NAME KEY

Math 1212
Test 2
Summer 2015

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x)=2x^3+6x^2-18x+5$. Find all extreme points and list the intervals of increase and decrease. Then list the intervals where the function is concave up and where it is concave down, and find all inflection points.

$f'(x) = 6x^2+12x-18$
$= 6(x^2+2x-3)$
$= 6(x+3)(x-1)=0$

CN: $x=-3, 1$

f': $\oplus$ on $(-\infty,-3)$, $\ominus$ on $(-3,1)$, $\oplus$ on $(1,\infty)$

$f(-3) = -54+54+54+5 = 59$
$f(1) = 2+6-18+5 = -5$

$f''(x) = 12x+12$
$= 12(x+1)=0$

IN: $x=-1$

f'': $\ominus$ on $(-\infty,-1)$, $\oplus$ on $(-1,\infty)$

$f(-1) = -2+6+18+5$
$= 27$

Results
inc on $(-\infty,-3)\cup(1,\infty)$
dec on $(-3,1)$
max $(-3,59)$
min $(1,-5)$
conc up on $(-1,\infty)$
conc down on $(-\infty,-1)$
inf. pt $(-1,27)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

(a) $f(x)=\dfrac{1-3x^2}{2x^2-4x+2} = \dfrac{1-3x^2}{2(x^2-2x+1)} = \dfrac{1-3x^2}{2(x-1)(x-1)}$

VA: $x=1$
HA: $y=-3/2$

(b) $f(x)=\dfrac{x-3}{x^2+x-12} = \dfrac{(x-3)}{(x-3)(x+4)}$

VA: $x=-4$
(notice $x=3$ gives a hole)
HA: $y=0$

(c) $f(x)=(1+x^2)^3$

VA: none
HA: none

3. Suppose the demand for a product is given by $q(p) = 500 - 2p$ where p is price. When $p = 100$, find the price elasticity of demand. Is demand elastic or inelastic at this price? Give an example of a product that might behave in this way.

$$E(p) = \frac{p}{q} \cdot q' = \frac{p}{500-2p} \cdot (-2) = \frac{-p}{250-p}$$

$$E(100) = \frac{-100}{250-100} = \frac{-100}{150} = -2/3$$

$|E(100)| = 2/3 < 1$, so inelastic

for \$100, inelastic demand could be ... medication, electric bill, car payment (should be an obvious necessity.)

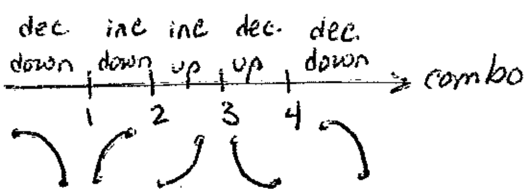
4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

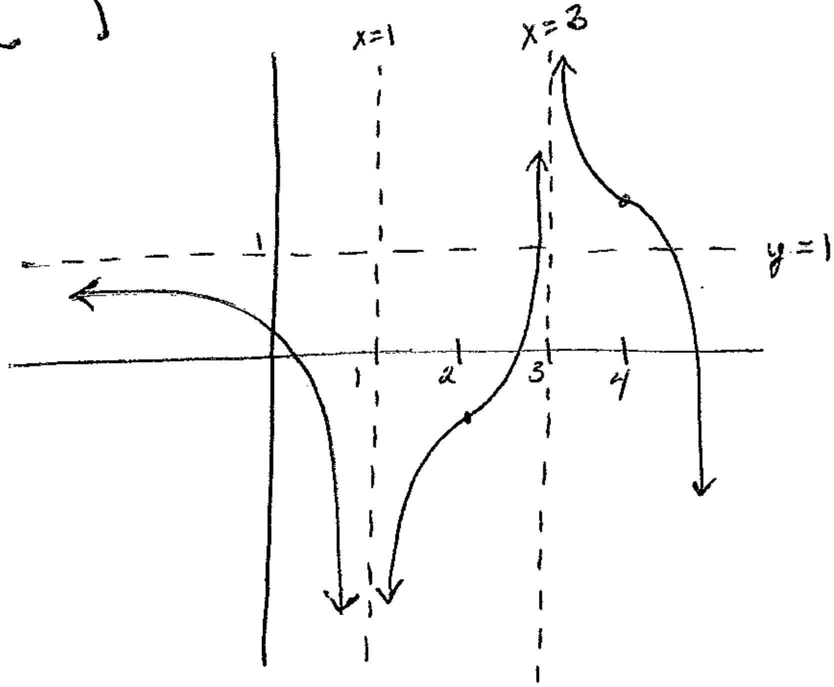
a) $f'(x) > 0$ when $1 < x < 3$, and $f'(x) \le 0$ otherwise

b) $f''(x) > 0$ when $2 < x < 3$ and when $3 < x < 4$, but $f''(x) \ge 0$ otherwise

c) $f(1)$ and $f(3)$ are undefined (holes or asymp)

d) $\lim_{x \to -\infty} f(x) = 1$. HA: $y=1$, on left side.





5. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x)=\dfrac{1-5x^2}{\sqrt{3+2x}} = \dfrac{1-5x^2}{(3+2x)^{1/2}}$

$$f'(x) = \frac{(-10x)(3+2x)^{1/2} - (1-5x^2)\left(\frac{1}{2}\right)(3+2x)^{-1/2}(2)}{3+2x}$$

(b) $f(x)=x^3(2x^2+x-3)^2$

$$f'(x) = (2x)(2x^2+x-3)^2 + (x^3)(2)(2x^2+x-3)(4x+1)$$

6. Find y' if $5x - x^2y^3 = 2y$.

$$5x - (x^2)(y^3) = 2y$$

Take derivative on both sides:

$$5 - [(2x)(y^3) + (x^2)(3y^2y')] = 2y'$$

$$5 - 2xy^3 - 3x^2y^2y' = 2y'$$

$$-3x^2y^2y' - 2y' = -5 + 2xy^3$$

$$y'(-3x^2y^2-2) = 2xy^3-5$$

$$y' = \frac{2xy^3-5}{-3x^2y^2-2}$$

7. Find the absolute minimum and absolute maximum *points* of $f(x)=3x^5-5x^3$ on the interval $-2 \le x \le 0$.

$$f'(x) = 15x^4 - 15x^2$$
$$= 15x^2(x^2-1)$$
$$= 15x^2(x-1)(x+1)$$

CN : $x = 1, -1$

not in interval, discard $x=1$.

$$f(-1) = 3(-1)^5 - 5(-1)^3 = -3+5 = 2$$
$$f(-2) = 3(-2)^5 - 5(-2)^3 = 3(-32) - 5(-8) = -96+40 = -56$$
$$f(0) = 0$$

abs max $(-1, 2)$

abs min $(-2, -56)$

8. Suppose that when the price of a commodity is $p(q)=37-2q$, all q units will be sold. The total cost of producing q units is given by $C(q)=3q^2+5q+75$. Find the quantity that should be produced in order to maximize profit. (Be sure your answer gives a *maximum*.)

Profit = Revenue − Cost

= price · quantity − cost

$$P = (37-2q)(q) - (3q^2+5q+75)$$
$$P = 37q - 2q^2 - 3q^2 - 5q - 75$$
$$P = -5q^2 + 32q - 75$$

$q = 3.2$ will maximize profit.

$$P' = -10q + 32 = 0$$
$$32 = 10q$$

CN: $q = 3.2$

$\oplus$ | $\ominus$ → P'

3.2

max

do one ⟷

$$P'' = -10$$
$$P''(3.2) = -10 < 0$$

max