}^{-1}x\right) = \sqrt{1-x^2}$

117) $\cot\left(\text{Cos}^{-1}x\right) = \frac{x}{\sqrt{1-x^2}}$

118) $\sin\left(\text{Tan}^{-1}x\right) = \frac{x}{\sqrt{1+x^2}}$

119) $\cos\left(2\text{Cot}^{-1}x\right) = \frac{x^2-1}{x^2+1}$

120) $\text{Tan}^{-1}x + \text{Tan}^{-1}(-x) = 0$

121) $\frac{1}{2}\text{Cos}^{-1}x = \text{Cot}^{-1}\frac{\sqrt{1-x^2}}{1-x}$

122) $2\ \text{Sin}^{-1}x = \text{Cos}^{-1}\left(1-2x^2\right)$

123) $2\ \text{Tan}^{-1}\frac{1}{x} = \text{Sin}^{-1}\left(\frac{2x}{x^2+1}\right)$

124) $\cos\left(\frac{1}{2}\text{Cos}^{-1}x\right) = \sqrt{\frac{1+x}{2}}$

125) $\text{Sin}^{-1}x + \text{Cos}^{-1}x = \text{Sin}^{-1}1$

Exercises: Section 6

1) Use the law of sines to find the unknown parts of the triangle.

a) c = 70, A = 44°, B = 65°
b) b = 2.8, A = 33°, C = 72°
c) a = 22, B = 25°, C = 95°
d) a = 10.5, A = 41°, C = 77°
e) b = 200, A = 56.7°, C = 40.0°
f) a = 5.6, A = 38°, B = 82°
g) c = 946, A = 18°, C = 111°
h) b = 675, A = 35.3°, B = 52.8°
i) a = 50, b = 26, A = 95°
j) c = 60, b = 82, B = 100°
k) a = 31, b = 26, B = 48°
l) b = 36, c = 48, B = 37°
m) a = 54, c = 61, A = 58°
n) c = 258, b = 386, C = 68.5°
o) a = 4.5, b = 6.0, A = 29°
p) a = 66, b = 77, A = 59°

2) Use the law of cosines to find the unknown parts of the triangle.

a) a = 4.0, b = 5.0, C = 53°
b) a = 12, c = 17, B = 98°
c) b = 13, c = 24, A = 27°
d) a = 18, c = 23, B = 106°
e) a = 380, b = 460, C = 71.4°
f) a = 327, b = 251, C = 72°
g) b = 37.9, c = 40.8, A = 67.3°
h) b = 143, c = 89.6, A = 81°
i) a = 21, b = 33, c = 26
j) a = 230, b = 306, c = 400
k) a = 22, b = 26, c = 33
l) a = 3.00, b = 1.30, c = 1.60
m) a = 3702, b = 3015, c = 1122
n) a = 40, b = 70, c = 50
o) a = 9.6, b = 6.2, c = 4.3
p) a = 4, b = 5, c = 8

Use the law of sines and the law of cosines as necessary to solve each of the following problems.

3) To find the distance between two points A and B on opposite sides of a river, we measure the distance from A to C to be 220 feet, the angle CAB to be 98°40′, and the angle ACB to be 41°30′. Compute the distance AB.

4) The sides of a parallelogram are 4 cm and 6 cm. One angle is 58° while another is 122°. Find the lengths of the diagonals of the figure.

5) A parallelogram has sides of length 25.9 cm and 32.5 cm. The longer diagonal has a length of 57.8 cm. Find the angle opposite the diagonal.

6) Two ships leave a harbor together, traveling on courses that have an angle of 135°40′ between them. If they each travel 402 mi, how far apart are they?

7) Town B is due east of town A and town C is in the direction N68°E of town A. If towns A and C are 19 miles apart and towns B and C are 11 miles apart, find the distance between A and B.

8) Two sides of a parallelogram make an angle of 54°. The lengths of the two sides are 21 and 33 feet. Find the length of each diagonal.

9) An airplane files 165 miles from point A with bearing N 40°W and then travels 80 miles S25°W. Approximately how far is the airplane from A?

10) A triangular plot of land has sides of lengths 420 feet, 350 feet, and 180 feet. Approximate the smallest angle between the sides.

11) The angles of elevation of a balloon from two points A and B on level ground are 24°10′ and 47°40′, respectively. Points A and B are 8.4 miles apart and the balloon is between the points, in the same vertical plane. Approximate the height of the balloon above the ground.

12) A tree stands vertically on a hillside which makes an angle of 19° with the horizontal. From a point 170 feet directly up the hill from the tree the angle of depression of the top of the tree is 10°. Find the height of the tree.

13) An athlete runs at a constant speed of one mile every eight minutes in the direction S40°E for 20 minutes and then in the direction N20°E for the next 16 minutes. Approximate, to the nearest tenth of a mile, the distance from the endpoint to the starting point of the athlete's course.

14) To determine the distance between two points A and B, separated by a lake, a surveyor measures a distance of 304 feet from A to C such that angle BAC is 40°20′. He then measures CB as 425 feet. Find the distance AB.

15) Find the area of each triangle

a) b = 20, c = 30, A = 60°
b) a = 15, b = 19, c = 24
c) a = 12, b = 17, C = 150°
d) b = 3.2, c = 4.0, A = 115°
e) b = 55, A = 80°, C = 98°
f) a = 27, b = 36, c = 49
g) a = 7, b = 6, A = 64°
h) a = 87, c = 14, B = 74°
i) c = 1.27, B = 27.8°, C = 19.3°
j) a = 17.33, A = 52.68°, C = 63.15°
k) a = 10, A = 53°, B = 74°
l) a = 12, b = 11, C = 30°

Exercises: Section 7

1) Simplify each of the following expressions.

a) $5i^{11} + 3i^3 + 6i^5$
b) $4i^6 + 9i^{13} - 5i^{14}$
c) $2i^2 + 5i^8 - 6i^{23}$

2) Perform the indicated operations, leaving each result in the form $a + bi$.

a) $(2-3i)+(-4+5i)$
b) $(6+2i)+(3-4i)$
c) $(7-2i)-(8-3i)$
d) $(4-4i)-(-3-4i)$
e) $4\sqrt{-9}+2\sqrt{-4}-6\sqrt{-25}+3\sqrt{-36}$
f) $4+\sqrt{-81}+5-2\sqrt{-49}-1+\sqrt{-1}$
g) $(3-i)(5+2i)$
h) $(5+4i)(5-4i)$
i) $(2-4i)(4+2i)(1+i)$
j) $\dfrac{4+i}{i}$
k) $\dfrac{2+i}{6-i}$
l) $\dfrac{2-3i}{17-6i}$

3) Prove that the sum of two conjugate complex numbers is a real number.

4) Prove that the product of two conjugate complex numbers is a positive real number.

5) Write the conjugate of each complex number. Plot the number and its conjugate.

a) $3-2i$
b) $4+i$
c) $-4+2i$
d) $3i$
e) $-2-3i$
f) $6+5i$
g) $4-7i$
h) $1-i$

6. Write each of the following complex numbers in trigonometric form.

a) $-2+2i\sqrt{3}$
b) $1+\sqrt{3}i$
c) $-5-5i$
d) $-\sqrt{3}+i$
e) $-\sqrt{2}+2i$
f) $\sqrt{3}-i$
g) $4\sqrt{3}+4i$
h) $-\sqrt{15}-i\sqrt{5}$
i) $2\sqrt{3}+2i$
j) $\sqrt{7}-\sqrt{21}i$

7) Change each number to algebraic form.

a) $2(\cos 45° + i\sin 45°)$
b) $8(\cos 135° + i\sin 135°)$
c) $\cos 90° + i\sin 90°$
d) $4(\cos 150° + i\sin 150°)$
e) $5(\cos 300° + i\sin 300°)$
f) $\sqrt{3}(\cos 315° + i\sin 315°)$
g) $\cos 77° + i\sin 77°$
h) $4(\cos 240° + i\sin 240°)$
i) $2(\cos 60° + i\sin 60°)$
j) $\sqrt{2}(\cos 330° + i\sin 330°)$

8) Perform the indicated operation, leaving each result in the form $a+bi$.

a) $2(\cos 40° + i\sin 40°)\cdot 3(\cos 80° + i\sin 80°)$
b) $3(\cos 60° + i\sin 60°)\cdot 2(\cos 90° + i\sin 90°)$
c) $2(\cos 125° + i\sin 125°)\cdot 7(\cos 100° + i\sin 100°)$
d) $8(\cos 300° + i\sin 300°)\cdot 5(\cos 120° + i\sin 120°)$
e) $\sqrt{3}(\cos 45° + i\sin 45°)\cdot \sqrt{3}(\cos 225° + i\sin 225°)$
f) $3(\cos 170° + i\sin 170°)\cdot 9(\cos 280° + i\sin 280°)$
g) $5(\cos 90° + i\sin 90°)\cdot 3(\cos 45° + i\sin 45°)$
h) $\sqrt{2}(\cos 300° + i\sin 300°)\cdot \sqrt{2}(\cos 270° + i\sin 270°)$
i) $\dfrac{\cos 120° + i\sin 120°}{\cos 30° + i\sin 30°}$
j) $\dfrac{4(\cos 120° + i\sin 120°)}{2(\cos 150° + i\sin 150°)}$
k) $\dfrac{3(\cos 305° + i\sin 305°)}{9(\cos 65° + i\sin 65°)}$
l) $\dfrac{12(\cos 293° + i\sin 293°)}{6(\cos 23° + i\sin 23°)}$
m) $\dfrac{14(\cos 31° + i\sin 31°)}{2(\cos 91° + i\sin 91°)}$
n) $\dfrac{9(\cos 310° + i\sin 310°)}{3(\cos 265° + i\sin 265°)}$
o) $\dfrac{10(\cos 225° + i\sin 225°)}{5(\cos 45° + i\sin 45°)}$
p) $\dfrac{24(\cos 150° + i\sin 150°)}{2(\cos 30° + i\sin 30°)}$

9) Express each number in trigonometric form and then perform the indicated operations. Leave the result in trigonometric form.

a) $(1+i)(1-\sqrt{3}i)$

b) $5i(4+4i)$

c) $(2-2i)(3-3i)$

d) $(3+3i)(-\sqrt{3}+i)(1+\sqrt{3}i)$

e) $\dfrac{2\sqrt{3}+2i}{-1+\sqrt{3}i}$

f) $\dfrac{4+4i}{2-2i}$

g) $\dfrac{-i}{1+i}$

h) $\dfrac{4}{3\sqrt{3}+3i}$

10) Find the indicated powers in each of the following. Express each result in trigonometric and algebraic form.

a) $[3(\cos 30° + i\sin 30°)]^3$

b) $[4(\cos 202° + i\sin 202°)]^2$

c) $[\cos 100° + i\sin 100°]^9$

d) $[\cos 130° + i\sin 130°]^6$

e) $[\cos 24° + i\sin 24°]^{10}$

f) $[2(\cos 135° + i\sin 135°)]^4$

g) $(1+i)^6$

h) $(2\sqrt{3}-2i)^5$

11) Find the indicated roots of the given expression. Leave each result in trigonometric form.

a) The cube roots of unity.

b) The cube roots of i.

c) The fourth roots of $16(\cos 148° + i\sin 148°)$.

d) The fifth roots of $\cos 175° + i\sin 175°$.

e) The fourth roots of $81(\cos 88° + i\sin 88°)$.

f) The fifth roots of $1-i$.

g) The cube of roots of $1+\sqrt{3}i$.

h) The fourth roots of 32.

i) The square roots of $25(\cos 132° + i\sin 132°)$.

j) The sixth roots of unity.

k) The fourth roots of $16i$.

l) The square roots of $-\sqrt{2}+\sqrt{2}i$.

12) Solve the following equations for x. Leave results in algebraic form.

a) $x^3 - 8i = 0$

b) $x^4 + 16 = 0$

c) $x^5 - i = 0$

d) $x^3 - (4+4\sqrt{3}i) = 0$

e) $x^3 + 27 = 0$

f) $x^4 + i = 0$

g) $x^3 - 8 = 0$

h) $x^6 + 1 = 0$