NAME KEY

Math 1212
Test 2
Fall 2016

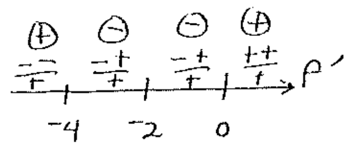
You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x)=\dfrac{x^2}{x+2}$. Find all vertical and horizontal asymptotes (if none, say so), list the intervals where the function is increasing and where it is decreasing, and find all of the maximum and minimum points.

Vertical asymp: $x=-2$
Horizontal asymp: none

$$f'(x)=\frac{(2x)(x+2)-(x^2)(1)}{(x+2)^2}=\frac{2x^2+4x-x^2}{(x+2)^2}=\frac{x^2+4x}{(x+2)^2}=\frac{x(x+4)}{(x+2)^2}$$

$\underline{CN}$: $x=-4,-2,0$



increasing on $(-\infty,-4)\cup(0,\infty)$
decreasing on $(-4,-2)\cup(-2,0)$

max at $(-4,-8)$
min at $(0,0)$

2. For the function $f(x)=x^4-2x^2+3$, find all critical numbers. Then use the second derivative test to tell whether each critical number will result in a maximum or a minimum (you don't need to find y-values).

$$\begin{aligned} f'(x) &= 4x^3-4x \\ &= 4x(x^2-1) \\ &= 4x(x+1)(x-1) \end{aligned}$$

$\underline{CN}$: $x=0,-1,1$

$f''(x)=12x^2-4$

$f''(0)=-4<0$ concave $\underline{\text{down}}$ $\frown$ so $x=0$ gives a max

$\left.\begin{aligned} f''(-1) &= 8 > 0 \\ f''(1) &= 8 > 0 \end{aligned}\right\}$ concave $\underline{\text{up}}$ $\smile$ so $x=1$, $x=-1$ give minima.

3. Suppose $q(p) = 500 - 2p$ units of a product are demanded when price is p dollars per unit.

(a) Calculate the elasticity of demand when $p = 200$. At this price, is the demand elastic or inelastic?

$$E(p) = \frac{p}{q} \cdot q' = \frac{p}{500-2p}(-2) = \frac{-2p}{500-2p} = \frac{-p}{250-p} = \frac{p}{p-250}$$

$$E(200) = \frac{200}{200-250} = \frac{200}{-50} = -4$$ ← elasticity of demand

$|E(200)| = |-4| = 4 > 1$, so demand is elastic.

(b) Write a sentence explaining the meaning of your answer in (a). Be as specific as possible.

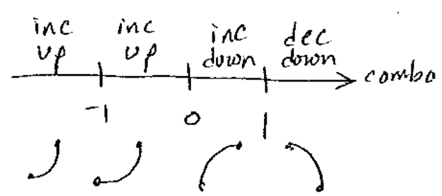
If price goes up 1%, demand will go down 4%.
(In this case, $q(200) = 100$, so if price goes from \$200 to \$202, demand will go from 100 to 96.)

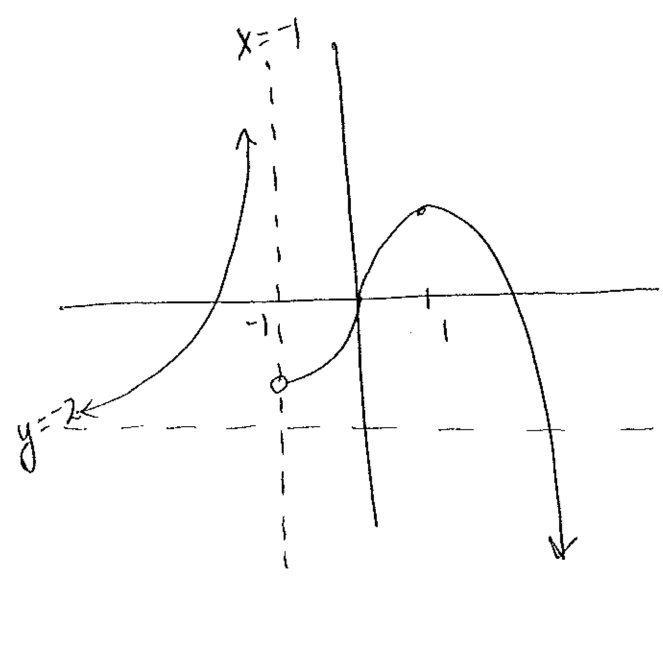
(c) Give an example of a product in this price range that might behave this way.

"Luxury" items that cost about \$200.
Maybe jewelry, really good wine ...

4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

(a) $f'(x) > 0$ when $x < 1$, but $f'(x) \le 0$ otherwise increasing / decreasing
(b) $f''(x) > 0$ when $x < -1$ and when $-1 < x < 0$, but $f''(x) \le 0$ otherwise conc up / down
(c) $f(x)$ is undefined when $x = -1$ ← hole or asymp
(d) $\lim_{x \to -\infty} f(x) = -2$. ← HA $y = -2$ on left





5. A retailer has purchased several cases of imported wine as an investment. As the wine ages, its value initially increases, but eventually the wine will pass its prime and its value will decrease. Suppose that x years from now, the value of a case will be ***changing at a rate*** of $53-10x$ dollars per year. Suppose in addition that storage rates for the cases of wine stay the same at \$3 per case per year. When should the retailer sell the wine in order to make the most profit?

Profit = value − storage cost

$P = V - 3x$ (for one case)

$P' = V' - 3$ V' is rate value changes, $= 53-10x$

$P' = 53-10x-3 = 50-10x = 10(5-x)$

CN: $x=5$.

max?

(1) $P'' = -10$

$P''(5) = -10$ ∩ conc down,

so $x=5$ years will maximize profit

or (2)

max

$+ \quad - \quad P'$

5

6. Find the equation of the line tangent to the graph of $x^2y - 2xy^3 + 6 = 2x + 2y$ at the point $(0,3)$.

$$(x^2)(y) - (2x)(y^3) + 6 = 2x + 2y$$

$$2xy + x^2y' - (2y^3 + 2x(3y^2y')) = 2 + 2y'$$

$$2(0)(3) + (0)(y') - (2(27) + 2(0)(3(9)y')) = 2 + 2y'$$

$$-54 = 2 + 2y'$$

$$-56 = 2y'$$

$$-28 = y' = m$$

Line: $y - 3 = -28(x - 0)$

or $y = -28x + 3$

7. Find the absolute minimum and absolute maximum *points* of $f(x)=(x^2-4)^3$ on the interval $[-3,1]$.

$$f'(x)=3(x^2-4)^2(2x)$$
$$=3\left[(x+2)(x-2)\right]^2(2x)$$

CN : $x=-2,\cancel{2},0$

not in interval.

check CN's and endpoints:

$f(-2)=0$

$f(0)=-64$ ← abs. min at $(0,-64)$

$f(-3)=125$ ← abs max at $(-3,125)$

$f(1)=-27$

8. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x)=\left(x+\dfrac{1}{x}\right)^2-\dfrac{5}{\sqrt{3x}}$ $=(x+x^{-1})^2-\dfrac{5}{\sqrt{3}}x^{-1/2}$

$$f'(x)=2(x+x^{-1})^1(1-x^{-2})+\frac{5}{2\sqrt{3}}x^{-3/2}$$

(b) $f(x)=\dfrac{(3x+1)^3}{(1-3x)^4}$

$$f'(x)=\frac{3(3x+1)^2(3)(1-3x)^4-(3x+1)^3(4)(1-3x)^3(-3)}{(1-3x)^8}$$