Mathematics 325
Homework 11

Due Date: ____________________

Name: _________________________

Compute the Fourier transform of the function $f(x) = e^{-|x|}$.

$$f(x) = e^{|x|}$$

$$\hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-|x|} e^{-i\xi x}\, dx = \frac{1}{\sqrt{2\pi}} \lim_{\substack{M\to\infty \\ N\to-\infty}} \left(\int_N^0 e^{+x-i\xi x}\, dx + \int_0^M e^{-x-i\xi x}\, dx \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left(\lim_{M\to-\infty} \left. \frac{e^{(+1-i\xi)x}}{+1-i\xi} \right|_N^0 + \lim_{M\to\infty} \left. \frac{e^{(-1-i\xi)x}}{-1-i\xi} \right|_0^M \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left(\lim_{N\to-\infty} \frac{1 - e^{(+1-i\xi)N}}{+1-i\xi} + \lim_{M\to\infty} \frac{e^{(-1-i\xi)M} - 1}{-1-i\xi} \right)$$

$$\left| e^{(1-i\xi)N} \right| = e^{N} \longrightarrow 0 \quad \text{as} \quad N \to -\infty$$

$$\left| e^{(-1-i\xi)M} \right| = e^{-M} \longrightarrow 0 \quad \text{as} \quad M \to \infty.$$

$$\therefore \quad \hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{1-i\xi} + \frac{1}{1+i\xi} \right) = \frac{2}{\sqrt{2\pi}\,(1+\xi^2)} = \boxed{\sqrt{\frac{2}{\pi}} \cdot \frac{1}{1+\xi^2}}$$