You have 50 minutes to complete this test. You must show all work to receive full credit. Work any 8 of the following 9 problems. Clearly CROSS OUT the problem you do not wish me to grade. Each problem is worth 12 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve \( \frac{dy}{dx} = \frac{xy}{\sqrt{1-x^2}} \) if \( y = 2 \) when \( x = 0 \).

\[ \begin{align*}
\frac{1}{y} \frac{dy}{dx} &= \frac{x}{\sqrt{1-x^2}} \\
\frac{1}{y} \int \frac{dy}{dx} &= \int \frac{x}{\sqrt{1-x^2}} \, dx \\
\frac{1}{y} \int \, dy &= \int \frac{x}{\sqrt{1-x^2}} \, dx \\
\ln |y| &= \frac{1}{2} \int u^{-1/2} \, du \\
\ln |y| &= -\frac{1}{2} u^{1/2} + C \\
\ln |y| &= -\sqrt{1-x^2} + C
\end{align*} \]

\[ \begin{align*}
\ln 2 &= C - \sqrt{1-0} \\
\ln 2 + 1 &= C \\
\ln |y| &= \ln 2 + 1 - \sqrt{1-x^2} \\
1y &= e^{\frac{\ln 2}{2} - \sqrt{1-x^2}} \\
y &= \pm e^{\frac{\ln 2}{2}} e^{-\sqrt{1-x^2}} \\
y &= \pm 2 e^{1-\sqrt{1-x^2}}
\end{align*} \]

2. Evaluate the following.

(a) \( \int 5e^{3x} \, dx = \frac{5}{3} e^{3x} + C \)

(b) \( \int \frac{3x + 6}{2x^2 + 8x + 3} \, dx \)

\[ \begin{align*}
\int \frac{3x + 6}{2x^2 + 8x + 3} \, dx &= \frac{3}{4} \int \frac{1}{u} \, du \\
&= \frac{3}{4} \ln |u| + C \\
&= \frac{3}{4} \ln |2x^2 + 8x + 3| + C \\
&= \frac{3}{4} \ln |u| + C \\
&= \frac{3}{4} \ln |(x+2)^2 + 1| + C
\end{align*} \]
3. Find all maxima, minima and inflection points of \( f(x) = xe^{-2x} \). Also give the intervals where \( f \) is increasing, decreasing, concave up, and concave down. Find all asymptotes. Then carefully sketch the graph of \( f \).

\[
\begin{align*}
f'(x) &= e^{-2x} - 2xe^{-2x} \\
&= e^{-2x}(1 - 2x) = 0 \\
C_N: x = \frac{1}{2} & \quad + - \rightarrow f' \\
\end{align*}
\]

\[
\begin{align*}
f''(x) &= -2e^{-2x}(1 - 2x) - 2e^{-2x} = 0 \\
&= -2e^{-2x}(1 - 2x + 1) = 0 \\
&= -4e^{-2x}(1 - x) = 0 \\
N: x = 1 & \quad + - \rightarrow f'' \\
\end{align*}
\]

Inc dec dec dec down down up combo

4. Find \( f''(x) \) for the following functions. DO NOT simplify!

(a) \( f(x) = x^2 e^{-x} \)

\[
f'(x) = 2xe^{-x} + x^2 e^{-x}(-1) \]

(b) \( f(x) = \ln(x^2 + 4x + 1) \)

\[
f'(x) = \frac{1}{\sqrt{x^2 + 4x + 1}} \cdot \frac{1}{2} (x^2 + 4x + 1)^{-1/2} (2x + 4) \]
5. How long will it take for a $2000 investment to be worth $5000 if it grows at an annual rate of 8% compounded continuously?

\[ B = Pe^{rt} \]

\[ 5000 = 2000e^{0.08t} \]

\[ \frac{5}{2} = e^{0.08t} \]

\[ \ln 2.5 = 0.08t \]

\[ t = \frac{\ln 2.5}{0.08} \approx 11.454 \text{ years} \approx 11 \frac{1}{2} \text{ years} \]

6. A large turkey is placed in a 350°F oven at noon on Thanksgiving Day. The original temperature of the turkey is 70°F. Newton’s Law of Cooling states that the temperature of the turkey \( t \) minutes later is given by a function of the form \( f(t) = 350 - Ae^{kt} \). Suppose that after 1 hour, the temperature of the turkey is 100°F. A turkey is done when its temperature is 165°F. What time is dinner?

\[ f(t) = 70 \text{ for } t = 0 \]

\[ f(t) = 100 \text{ for } t = 1 \]

\[ f(t) = 165 \text{ for } t = ? \]

\[ f(t) = 350 - 280e^{kt} \]

\[ 70 = 350 - 280e^{k(0)} \]

\[ A = 350 - 70 = 280 \]

\[ f(t) = 350 - 280e^{kt} \]

\[ 100 = 350 - 280e^{k(1)} \]

\[ 280e^{k} = 250 \]

\[ e^{k} = \frac{25}{28} \approx 0.11333 \]

\[ K = \ln \frac{25}{28} \approx -0.11333 \]

\[ f(t) = 350 - 280e^{-0.11333t} \]

\[ 165 = 350 - 280e^{-0.11333t} \]

\[ 280e^{-0.11333t} = 185 \]

\[ e^{-0.11333t} = \frac{185}{280} \]

\[ -0.11333t = \ln \frac{185}{280} \]

\[ t = \frac{\ln 185}{280} \approx 3.657 \text{ hrs} \]

\[ 3 \text{ hrs} + 0.657 \text{ hrs} \times \frac{60 \text{ min}}{1 \text{ hr}} \approx 3 \text{ hrs} 30 \text{ min.} \]

Dinner is around 3:30 pm.
7. a) Evaluate $e^{3 \ln 4 - \ln 2}$. Your answer should be an integer.

\[
e^{3 \ln 4 - \ln 2} = e^{\ln 4^3 - \ln 2}
\]

\[
= e^{\ln 64 - \ln 2}
\]

\[
= e^{\ln \frac{64}{2}}
\]

\[
= 32
\]

b) Find $\frac{1}{a} \ln \left( \frac{\sqrt{b}}{c} \right)$ if $\ln b = 6$ and $\ln c = 2$.

\[
\frac{1}{a} \ln \left( \frac{b^{\frac{1}{2}}}{c} \right) = \ln \left( \frac{b^{\frac{1}{2}}}{c} \right)
\]

\[
= \frac{1}{2} \ln b - \ln c
\]

\[
= \frac{1}{2} (6) - 2
\]

\[
= 3 - 2 = 1
\]

8. A manufacturer estimates marginal revenue to be $200q^{-\frac{1}{2}}$ dollars per unit when the level of production is $q$ units. The corresponding marginal cost has been found to be $0.4q$ dollars per unit. If the manufacturer's profit is $\$2000$ when the level of production is 25 units, what is the profit when the level of production is 36 units?

\[
\text{Profit} = \text{Rev} - \text{Cost}
\]

\[
P' = R' - C'
\]

\[
P' = 200 q^{-\frac{1}{2}} - 0.4 q
\]

\[
P = 400 q^{\frac{1}{2}} - 0.2 q^2 + K
\]

\[
P = 2000 \text{ when } q = 25, \text{ so}
\]

\[
2000 = 400 \sqrt{25} - 0.2 (625) + K
\]

\[
2000 = 2000 - 125 + K
\]

\[
K = 125
\]

\[
P = 400 q^{\frac{1}{2}} - 0.2 q^2 + 125
\]

when $q = 36$,

\[
P = 400 (6) - 0.2 (1296) + 125
\]

\[
= 2400 - 259.2 + 125
\]

\[
= 2265.80
\]

If $q = 36$ units, profit will be

\[
P = \$2265.80
\]
9. Evaluate \( \int x \ln 3x \, dx \).

\[
\begin{align*}
\ u &= \ln 3x \\
\ du &= \frac{1}{3x} \cdot 3 \, dx \\
&= \frac{1}{x} \, dx \\
\ dv &= x \, dx \\
\ v &= \int x \, dx \\
&= \frac{1}{2} x^2
\end{align*}
\]

\[
\int u \, dv = uv - \int v \, du
\]

\[
\int x \cdot \ln 3x \, dx = (\ln 3x) \left( \frac{1}{2} x^2 \right) - \int \left( \frac{1}{2} x^2 \right) \left( \frac{1}{x} \, dx \right)
\]

\[
= \frac{1}{2} x^2 \ln 3x - \frac{1}{2} \int x \, dx
\]

\[
= \frac{1}{2} x^2 \ln 3x - \frac{1}{4} x^2 + C
\]