You have 50 minutes to complete this test. You must show all work to receive full credit. Work any 7 of the following 8 problems. Clearly Cross Out the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve \( \frac{dy}{dx} = \sqrt{xy} \) for \( y \).

\[
\frac{dy}{dx} = x^{1/2} y^{1/2}
\]

\[
\int y^{-1/2} \, dy = \int x^{-1/2} \, dx
\]

\[
2y^{1/2} = \frac{2}{3} x^{3/2} + C
\]

\[
y^{1/2} = \frac{1}{3} x^{3/2} + \frac{1}{2} C
\]

\[
y = \left( \frac{1}{3} x^{3/2} + \frac{1}{2} C \right)^2
\]

2. Evaluate \( \int \frac{(\ln 2x)^3}{5x} \, dx \).

Let \( u = \ln 2x \).
Then \( du = \frac{1}{2x} \cdot 2 \, dx \)

\[
du = \frac{1}{x} \, dx
\]

\[
\int \frac{(\ln 2x)^3}{5x} \, dx = \frac{1}{5} \int (\ln 2x)^3 \cdot \frac{1}{x} \, dx
\]

\[
= \frac{1}{5} \int u^3 \, du
\]

\[
= \frac{1}{5} \left( \frac{1}{4} u^4 \right) + C
\]

\[
= \frac{1}{20} (\ln 2x)^4 + C
\]
3. Find all maxima, minima and inflection points of $f(x) = 5 - 2e^{-x}$. Also give the intervals where $f$ is increasing, decreasing, concave up, and concave down. Find all vertical and horizontal asymptotes, or state that none exist. Then carefully sketch the graph of $f$.

$$f'(x) = -2e^{-x} \quad (-1) = 2e^{-x} = 0$$
never happens, no CNS, so
$$\quad + \quad \rightarrow f'$$

$$f''(x) = 2e^{-x} (-1) = -2e^{-x} = 0$$
never happens, no INS, so
$$\quad \rightarrow f''$$

Results: inc. on $(-\infty, 0)$
dec. never
no max. or min. (don't need)
come up never
come down $(-\infty, 0)$
no intercepts. (don't need)

VA: none, always defined
HA: If $x \to -\infty$, $5 - 2e^{-x} = 5 - \frac{x}{e^{\ln 2}} = 5$
If $x \to +\infty$, $5 - 2e^{-x} = 5 - 2e^{-y} \to -\infty$
y = 5 on right side

Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^3 \ln(x^2 + 3)$
$$f'(x) = 3x^2 \ln(x^2 + 3) + (x^3) \left(\frac{1}{x^2 + 3}\right)(2x)$$

(b) $f(x) = \frac{\ln \sqrt{x}}{e^{-2x} + 1}$
$$f'(x) = \frac{\left(\frac{1}{2x^{3/2}}\right)\left(\frac{1}{x^{1/2}}\right)(e^{-2x} + 1) - (\ln \sqrt{x})(-2x)(e^{-2x} - 2)}{(e^{-2x} + 1)^2}$$
Suppose you win a sweepstakes, and you get to choose how your prize money will be distributed to you. You can either take a lump sum payment of $10,000 now, or you can receive your prize as three payments: $4000 now, $4000 in one year, and $4000 in two years. The prevailing annual interest rate is 8% compounded continuously. Which method of payment will you choose, and why?

\[ B = Pe^{rt} \]

<table>
<thead>
<tr>
<th>( t )</th>
<th>( B )</th>
<th>( \text{Amount} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$10,000</td>
<td>( e^{0 \cdot 0.08} = 1 )</td>
</tr>
<tr>
<td>3</td>
<td>( B )</td>
<td>( e^{3 \cdot 0.08} \approx 11,735.11 )</td>
</tr>
</tbody>
</table>

\[ 3 \text{ payments} \]

\[ \begin{align*}
&\text{\$4000 now} \quad B_1 = 4000 e^{0.08(0)} \approx 4694.04 \\
&\text{\$4000 in 1yr} \quad B_2 = 4000 e^{0.08(1)} \approx 4333.15 \\
&\text{\$4000 in 2yrs} \quad B_3 = 4000 e^{0.08(2)} \approx 13,027.19
\end{align*} \]

Better to do the payment plan

Suppose 30 grams of a radioactive substance is sitting in a lab. Two years later, there are only 22 grams of radioactive material left. How long will it take until there are just 20 grams left?

\[ B = Pe^{rt} \]

<table>
<thead>
<tr>
<th>( t )</th>
<th>( B )</th>
<th>( \text{Amount} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>( e^{0 \cdot r} = 1 )</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>( \frac{22}{30} = e^{2r} )</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>( \frac{20}{30} = e^{3r} )</td>
</tr>
</tbody>
</table>

New equation: \( B = 30e^{rt} \)

\[ \begin{align*}
22 &= 30e^{2r} \\
\frac{22}{30} &= e^{2r} \\
\ln \frac{22}{30} &= 2r \\
r &= \frac{1}{2} \ln \frac{22}{30} \approx -0.1551
\end{align*} \]

Best equation: \( B = 30e^{-0.1551t} \)

\[ \begin{align*}
20 &= 30e^{-0.1551t} \\
\frac{2}{3} &= e^{-0.1551t} \\
\ln \frac{2}{3} &= -0.1551t \\
t &= \frac{\ln \frac{2}{3}}{-0.1551} \approx 2.6 \text{ years}
\end{align*} \]
7. a) If \( \ln x = \frac{1}{3}(\ln 16 + 2 \ln 2) \), solve for \( x \). Your answer should be an integer, and be sure to show all your steps (i.e., don’t just use your calculator).

\[
x = e^{\frac{1}{3}(\ln 16 + 2 \ln 2)} = e^{\ln 16^{\frac{1}{3}} + \ln 2^{\frac{2}{3}}}
= e^{\ln (16^{\frac{1}{3}} \cdot 2^{\frac{2}{3}})}
= 16^{\frac{1}{3}} \cdot 2^{\frac{2}{3}}
= 2^{\frac{4}{3}} \cdot 2^{\frac{2}{3}}
= 2^{\frac{4}{3} + \frac{2}{3}}
= 2^{\frac{6}{3}}
= 2^2
= 4.
\]

b) If \( \log_3 a = 3 \), \( \log_3 b = 2 \), and \( \log_3 c = -4 \), find \( \log_3 \frac{a^3 \sqrt{b}}{c^2} \).

\[
\log_3 \left( \frac{a^3 \sqrt{b}}{c^2} \right) = \log_3 a^3 + \log_3 b^{\frac{1}{2}} - \log_3 c^2
= 3 \log_3 a + \frac{1}{2} \log_3 b - 2 \log_3 c
= 3(3) + \frac{1}{2}(2) - 2(-4)
= 9 + 1 + 8
= 18.
\]

8. Evaluate \( \int x \ln(x^2) \, dx \).

Let \( u = \ln(x^2) \)
\[
du = \frac{2}{x^2} \cdot 2x \, dx
= \frac{2}{x} \, dx
\]

\[
\int u \, dv = uv - \int v \, du
\int x \ln(x^2) \, dx = (\ln(x^2)) \cdot \left(\frac{1}{2} x^2\right) - \int \frac{1}{2} x^2 \cdot \frac{2}{x} \, dx
= \frac{1}{2} x^2 \ln(x^2) - \int \frac{1}{2} x \, dx
= \frac{1}{2} x^2 \ln(x^2) - \frac{1}{2} x^2 + C.
\]