You have 60 minutes to complete this test. You must show all work to receive full credit. Work any 7 of the following 8 problems. Clearly cross out the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find \( f'(x) \) if \( f(x) = 2 - \frac{x}{4} - x^2 \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(2 - \frac{x+h}{4} - (x+h)^2) - (2 - \frac{x}{4} - x^2)}{h}
\]

\[
= \lim_{h \to 0} \frac{-x-h}{4} - x^2 - 2xh - h^2 + \frac{x}{4} + x^2
\]

\[
= \lim_{h \to 0} \frac{-h}{4} - 2x - h
\]

\[
= \lim_{h \to 0} \left( -\frac{1}{4} - 2x \right) = \left[ -\frac{1}{4} - 2x \right]
\]

2. Evaluate the following limits. If any of them do not exist, explain why not (“because it's undefined” and “denominator is zero” are not sufficient explanations).

(a) \( \lim_{x \to 3} \frac{x+3}{x^2-9} \)

\[
= \lim_{x \to 3} \frac{x+3}{(x+3)(x-3)} = \lim_{x \to 3} \frac{1}{x-3}
\]

\[
= \lim_{x \to 3} \frac{1}{-3} = \frac{1}{-3}
\]

(b) \( \lim_{x \to 5} \frac{2x^2 + 9x - 5}{x^2 + 5x} \)

\[
= \lim_{x \to 5} \frac{(2x-1)(x+5)}{x(x+5)}
\]

\[
= \lim_{x \to 5} \frac{2x-1}{x} = \frac{-10-1}{5} = \frac{-11}{5}
\]

(c) \( \lim_{x \to 3} \frac{2x^2 + x - 3}{x^3 + 4} \)

\[
= \lim_{x \to 3} \frac{2x+1-3}{1+4} = \frac{0}{5} = \frac{0}{5}
\]
3. Find the equation of the line through \((4, -7)\) and perpendicular to the line \(3x + 2y = 1\).

\[
\begin{align*}
3x + 2y &= 1 \\
2y &= -3x + 1 \\
y &= \frac{-3}{2}x + \frac{1}{2}
\end{align*}
\]

old \(m = -\frac{3}{2}\), so perpendicular \(m = \frac{2}{3}\), point is \((4, -7)\).

\[
\text{Line: } y + 7 = \frac{2}{3}(x - 4)
\]

\[
y = \frac{2}{3}x - \frac{8}{3} - 7
\]

\[
y = \frac{2}{3}x - \frac{29}{3}
\]

4. Find \(y'\) for the following functions (do not simplify):

a) \(y = \left(8x^2 - 3\sqrt{x} + \frac{3}{4x^2}\right)(5x^{-3} + 7) = \left(8x^2 - 3x^{\frac{3}{2}} + \frac{3}{4}x^{-2}\right)(5x^{-3} + 7)\)

\[
y' = \left(16x - \frac{3}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-3}\right)(5x^{-3} + 7) + \left(8x^2 - 3x^{\frac{3}{2}} + \frac{3}{4}x^{-2}\right)(-15x^{-4})
\]

b) \(y = \frac{\sqrt{x} + 1}{3x^4 - 5} = \frac{x^{\frac{1}{2}} + 1}{3x^4 - 5}\)

\[
y' = \left(\frac{1}{2}x^{-\frac{1}{2}}\right)(3x^4 - 5) - (x^{\frac{1}{2}} + 1)(12x^3)
\]

\[
\frac{1}{(3x^4 - 5)^2}
\]
5. Suppose a company produces $x$ custom tablets each week, and it costs the company $350 per tablet to produce them. Suppose the company sells each tablet for $800 - x$ dollars, and at that price all of the tablets will be sold.

a) Find the revenue equation. \[ R(x) = (800-x)(x) = 800x - x^2 \]

b) Find the profit equation. \[ P(x) = (800x - x^2) - 350x = 450x - x^2 \]

c) What is marginal profit?
\[ P'(x) = 450 - 2x \]

d) If the company is currently producing 160 netbooks per week, should it increase or decrease production in order to raise its profit? Explain your answer.
\[ P'(160) = 450 - 320 = 130 \]
Production should be increased, since the next tablet will bring in $130$ extra profit. (Keep making more until marginal profit is zero.)

6. Find the equation of the line tangent to \( f(x) = \frac{\sqrt{x} (2-x^2)}{x} \) at the point where \( x = 4 \).

\[ \text{point} : \ x = 4, \ y = f(4) = \frac{\sqrt{4} (2-16)}{4} = \frac{2-14}{4} = -2 \]
\[ (4, -2) \]

\[ \text{slope} : \ f(x) = x^{\frac{1}{2}} (2-x^2) x^{-1} = x^{\frac{1}{2}} (2x^{-1} - x) = 2x^{-\frac{1}{2}} - x^{\frac{3}{2}} \]
\[ f'(x) = -x^{-\frac{3}{2}} - \frac{3}{2} x^{-\frac{1}{2}} \]
\[ f'(4) = -4^{-\frac{3}{2}} - \frac{3}{2} \sqrt{4} = -\frac{1}{(\sqrt{4})^3} - 3 = -\frac{1}{2^3} - 3 \]
\[ = -\frac{1}{8} - 3 = -\frac{25}{8} = m \]

\[ \text{line} : \ y + 2 = -\frac{25}{8} (x-4) \]
7. Consider the graph of the function $f(x)$ given below.

a) Find $\lim_{x \to -1} f(x)$. 2
b) Find $\lim_{x \to -2} f(x)$. $-\infty$
c) Find $\lim_{x \to \infty} f(x)$. 2
d) Find $\lim_{x \to 2} f(x)$. DNE
e) Find $\lim_{x \to 1} f(x)$. 2
f) Find $\lim_{x \to -\infty} f(x)$. -2

g) List the intervals where $f(x)$ is continuous.

$f(x)$ is continuous on $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

8. Graph the function $f(x) = \begin{cases} 
\frac{1}{x} & \text{if } x \leq 2 \\
x & \text{if } x > 2 
\end{cases}$.

Be sure your graph is large enough for me to see and that it is clearly labeled. Then describe the continuity of the function based on your graph.

$f$ is continuous on $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$