NAME Key

Math 1212
Test 2
Fall 2014

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x)=\frac{1-x}{x^2}$. List the intervals where the function is increasing and where it is decreasing, and find all of the maximum and minimum *points*.

$$f'(x)=\frac{(-1)(x^2)-(1-x)(2x)}{x^4}=\frac{-x^2-2x+2x^2}{x^4}=\frac{x^2-2x}{x^4}=\frac{x(x-2)}{x^4}$$

critical #'s : $x=0,\ x=2$

f': $\oplus$ on $(-\infty,0)$, $\ominus$ on $(0,2)$, $\oplus$ on $(2,\infty)$

increasing on $(-\infty,0)\cup(2,\infty)$

decreasing on $(0,2)$

$f(0)$ is undefined

$f(2)=-1/4$

minimum at $(2,-1/4)$

no maximum

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

(a) $f(x)=\frac{2x^2}{x^2+x-6}=\frac{2x^2}{(x+3)(x-2)}$

vertical: $x=-3$, $x=2$

horizontal: $y=2$

(b) $f(x)=\frac{x}{x^2-4x}=\frac{x}{x(x-4)}$

vertical: $x=4$

(notice $x=0$ gives a HOLE)

horizontal: $y=0$

(c) $f(x)=\frac{x^2-5x+5}{1}$

vertical: none

horizontal: none

3. Suppose $q(p) = p^2 - 40p + 400$ units of a product are demanded when price is p (in thousands of dollars) per unit.

a) Calculate the price elasticity of demand when $p = 15$. At this price, is the demand elastic or inelastic?

$$E(p) = \frac{p}{q} \cdot q' = \frac{p}{p^2 - 40p + 400} \cdot (2p - 40)$$

$$E(15) = \frac{15}{225 - 600 + 400}(-10) = \frac{-150}{25} = -6$$

$|E(15)| = |-6| = 6 > 1$, so demand is elastic

b) Write a sentence explaining the meaning of your answer in (a) in plain language. Be as specific as possible.

when the price is \$15,000 and increases 1%, to \$15,150, the demand will decrease 6%, from $q(15) = 25$ to 23.5 units.

c) Give an example of a product that might behave this way.

An "elastic" product priced around \$15,000 would be a luxury item, like jewelry, a boat, etc.

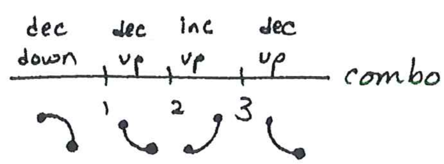
4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

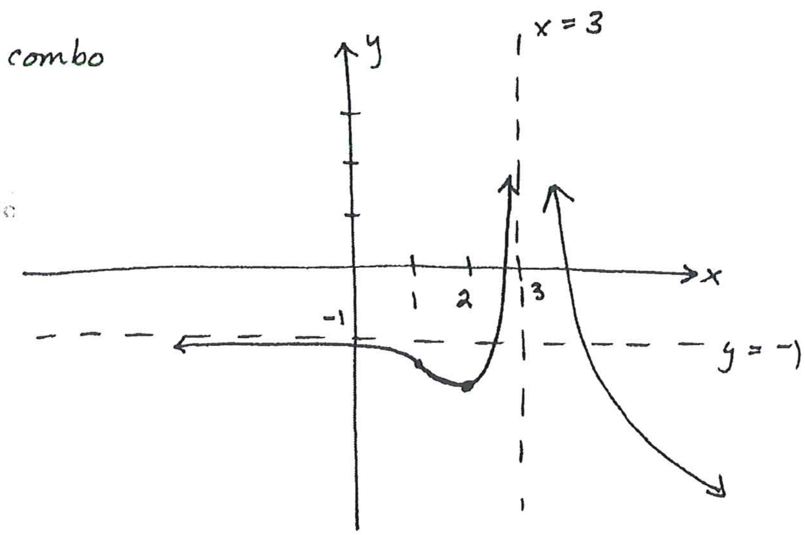
a) $f'(x) > 0$ on the interval $(2,3)$, but $f'(x) \le 0$ otherwise

b) $f''(x) > 0$ on the interval $(1,3) \cup (3,\infty)$, but $f''(x) \le 0$ otherwise

c) $f(x)$ is undefined when $x = 3$ ← asymp. or hole when $x = 3$

d) $\lim_{x \to -\infty} f(x) = -1$. ← horiz. asymp $y = -1$, gets close as $x \to -\infty$





5. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \left(\dfrac{2x+5}{x^2+1}\right)^4$

$$f'(x) = 4\left(\frac{2x+5}{x^2+1}\right)^3\left(\frac{(2)(x^2+1)-(2x+5)(2x)}{(x^2+1)^2}\right)$$

(b) $f(x) = \sqrt{2x} + \dfrac{1}{\sqrt{2x}} = \sqrt{2}\, x^{1/2} + \dfrac{1}{\sqrt{2}} x^{-1/2}$

$$f'(x) = \frac{\sqrt{2}}{2} x^{-1/2} - \frac{1}{2\sqrt{2}} x^{-3/2}$$

Alternate method:

$f(x) = (2x)^{1/2} + (2x)^{-1/2}$

$f'(x) = \frac{1}{2}(2x)^{-1/2}(2) + \frac{-1}{2}(2x)^{-3/2}(2)$

(c) $f(x) = (6x+1)^7(2x-3)^3$

$$f'(x) = 7(6x+1)^6(6)(2x-3)^3 + (6x+1)^7(3)(2x-3)^2(2)$$

6. Find the equation of the line tangent to the graph of $y^2 + xy - x^2 = 5$ at the point $(4,3)$.

$2yy' + (1)(y) + (x)(y') - 2x = 0$

$x = 4,\ y = 3$, so

$2(3)y' + 3 + 4y' - 2(4) = 0$

$10y' + 3 - 8 = 0$

$10y' = 5$

$y' = 1/2 = m$

$y - 3 = 1/2\,(x-4)$

$y = \frac{1}{2}x - 2 + 3$

$y = \frac{1}{2}x + 1$

7. Find the absolute minimum and absolute maximum *points* of $f(x)=\frac{x}{x^2+1}$ on the interval $0 \le x \le 2$.

$$f'(x) = \frac{(1)(x^2+1)-(x)(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{(1-x)(1+x)}{(x^2+1)^2}$$

critical #'s : $x = 1, -1$

$f(0) = 0$

$f(1) = 1/2$

$f(2) = 2/5$

↑

check critical #'s and endpts.
Don't check $x=-1$, since it's outside the Interval

absolute minimum $(0,0)$
absolute maximum $(1, 1/2)$

8. A satellite TV company has 4800 subscribers to an add-on package who are each paying \$18 per month for the bonus channels. The company can get 150 more subscribers for each \$0.50 decrease in the monthly fee. What rate will yield the maximum revenue (be sure your solution is a *maximum*), and what will this maximum revenue be?

Revenue = price · quantity

Let x = # of \$0.50 decreases

$R = (18 - 0.50x)(4800 + 150x)$

$= 86400 - 2400x + 2700x - 75x^2$

$= -75x^2 + 300x + 86400$

$R' = -150x + 300 = 0$

$x = 2$ is crit #.

$$\begin{array}{ccc} + & & - \quad R' \\ \hline & 2 & \end{array}$$

↑
max

OR : use 2nd deriv test
$R'' = -150$
$R''(2) < 0$, concave down, max.

max revenue is when $x=2$, so the rate should be \$17.

max revenue is

$R(2) = (18-1)(4800+300)$
$= 17(5100)$
$= \$86700$