NAME KEY

Math 1212
Test 3
Fall 2015

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 8 of the following 9 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 12 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $\frac{dy}{dx} = \frac{xy}{\sqrt{1-x^2}}$ if $y = 2$ when $x = 0$.

$\frac{1}{y}\,dy = \frac{x}{(1-x^2)^{1/2}}\,dx$

$\int \frac{1}{y}\,dy = \int x(1-x^2)^{-1/2}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$\ln|y| = \frac{-1}{2}\int u^{-1/2}\,du$

$\ln|y| = -u^{1/2} + C$

$\ln|y| = -\sqrt{1-x^2} + C$

$\ln 2 = C - \sqrt{1-0}$

$\ln 2 + 1 = C$

$\ln|y| = \ln 2 + 1 - \sqrt{1-x^2}$

$|y| = e^{\ln 2 + 1 - \sqrt{1-x^2}}$

$y = \pm e^{\ln 2} \cdot e^{1-\sqrt{1-x^2}}$

$y = \pm 2e^{1-\sqrt{1-x^2}}$

2. Evaluate the following.

(a) $\int 5e^{3x}\,dx = \frac{5}{3}e^{3x} + C$

(b) $\int \frac{3x+6}{2x^2+8x+3}\,dx = \frac{3}{4}\int \frac{1}{u}\,du = \frac{3}{4}\ln|u| + C$

$= \frac{3}{4}\ln|2x^2+8x+3| + C$

$u = 2x^2+8x+3$

$du = (4x+8)dx$

$\frac{1}{4}\,du = (x+2)\,dx \rightarrow \frac{3}{4}\,du = (3x+6)dx$

~~$\frac{3}{4}du = ($[illegible]$+6)dx$~~

3. Find all maxima, minima and inflection points of $f(x) = xe^{-2x}$. Also give the intervals where f is increasing, decreasing, concave up, and concave down. Find all asymptotes. Then carefully sketch the graph of f.

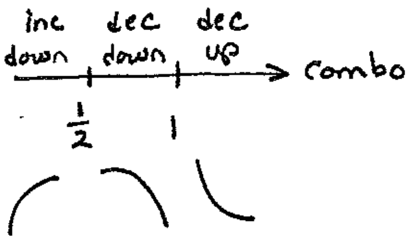
$$f'(x) = e^{-2x} - 2xe^{-2x}$$
$$= e^{-2x}(1-2x) = 0$$

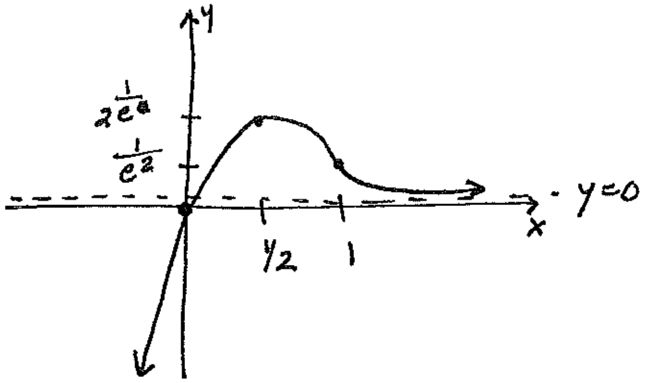
CN : $x = 1/2$ $\quad \xrightarrow[\frac{1}{2}]{+\ \ |\ \ -} f'$

$$f''(x) = -2e^{-2x}(1-2x) - 2e^{-2x} = 0$$
$$= -2e^{-2x}(1-2x+1) = 0$$
$$= -4e^{-2x}(1-x) = 0$$

IN : $x = 1$ $\quad \xrightarrow[1]{-\ \ |\ \ +} f''$

Results : inc on $(-\infty, 1/2)$
dec on $(1/2, \infty)$
max $(\frac{1}{2}, \frac{1}{2e})$
min none
conc up $(1, \infty)$
conc down $(-\infty, 1)$
infl. pt. $(1, \frac{1}{e^2})$
VA: none
HA : $y = 0$ as $x \to \infty$





4. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^2e^{-x}$

$$f'(x) = 2xe^{-x} + x^2e^{-x}(-1)$$

(b) $f(x) = \ln\sqrt{x^2+4x+1}$

$$f'(x) = \frac{1}{\sqrt{x^2+4x+1}} \cdot \frac{1}{2}(x^2+4x+1)^{-1/2}(2x+4)$$

5. How long will it take for a \$2000 investment to be worth \$5000 if it grows at an annual rate of 8% compounded continuously?

$$B = Pe^{rt}$$

$$5000 = 2000e^{.08t}$$

$$\frac{5}{2} = e^{.08t}$$

$$\ln 2.5 = .08t$$

$$t = \frac{\ln 2.5}{.08} \approx 11.454 \text{ years} \approx 11\tfrac{1}{2} \text{ years}$$

6. A large turkey is placed in a 350°F oven at noon on Thanksgiving Day. The original temperature of the turkey is 70°F. Newton's Law of Cooling states that the temperature of the turkey t minutes later is given by a function of the form $f(t) = 350 - Ae^{kt}$. Suppose that after 1 hour, the temperature of the turkey is 100°F. A turkey is done when its temperature is 165°F. What time is dinner?

(noon) $t = 0$ $f(t) = 70$ ①

$t = 1$ $f(t) = 100$ ②

$t = ?$ $f(t) = 165$ ③

① $70 = 350 - Ae^{k(0)}$

$A = 350 - 70 = 280$

$f(t) = 350 - 280e^{kt}$

② $100 = 350 - 280e^{k(1)}$

$280e^{k} = 250$

$e^{k} = \frac{25}{28}$

$k = \ln \frac{25}{28} \approx -0.11333$

$f(t) = 350 - 280e^{-0.11333t}$

③ $165 = 350 - 280e^{-0.11333t}$

$280e^{-0.11333t} = 185$

$e^{-0.11333t} = \frac{185}{280}$

$-0.11333t = \ln \frac{185}{280}$

$t = \dfrac{\ln \frac{185}{280}}{-0.11333} \approx 3.657 \text{ hrs}$

$3 \text{ hrs} + .657 \text{ hrs} \times \frac{60 \text{ min}}{1 \text{ hr}}$

$\approx 3 \text{ hrs } 30 \text{ min.}$

Dinner is around 3:30 pm.

7. a) Evaluate $e^{3\ln 4-\ln 2}$. Your answer should be an integer.

$$\begin{aligned} e^{3\ln 4-\ln 2} &= e^{\ln 4^3-\ln 2} \\ &= e^{\ln 64-\ln 2} \\ &= e^{\ln \frac{64}{2}} \\ &= 32 \end{aligned}$$

b) Find $\frac{1}{a}\ln\left(\frac{\sqrt{b}}{c}\right)^a$ if $\ln b = 6$ and $\ln c = 2$.

$$\begin{aligned} \frac{1}{a}\ln\left(\frac{b^{1/2}}{c}\right)^a &= \ln\left(\frac{b^{1/2}}{c}\right) \\ &= \tfrac{1}{2}\ln b - \ln c \\ &= \tfrac{1}{2}(6) - 2 \\ &= 3 - 2 = 1 \end{aligned}$$

8. A manufacturer estimates marginal revenue to be $200q^{-\frac{1}{2}}$ dollars per unit when the level of production is q units. The corresponding marginal cost has been found to be $0.4q$ dollars per unit. If the manufacturer's profit is \$2000 when the level of production is 25 units, what is the profit when the level of production is 36 units?

Profit = Rev − Cost

$P' = R' - C'$

$P' = 200q^{-1/2} - 0.4q$

$P = 400q^{1/2} - 0.2q^2 + K$

$P = 2000$ when $q = 25$, so

$2000 = 400\sqrt{25} - 0.2(625) + K$

$2000 = 2000 - 125 + K$

$K = 125$

$P = 400q^{1/2} - 0.2q^2 + 125$

when $q = 36$,

$P = 400(6) - 0.2(1296) + 125$

$= 2400 - 259.2 + 125$

$= 2265.80$

If $q = 36$ units, profit will be

$P = \$2265.80$

9. Evaluate $\int x \ln 3x \, dx$.

$u = \ln 3x$

$du = \frac{1}{3x} \cdot 3 \, dx$

$= \frac{1}{x} \, dx$

$dv = x \, dx$

$v = \int x \, dx$

$= \frac{1}{2}x^2$

$\int u \, dv = uv - \int v \, du$

$$\int x \ln 3x \, dx = (\ln 3x)\left(\tfrac{1}{2}x^2\right) - \int \left(\tfrac{1}{2}x^2\right)\left(\tfrac{1}{x}\, dx\right)$$

$$= \tfrac{1}{2}x^2 \ln 3x - \tfrac{1}{2} \int x \, dx$$

$$= \tfrac{1}{2}x^2 \ln 3x - \tfrac{1}{4} x^2 + C$$