NAME KEY

Math 1212
Test 3
Fall 2014

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $\frac{dy}{dx} = \sqrt{xy}$ for y.

$$\frac{dy}{dx} = x^{1/2} y^{1/2}$$

$$\int y^{-1/2}\,dy = \int x^{1/2}\,dx$$

$$2y^{1/2} = \frac{2}{3}x^{3/2} + C$$

$$y^{1/2} = \frac{1}{3}x^{3/2} + \frac{1}{2}C$$

$$y = \left(\frac{1}{3}x^{3/2} + \frac{1}{2}C\right)^2$$

2. Evaluate $\int \frac{(\ln 2x)^3}{5x}\,dx$.

Let $u = \ln 2x$.

Then $du = \frac{1}{2x} \cdot 2\,dx$

$$du = \frac{1}{x}\,dx$$

$$\int \frac{(\ln 2x)^3}{5x}\,dx = \frac{1}{5}\int (\ln 2x)^3 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{5}\int u^3\,du$$

$$= \frac{1}{5}\left(\frac{1}{4}u^4\right) + C$$

$$= \frac{1}{20}(\ln 2x)^4 + C$$

3. Find all maxima, minima and inflection points of $f(x) = 5 - 2e^{-x}$. Also give the intervals where f is increasing, decreasing, concave up, and concave down. Find all vertical and horizontal asymptotes, or state that none exist. Then carefully sketch the graph of f.

$f'(x) = -2e^{-x}(-1) = 2e^{-x} = 0$

never happens, no CNS, so

$+$ ⟶ f'

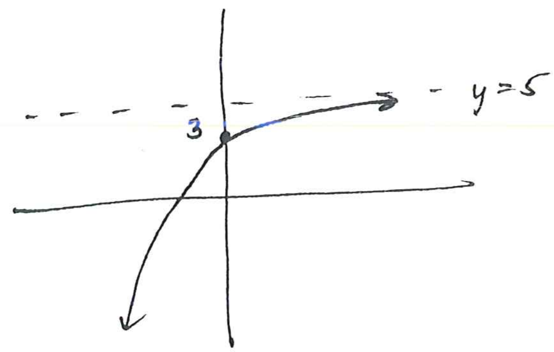
$f''(x) = 2e^{-x}(-1) = -2e^{-x} = 0$

never happens, no INS, so

$-$ ⟶ f''

inc down ⟶ combo

Results: inc on $(-\infty, \infty)$
dec never
~~no max or min~~ (don't need)
conc up never
conc down $(-\infty, \infty)$
~~no inf pts.~~ (don't need)
VA none
HA $y = 5$



VA: none, always defined

HA: If $x \to \infty$, $5 - 2e^{-x} = 5 - \frac{2}{e^{big}} = 5$

If $x \to -\infty$, $5 - 2e^{-x} = 5 - 2e^{big} \to -\infty$

$y = 5$ on right side

notice $f(0) = 5 - 2e^{0} = 5 - 2 = 3$

4. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^3 \ln(x^2 + 3)$

$f'(x) = 3x^2 \ln(x^2+3) + (x^3)\left(\frac{1}{x^2+3}\right)(2x)$

(b) $f(x) = \frac{\ln\sqrt{x}}{e^{-2x} + 1}$

$$f'(x) = \frac{\left(\frac{1}{x^{1/2}}\right)\cdot\left(\frac{1}{2}x^{-1/2}\right)\left(e^{-2x}+1\right) - \left(\ln\sqrt{x}\right)\left(e^{-2x}(-2)\right)}{\left(e^{-2x}+1\right)^2}$$

5. Suppose you win a sweepstakes, and you get to choose how your prize money will be distributed to you. You can either take a lump sum payment of $10,000 now, or you can receive your prize as three payments: $4000 now, $4000 in one year, and $4000 in two years. The prevailing annual interest rate is 8% compounded continuously. Which method of payment will you choose, and why?

How much will I have in 2 years? $B = Pe^{rt}$

$10000 now

$t=0$, $B=10000$
$t=3$, $B=?$ } → $B = 10000\, e^{.08(2)} \approx$ ~~$12,712.49~~ $11,735.11 in 2 years.

3 payments

$4000 now → $B_1 = 4000\, e^{.08(2)} \approx 4694.04$

$4000 in 1yr → $B_2 = 4000\, e^{.08(1)} \approx 4333.15$

$4000 in 2yrs → $B_3 = 4000 = 4000.00$

$\approx$ $13027.19 in 2 years

Better to do the payment plan

6. Suppose 30 grams of a radioactive substance is sitting in a lab. Two years later, there are only 22 grams of radioactive material left. How long will it take until there are just 20 grams left?

$B = Pe^{rt}$

① $t=0$ $B=30$
② $t=2$ $B=22$
③ $t=?$ $B=20$

① $30 = Pe^{r(0)} = P$

new equation: $B = 30e^{rt}$

② $22 = 30e^{r(2)}$

$\frac{22}{30} = e^{2r}$

$\ln\frac{22}{30} = 2r$

$r = \frac{1}{2}\ln\frac{22}{30} \approx -0.1551$

Best equation: $B \doteq 30e^{-0.1551\,t}$

③ $20 \doteq 30\, e^{-0.1551\,t}$

$\frac{2}{3} \doteq e^{-0.1551\,t}$

$\ln\frac{2}{3} \doteq -0.1551\,t$

$t \doteq \frac{\ln 2/3}{-0.1551} \approx 2.6$ years

7. a) If $\ln x = \frac{1}{3}(\ln 16 + 2\ln 2)$, solve for x. Your answer should be an integer, and be sure to show all your steps (i.e., don't just use your calculator).

$$x = e^{\frac{1}{3}(\ln 16 + 2\ln 2)}$$
$$= e^{(\ln 16^{1/3} + \ln 2^{2/3})}$$
$$= e^{\ln(16^{1/3}\cdot 2^{2/3})}$$

$$x = 16^{1/3}\cdot 2^{2/3}$$
$$= (2^4)^{1/3}\cdot 2^{2/3}$$
$$= 2^{4/3}\cdot 2^{2/3} = 2^{6/3} = 2^2 = 4.$$

b) If $\log_3 a = 3$, $\log_3 b = 2$, and $\log_3 c = -4$, find $\log_3 \frac{a^3\sqrt{b}}{c^2}$.

$$\log_3\left(\frac{a^3 b^{1/2}}{c^2}\right) = \log_3 a^3 + \log_3 b^{1/2} - \log_3 c^2$$
$$= 3\log_3 a + \tfrac{1}{2}\log_3 b - 2\log_3 c$$
$$= 3(3) + \tfrac{1}{2}(2) - 2(-4)$$
$$= 9 + 1 + 8$$
$$= 18$$

8. Evaluate $\int x \ln(x^2)\, dx$.

Let $u = \ln(x^2)$ $\qquad dv = x\,dx$

$du = \frac{1}{x^2}\cdot 2x\,dx$ $\qquad v = \int x\,dx = \frac{1}{2}x^2$

$= \frac{2}{x}\,dx$

$$\int u\,dv = uv - \int v\,du$$
$$\int x\ln(x^2)\,dx = (\ln(x^2))\cdot(\tfrac{1}{2}x^2) - \int \tfrac{1}{2}x^2\cdot\tfrac{2}{x}\,dx$$
$$= \tfrac{1}{2}x^2\ln(x^2) - \int x\,dx$$
$$= \tfrac{1}{2}x^2\ln x^2 - \tfrac{1}{2}x^2 + C$$