You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region bounded by the curves $y = 8 - x^2$, $y = x^2$, and $y = 7x$ in the first quadrant. Be sure to sketch a graph first! The region should use all three functions as its edges, and only be located in the first quadrant.

2. For $f(x, y) = 8x^3 + 2x^2y^2 + 5y^4$, show that $f_{xy}(x, y) = f_{yx}(x, y)$.
3. Find and classify the critical points of \( f(x, y) = x^3 + y^3 - xy \).

4. Suppose \( p_1 \) and \( p_2 \) are the prices of two products. Also suppose 
\[ D_1(p_1, p_2) = 1000 - 50p_1 + 2p_2 \quad \text{and} \quad D_2(p_1, p_2) = 500 + 4p_1 - 20p_2 \]
are the demand functions for the two products (quantities). Answer the following questions, showing your work below.

a) If the price of product 1 goes up by a dollar, the demand for product 2 will go up/down (circle one) by \( \underline{\quad} \) units.

b) If the price of product 2 goes up by a dollar, the demand for product 1 will go up/down (circle one) by \( \underline{\quad} \) units.

c) These two products are competitive/complementary/neither (circle one).
An example of two products that might behave this way are \( \underline{\quad} \) and \( \underline{\quad} \).
5. For each three-dimensional surface below, determine the matching set (a, b, c, or d) of level curves in the xy-plane.

6. Calculate \( \int_{1}^{\infty} e^{1-x} \, dx \).
7. Suppose a firm has an order for 200 units of its product and wishes to distribute its manufacture between two plants. Suppose \( x \) units will be produced at the Minneapolis location and \( y \) units will be produced at the Chicago location. If the total cost function is given by \( C(x, y) = 2x^2 + xy + y^2 + 200 \), how many units should be produced at each location in order to minimize cost?