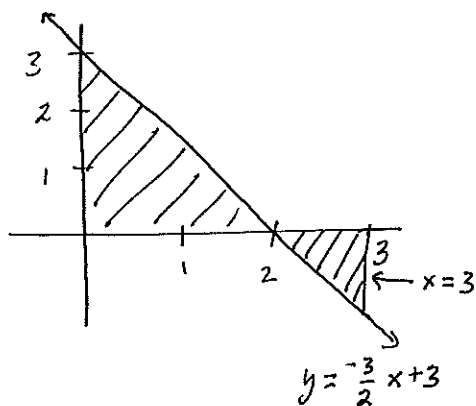


You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Use calculus to find the area of the region bounded by the curves $y = -\frac{3}{2}x + 3$, $x = 3$, and the coordinate axes. Be sure to sketch a graph first.



$$\begin{aligned}
 \text{Area} &= \text{Big } \Delta + \text{small } \Delta \\
 &= \int_0^2 \left(-\frac{3}{2}x + 3\right) dx + \int_2^3 \left[0 - \left(-\frac{3}{2}x + 3\right)\right] dx \\
 &= \left[-\frac{3}{4}x^2 + 3x\right]_0^2 + \left[\frac{3}{4}x^2 - 3x\right]_2^3 \\
 &= \left[(-3+6) - 0\right] + \left[\left(\frac{27}{4} - 9\right) - (3-6)\right] \\
 &= 3 + \left(-\frac{9}{4} + 3\right) \\
 &= 3 + \frac{3}{4} \\
 &= \frac{15}{4} \text{ or } 3.75
 \end{aligned}$$

2. For $f(x, y) = e^{x^2y}$, show that $f_{xy}(x, y) = f_{yx}(x, y)$.

$$f_x = (e^{x^2y})(2xy) = 2xy e^{x^2y}$$

$$f_y = (e^{x^2y})(x^2) = x^2 e^{x^2y}$$

$$f_{xy} = (2x)(e^{x^2y}) + (2xy)(e^{x^2y})(x^2) = 2x e^{x^2y} + 2x^3 y e^{x^2y} \quad \checkmark$$

$$f_{yx} = (2x)(e^{x^2y}) + (x^2)(e^{x^2y})(2xy) = 2x e^{x^2y} + 2x^3 y e^{x^2y}$$

We see that $f_{xy} = f_{yx}$.

3. Find and classify the critical points of $f(x, y) = x^3 - 3xy + y^2 + y - 5$.

$$\begin{aligned}
 f_x = 3x^2 - 3y = 0 &\longrightarrow y = x^2 \\
 f_y = -3x + 2y + 1 = 0 &\longrightarrow -3x + 2x^2 + 1 = 0 \\
 &2x^2 - 3x + 1 = 0 \\
 &(2x-1)(x-1) = 0 \\
 &x = \frac{1}{2}, 1 \\
 \left. \begin{aligned} f_{xx} &= 6x \\ f_{yy} &= 2 \\ f_{xy} &= -3 \end{aligned} \right\} D(x, y) = 12x - 9 \\
 \text{Critical points: } &(\frac{1}{2}, \frac{1}{4}), (1, 1)
 \end{aligned}$$

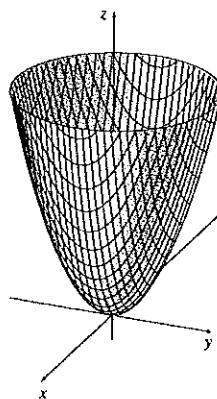
$$\begin{aligned}
 D(\frac{1}{2}, \frac{1}{4}) &= 6 - 9 < 0, \text{ so } (\frac{1}{2}, \frac{1}{4}) \text{ gives a saddle point.} \\
 D(1, 1) &= 12 - 9 > 0, \text{ so } f_{xx}(1, 1) = 6 > 0 \quad \cup \\
 &\text{so } (1, 1) \text{ gives a minimum.}
 \end{aligned}$$

4. Suppose p_1 and p_2 are the prices of two products. Also suppose $D_1(p_1, p_2) = 500 + \frac{10}{p_1 + 2} - 5p_2$ and $D_2(p_1, p_2) = 400 - 2p_1 + \frac{7}{p_2 + 3}$ are the demand functions for the two products (quantities). Determine whether these two products are competitive, complementary, or neither. Then give an example of two products that might behave in this way.

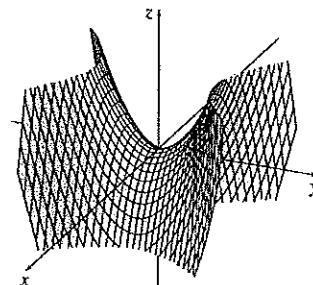
$$\left. \begin{aligned} \frac{\partial D_1}{\partial p_2} &= -5 < 0 \\ \frac{\partial D_2}{\partial p_1} &= -2 < 0 \end{aligned} \right\} \text{The products are complementary} \\
 &\text{(ex - peanut butter \& jelly)}$$

5. For each three-dimensional surface below, determine the matching equation (a, b, c, d, or e).

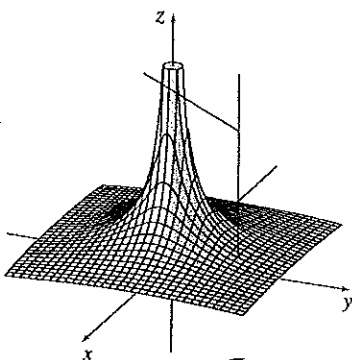
- a) $z = x^2 + y^2$
 b) $z = y^2 - x^2$
 c) $z^2 - y^2 - x^2 = 1$
 d) $\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{4} = 1$
 e) $z = 5(x^2 + y^2)^{-\frac{1}{2}}$



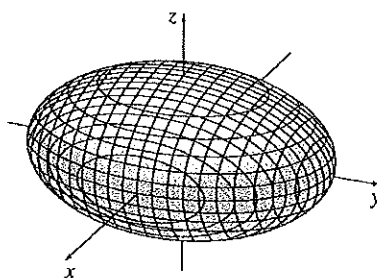
equation A



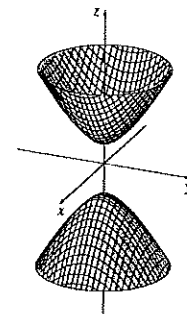
equation B



equation E



equation D



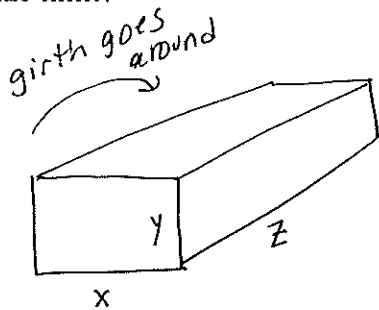
equation C

6. Calculate $\int_0^{\infty} 5e^{-2x} dx$.

$$\begin{aligned}
 \int_0^{\infty} 5e^{-2x} dx &= \lim_{n \rightarrow \infty} \int_0^n 5e^{-2x} dx \\
 &= \lim_{n \rightarrow \infty} \left[\frac{-5}{2} e^{-2x} \right]_0^n \\
 &= \lim_{n \rightarrow \infty} \left[\frac{-5}{2} e^{-2n} + \frac{5}{2} e^0 \right] \\
 &= \lim_{n \rightarrow \infty} \left[\frac{-5}{2e^{2n}} + \frac{5}{2} \right] \\
 &= \frac{5}{2}
 \end{aligned}$$

denom gets large
as $n \rightarrow \infty$

7. Postal regulations require that to avoid oversize rates, the length plus the girth (distance around) of a box-shaped package must not exceed 108 inches. What dimensions will maximize the volume of a rectangular package that is within the size limit?



maximize volume $V = xyz$
 constraint $z + 2x + 2y = 108$

$$F(x, y, z, \lambda) = xyz - \lambda(z + 2x + 2y - 108)$$

$$F_x = yz - 2\lambda = 0 \rightarrow \lambda = \frac{1}{2} yz \quad \left. \begin{array}{l} > \\ > \end{array} \right\} \begin{array}{l} \frac{1}{2} yz = \frac{1}{2} xz \quad (z \neq 0) \\ \textcircled{y = x} \end{array}$$

$$F_y = xz - 2\lambda = 0 \rightarrow \lambda = \frac{1}{2} xz \quad \left. \begin{array}{l} > \\ > \end{array} \right\} \begin{array}{l} \frac{1}{2} xz = xy \quad (x \neq 0) \\ \textcircled{z = 2y = 2x} \end{array}$$

$$F_z = xy - \lambda = 0 \rightarrow \lambda = xy$$

$$F_\lambda = -z - 2x - 2y + 108 = 0$$

$$-2x - 2x - 2x + 108 = 0$$

$$108 = 6x$$

$$x = 18$$

$$y = 18$$

$$z = 36$$

The box with maximum volume has dimensions 18" x 18" x 36".