You have 50 minutes to complete this test. You must show all work to receive full credit. Work any 7 of the following 8 problems. Clearly CROSS OUT the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = x^2 - 3x + 1$.

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" or "denominator is zero" are not sufficient explanations).

   (a) $\lim_{x \to 3} \frac{x + 1}{x + 5}$

   (b) $\lim_{x \to 3} \frac{9 - x^2}{x - 3}$

   (c) $\lim_{x \to -2} \frac{x + 1}{x + 2}$
3. Suppose all $x$ units of a product can be sold if the price is set at $p(x) = -x^2 + 4x + 10$. Also assume that the total cost to produce all $x$ units is $C(x) = \frac{1}{3}x^2 + 2x + 39$.

(a) Find an equation for profit when $x$ units are produced.
(b) Using marginal analysis, estimate the change in profit derived from the production and sale of the $5^{th}$ unit.

4. Find $f'(x)$ if:

a) $f(x) = \frac{2x - 3}{x^3}$

b) $f(x) = x^3 - \frac{1}{3x^5} + 2\sqrt{x} + \sqrt{2}$
5. Find the equation of the line tangent to \( f(x) = (2x + 1)(x^2 - x + 3) \) at the point where \( x = 0 \).

6. Graph the function \( f(x) = \begin{cases} x^2 - 3x + 2 & \text{if } x \leq 3 \\ x + 1 & \text{if } x > 3 \end{cases}. \) Your graph should be clearly labeled and large enough for me to see everything easily.

(a) For what values of \( x \) is \( f(x) \) discontinuous?

(b) Find \( \lim_{x \to 0} f(x) \).

(c) Find \( \lim_{x \to 3^-} f(x) \).

(d) Find \( \lim_{x \to 3^+} f(x) \).

(e) Find \( \lim_{x \to 3} f(x) \).
7. Suppose that the total cost to produce \( x \) units of a commodity is given by 
\[ C(x) = 2x^2 - 12x + 30 \text{ dollars}. \] Using calculus, determine how many units should be produced in order to minimize cost. What is the minimum cost?

8. Find the derivative of 
\[ y = \frac{(3x + 1)^2}{\left(\sqrt{x^2} + 10x^3\right)\left(2x^4 - 6\right)}. \]