NAME Key

Math 1212
Test 1
Fall 2014

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = 3 - \sqrt{x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(3-\sqrt{x+h})-(3-\sqrt{x})}{h}$$

$$= \lim_{h\to 0} \frac{-\sqrt{x+h}+\sqrt{x}}{h} \cdot \frac{-\sqrt{x+h}-\sqrt{x}}{-\sqrt{x+h}-\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(-\sqrt{x+h}-\sqrt{x})} = \lim_{h\to 0} \frac{h}{h(-\sqrt{x+h}-\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{-\sqrt{x+h}-\sqrt{x}} = \frac{1}{-\sqrt{x}-\sqrt{x}} = \frac{-1}{2\sqrt{x}}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $$\lim_{x\to -4} \frac{x^2+x-12}{x^2-2x-24} = \lim_{x\to -4} \frac{(x+4)(x-3)}{(x+4)(x-6)} = \lim_{x\to -4} \frac{x-3}{x-6} = \frac{-4-3}{-4-6}$$

fill in, get $\frac{0}{0}$, so factor

$$= \frac{-7}{-10} = \frac{7}{10}$$

(b) $$\lim_{x\to 4^+} \frac{x-3}{x-4} = \infty$$

fill in, get $\frac{1}{0}$, so use chart

x	y
5	$\frac{2}{1} = 2$
4.5	$\frac{1.5}{.5} = 3$
4.1	$1.1/.1 = 11$
4.01	$1.01/.01 = 101$

$\downarrow$ ∞

(c) $$\lim_{x\to 1} \frac{x-1}{(x+1)^2} = \frac{1-1}{(1+1)^2} = \frac{0}{4} = 0$$

3. During the summer, a group of students runs a lawn care business. Suppose it costs them \$1450 for a riding mower, and that the gas for the mower for an average lawn will cost \$2. The price they charge to cut an average lawn is \$60.

a) How many lawns must the students cut to break even?
b) How many lawns must the students cut to make a profit of \$1000?

a) "Break even" means Profit = 0, or Revenue = cost
money in = money out.

Let x = # lawns.

Revenue = price · quantity = $(60)(x)$

Cost = cost for mower + cost for gas = $1450 + 2(x)$

$60x = 1450 + 2x$
$58x = 1450$
$x = \frac{1450}{58} = \boxed{25 \text{ lawns}}$ to breakeven.

b) Profit = Rev − Cost = $60x - (1450 + 2x) = 1000$
$58x = 2450$
$x = \frac{2450}{58} \approx 42.24$ lawns, so

$\boxed{43 \text{ lawns}}$ to make \$1000.

4. Find $f'(x)$ (do not simplify!) if:

a) $f(x) = (3x^2 - 2)(\sqrt{x}^3 + 10x)$

$f(x) = (3x^2 - 2)(x^{3/2} + 10x)$

$f'(x) = (6x)(x^{3/2} + 10x) + (3x^2 - 2)(\frac{3}{2}x^{1/2} + 10)$

b) $f(x) = 2x^{\frac{-1}{2}} + 3 - 15x^3 - \frac{1}{3x}$

$f(x) = 2x^{-1/2} + 3 - 15x^3 - \frac{1}{3}x^{-1}$

$f'(x) = -x^{-3/2} - 45x^2 + \frac{1}{3}x^{-2}$

5. Suppose the total cost of manufacturing q units is $C(q) = 3q^2 + q + 500$ dollars.

a) Use marginal analysis to ***estimate*** the cost of manufacturing the 41st unit.
b) Calculate the ***actual*** cost of manufacturing the 41st unit.

a) marginal cost $= c'(q) = 6q + 1$

$c'(40) = 6(40) + 1 = \boxed{\$241}$ to produce the 41^{st} unit

b) actual cost of 41^{st} $= c(41) - c(40)$

$= (3(41)^2 + 41 + 500) - (3(40)^2 + 40 + 500)$

$= (5043 + 41 + 500) - (4800 + 40 + 500)$

$= 5584 - 5340$

$= \boxed{\$244}$

6. Find the equation of the line tangent to $f(x) = \dfrac{x^2 - 1}{\left(3x^{\frac{2}{3}} + x^2\right)(2x - 5)}$ at the point where $x = 1$.

point: $x = 1$

$y = f(1) = \dfrac{1-1}{(3+1)(2-5)} = \dfrac{0}{4 \cdot {}^{-}3} = \dfrac{0}{-12} = 0 \qquad (1, 0)$

Slope: $f'(x) = \dfrac{(2x)(3x^{2/3} + x^2)(2x-5) - (x^2-1)\left[(2x^{-1/3} + 2x)(2x-5) + (3x^{2/3} + x^2)(2)\right]}{\left[(3x^{2/3} + x^2)(2x-5)\right]^2}$

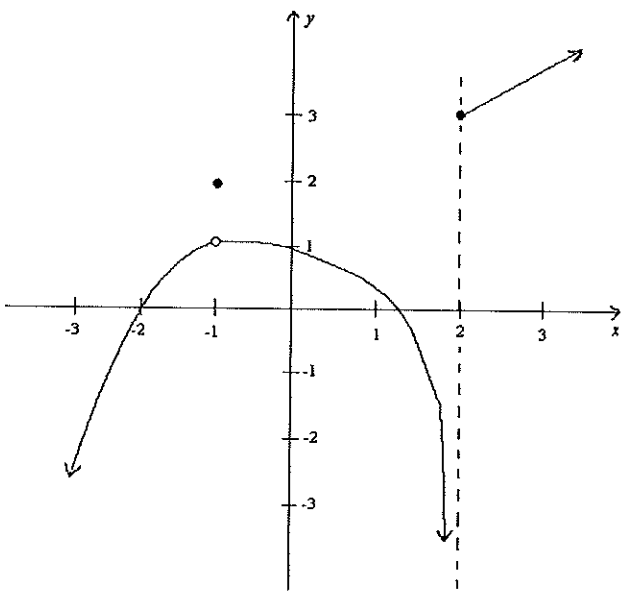
$m = f'(1) = \dfrac{(2)(3+1)(2-5) - (1-1)\left[(2+2)(2-5) + (3+1)(2)\right]}{\left[(3+1)(2-5)\right]^2}$

$= \dfrac{2 \cdot 4 \cdot {}^{-}3}{(4 \cdot -3)^2} = \dfrac{-24}{144} = \dfrac{-1}{6}$

line: $y - 0 = \frac{-1}{6}(x - 1)$

$y = \frac{-1}{6}(x - 1)$

7. Consider the graph of the function $f(x)$ given below.



a) For what values of x is $f(x)$ discontinuous? $x = -1$ and $x = 2$

b) Find $\lim_{x \to -2} f(x)$. $= 0$

c) Find $\lim_{x \to 2^-} f(x)$. $= -\infty$

d) Find $\lim_{x \to 2^+} f(x)$. $= 3$

e) Find $\lim_{x \to 2} f(x)$. DNE

f) Find $\lim_{x \to -1} f(x)$. $= 1$

8. Is the function $f(x) = \begin{cases} 2x^2 + 1 & \text{if } x < 3 \\ 6x + 2 & \text{if } x \geq 3 \end{cases}$ continuous at $x = 3$? Explain why or why not.

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (2x^2+1) = 2(9)+1 = 19$ (hole at $(3,19)$)

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x+2) = 6(3)+2 = 20$ (point at $(3,20)$)

To be continuous at $x=3$, $\lim_{x \to 3} f(x)$ must exist, these do not, so f is not continuous at $x=3$. (Point & hole don't match).