NAME Key

Math 1212
Test 1
Spring 2016

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = x^2 - 3x + 1$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2 - 3(x+h)+1] - [x^2-3x+1]}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-3x-3h+1-x^2+3x-1}{h} = \lim_{h\to 0} \frac{2xh+h^2-3h}{h}$$

$$= \lim_{h\to 0} (2x+h-3) = 2x-3$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" or "denominator is zero" are not sufficient explanations).

(a) $\displaystyle\lim_{x\to 5} \frac{x+1}{x+5} = \frac{5+1}{5+5} = \frac{6}{10} = \frac{3}{5}$

(b) $\displaystyle\lim_{x\to 3} \frac{9-x^2}{x-3} = \lim_{x\to 3} \frac{-(x^2-9)}{x-3} = \lim_{x\to 3} \frac{-(x+3)(x-3)}{x-3}$

$$= \lim_{x\to 3} -(x+3) = -6$$

(c) $\displaystyle\lim_{x\to -2^-} \frac{x+1}{x+2}$ filling in gives $\frac{-1}{0}$, must use a chart

x	y
-2.1	$-1.1/-.1 = 11$
-2.01	$-1.01/-.01 = 101$
-2.001	$-1.001/-.001 = 1001$

$\downarrow \infty$

so $\displaystyle\lim_{x\to -2^-} \frac{x+1}{x+2} = \infty$

3. Suppose all x units of a product can be sold if the price is set at $p(x) = -x^2 + 4x + 10$. Also assume that the total cost to produce all x units is $C(x) = \frac{1}{3}x^2 + 2x + 39$.

(a) Find an equation for profit when x units are produced.
(b) Using marginal analysis, estimate the change in profit derived from the production and sale of the 5th unit.

a) Profit = Revenue − Cost
= price · quantity − Cost

$P(x) = (-x^2 + 4x + 10)(x) - (\frac{1}{3}x^2 + 2x + 39)$

$P(x) = -x^3 + \frac{11}{3}x^2 + 8x - 39$

b) estimate, so use derivative.

Change in profit $\approx P'(x) = -3x^2 + \frac{22}{3}x + 8$

Change for 5th unit $\approx P'(4) = -3(16) + \frac{88}{3} + 8$

$= \frac{88}{3} - 40 = \frac{-32}{3}$

(profit decreases about \$10.67)

4. Find $f'(x)$ if:

a) $f(x) = \dfrac{2x-3}{x^3}$

$f'(x) = \dfrac{(2)(x^3) - (2x-3)(3x^2)}{x^6} = \dfrac{2x^3 - 6x^3 + 9x^2}{x^6}$

$= \dfrac{-4x^3 + 9x^2}{x^6} = \dfrac{-4x + 9}{x^4}$

b) $f(x) = x^3 - \dfrac{1}{3x^5} + 2\sqrt{x} + \sqrt{2}$

$= x^3 - \frac{1}{3}x^{-5} + 2x^{1/2} + \sqrt{2}$

$f'(x) = 3x^2 + \frac{5}{3}x^{-6} + x^{-1/2}$

5. Find the equation of the line tangent to $f(x) = (2x+1)(x^2 - x + 3)$ at the point where $x = 0$.

Slope: $f'(x) = (2)(x^2 - x + 3) + (2x+1)(2x-1)$

$m = f'(0) = (2)(3) + (1)(-1) = 6 - 1 = 5$

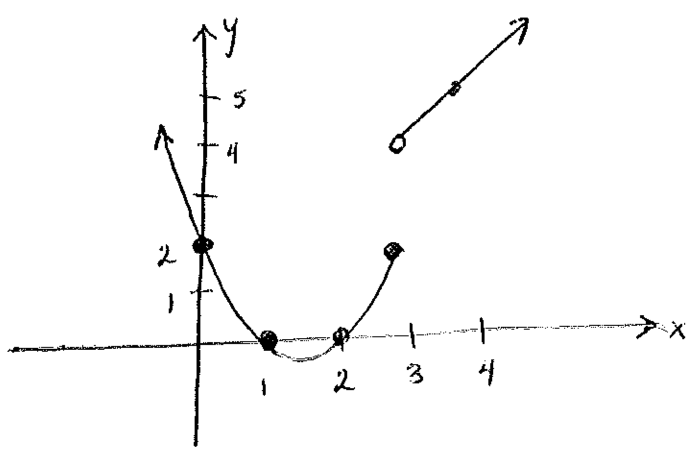
point: $x = 0$

$y = f(0) = (1)(3) = 3 \quad (0,3)$

Line: $y - 3 = 5(x - 0)$

$y = 5x + 3$

6. Graph the function $f(x) = \begin{cases} x^2 - 3x + 2 & \text{if} \quad x \le 3 \\ x+1 & \text{if} \quad x > 3 \end{cases}$. Your graph should be clearly labeled and large enough for me to see everything easily.



Note: to get the point where $x = 3$, fill in to top:
$f(3) = 3^2 - 3 \cdot 3 + 2$
$= 2$

to get the hole, fill $x = 3$ into the other part,
$y = 3 + 1 = 4$
hole at $(3, 4)$

(a) For what values of x is $f(x)$ discontinuous? $x = 3$

(b) Find $\lim_{x \to 0} f(x)$. $= 2$

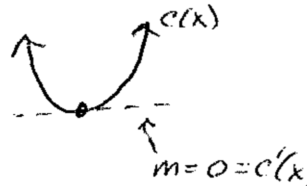
(c) Find $\lim_{x \to 3^-} f(x)$. $= 2$

(d) Find $\lim_{x \to 3^+} f(x)$. $= 4$

(e) Find $\lim_{x \to 3} f(x)$. DNE

7. Suppose that the total cost to produce x units of a commodity is given by $C(x) = 2x^2 - 12x + 30$ dollars. Using calculus, determine how many units should be produced in order to minimize cost. What is the minimum cost?

Notice the graph of $C(x)$ is a parabola that opens up. To find the minimum, we can find the value of x that makes $C'(x) = 0$, since that means the tangent line has slope zero (which happens at the bottom).



$C'(x) = 4x - 12 = 0$,
so $x = 3$ will give the minimum cost.

This minimum cost is then
$C(3) = 2(9) - 36 + 30 = \$12$

8. Find the derivative of $y = \dfrac{(3x+1)^2}{\left(\sqrt[3]{x^2} + 10x^3\right)\left(2x^4 - 6\right)}$. $= \dfrac{9x^2 + 6x + 1}{(x^{2/3} + 10x^3)(2x^4 - 6)}$

$$y' = \frac{(18x+6)\left[(x^{2/3}+10x^3)(2x^4-6)\right] - (9x^2+6x+1)\left[(x^{2/3}+10x^3)(2x^4-6)\right]'}{\left[(x^{2/3}+10x^3)(2x^4-6)\right]^2}$$

$$y' = \frac{(18x+6)\left[(x^{2/3}+10x^3)(2x^4-6)\right] - (9x^2+6x+1)\left[\left(\tfrac{2}{3}x^{-1/3} + 30x^2\right)(2x^4-6) + (x^{2/3}+10x^3)(8x^3)\right]}{\left[(x^{2/3}+10x^3)(2x^4-6)\right]^2}$$