You have 50 minutes to complete this test. You must show all work to receive full credit. Work any 7 of the following 8 problems. Clearly cross out the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x) = \frac{1-x}{x^2}$. List the intervals where the function is increasing and where it is decreasing, and find all of the maximum and minimum points.

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a line, not a number). If there are no asymptotes, say so.

(a) $f(x) = \frac{2x^2}{x^2 + x - 6}$

(b) $f(x) = \frac{x}{x^2 - 4x}$

(c) $f(x) = x^2 - 5x + 5$
3. Suppose \( q(p) = p^2 - 40p + 400 \) units of a product are demanded when price is \( p \) (in thousands of dollars) per unit.

a) Calculate the price elasticity of demand when \( p = 15 \). At this price, is the demand elastic or inelastic?

b) Write a sentence explaining the meaning of your answer in (a) in plain language. Be as specific as possible.

c) Give an example of a product that might behave this way.

4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

a) \( f'(x) > 0 \) on the interval \((2,3)\), but \( f'(x) \leq 0 \) otherwise

b) \( f''(x) > 0 \) on the interval \((1,3) \cup (3,\infty)\), but \( f''(x) \leq 0 \) otherwise

c) \( f(x) \) is undefined when \( x = 3 \)

d) \( \lim_{x \to \infty} f(x) = -1 \).
5. Find $f''(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \left(\frac{2x + 5}{x^2 + 1}\right)^4$

(b) $f(x) = \sqrt{2x} + \frac{1}{\sqrt{2x}}$

(c) $f(x) = (6x + 1)^3 (2x - 3)^3$

6. Find the equation of the line tangent to the graph of $y^2 + xy - x^2 = 5$ at the point (4,3).
7. Find the absolute minimum and absolute maximum points of \( f(x) = \frac{x}{x^2 + 1} \) on the interval \( 0 \leq x \leq 2 \).

8. A satellite TV company has 4800 subscribers to an add-on package who are each paying $18 per month for the bonus channels. The company can get 150 more subscribers for each $0.50 decrease in the monthly fee. What rate will yield the maximum revenue (be sure your solution is a maximum), and what will this maximum revenue be?
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