You have 50 minutes to complete this test. You must show all work to receive full credit. Work any 6 of the following 7 problems. Clearly cross out the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region bounded by the curves $y = x^2$ and $y = 2x - x^2$. Be sure to sketch a graph first!

2. For $f(x, y, z) = x^2 y^3 - \frac{x}{z} + e^x \ln y$, find $f_x$, $f_y$, and $f_z$. 
3. Find and classify the critical points of \( f(x, y) = -x^4 + 4xy - 2y^2 + 1 \).

4. Suppose \( p_1 \) and \( p_2 \) are the prices of two products. Also suppose
\[
D_1(p_1, p_2) = 500 - 0.5p_1 - p_2^2 \quad \text{and} \quad D_2(p_1, p_2) = 10,000 - 8p_1 - 100p_2^2
\]
are the demand functions for the two products (quantities). Answer the following questions, showing your work below.

a) Are these two products competitive (substitutes), complementery, or neither?

b) An example of two products that might behave this way are
_________________________ and _______________________. 
5. Sketch at least three level curves for $z = x^2 + y^2$ on the first set of axes below. Then sketch at least three level curves for $z^2 = x^2 + y^2$ on the second set of axes. Describe the three dimensional surface represented by each and why you think each surface has that shape based on your level curves.

\[ z = x^2 + y^2 \quad \text{and} \quad z^2 = x^2 + y^2 \]

6. Calculate \( \int_0^\infty \frac{x}{(x^2 + 5)^2} \, dx \).
7. Suppose a manufacturing firm has budgeted $60,000 per month for labor and materials. If $x$ thousand is spent on labor and $y$ thousand is spent on materials, and if the monthly output (in units) is given by $N(x, y) = 4xy - 8x$, how should the budget be allocated in order to maximize the output $N$? What is the maximum output?