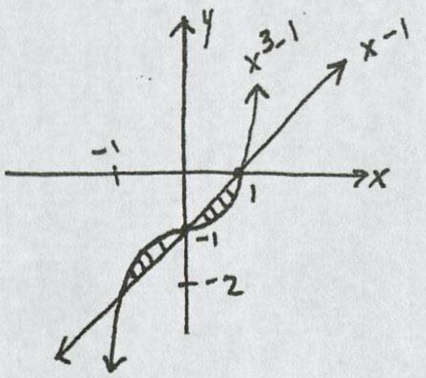
NAME KEY

Math 1212
Test 4
Fall 2014

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region bounded by the curves $y = x^3 - 1$ and $y = x - 1$. Be sure to sketch a graph first!



$$\text{Area} = \int_{-1}^{0} [(x^3-1)-(x-1)]\,dx + \int_0^1 [(x-1)-(x^3-1)]\,dx$$

$$= \int_{-1}^{0} (x^3-x)\,dx + \int_0^1 (x-x^3)\,dx$$

$$= \left[\tfrac{1}{4}x^4 - \tfrac{1}{2}x^2\right]_{-1}^{0} + \left[\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right]_0^1$$

$$= \left[0 - \left(\tfrac{1}{4} - \tfrac{1}{2}\right)\right] + \left[\left(\tfrac{1}{2} - \tfrac{1}{4}\right) - 0\right]$$

$$= \tfrac{1}{4} + \tfrac{1}{4}$$

$$= \tfrac{1}{2}$$

2. Suppose $z = 5x\ln(x^2 + y)$. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. Do not simplify.

$$\frac{\partial z}{\partial x} = z_x = (5)(\ln(x^2+y)) + (5x)\left(\frac{1}{x^2+y}\right)(2x)$$

$$\frac{\partial z}{\partial y} = z_y = (0)(\ln(x^2+y)) + (5x)\left(\frac{1}{x^2+y}\right)(1)$$

3. Find and classify the critical points of $f(x,y) = x^3 + y^3 - xy$.

$f_x = 3x^2 - y = 0 \rightarrow y = 3x^2$

$f_y = 3y^2 - x = 0 \longrightarrow 3(3x^2)^2 - x = 0$

$3(9x^4) - x = 0$

$27x^4 - x = 0$

$x(27x^3 - 1) = 0$

$x = 0 \rightarrow y = 0$

$x = \frac{1}{3} \rightarrow y = 3(\frac{1}{9}) = \frac{1}{3}$

$f_{xx} = 6x$

$f_{xy} = -1$

$f_{yy} = 6y$

crit pts: $(0,0), (\frac{1}{3}, \frac{1}{3})$

$D(x,y) = (6x)(6y) - (-1)^2$

$= 36xy - 1$

$D(0,0) = -1 < 0$, so $(0,0)$ is a saddle point

$D(\frac{1}{3}, \frac{1}{3}) = 36(\frac{1}{9}) - 1 > 0$

$f_{xx}(\frac{1}{3}, \frac{1}{3}) = 6(\frac{1}{3}) > 0$, so $(\frac{1}{3}, \frac{1}{3})$ is a minimum.

4. The demand functions for two products are given by $D_1 = \frac{100}{p_1\sqrt{p_2}}$ and $D_2 = \frac{500}{p_2\sqrt[3]{p_1}}$, where p_1 and p_2 are the respective prices of the products. Are the two products competitive, complementary, or neither? (show work!) Give an example of two products that might behave in this way.

$D_1 = 100 p_1^{-1} p_2^{-1/2}$

$\partial D_1 / \partial p_2 = -50 p_1^{-1} p_2^{-3/2}$

$D_2 = 500 p_2^{-1} p_1^{-1/3}$

$\partial D_2 / \partial p_1 = -\frac{500}{3} p_2^{-1} p_1^{-4/3}$

these are both negative, so (since $p_1, p_2 > 0$) the products are complementary

Examples: cameras & film

peanut butter & jelly

⋮

5. A computer company has a monthly advertising budget of \$60,000. Its marketing department estimates that if x dollars are spent each month on advertising in newspapers and y dollars per month on television advertising, then the monthly sales will be given by $S = 90x^{\frac{1}{4}}y^{\frac{3}{4}}$ dollars. If the profit is 10% of sales, less the advertising cost, determine how to allocate the advertising budget in order to maximize the monthly profit.

$x + y = 60000 \leftarrow$ constraint

profit $= P =$ 10% (sales) − advertising cost

$P = 9x^{1/4}y^{3/4} - 60000 \longleftarrow$ optimize this.

$F(x,y,\lambda) = 9x^{1/4}y^{3/4} - 60000 - \lambda(x+y-60000)$

$F_x = \frac{9}{4}x^{-3/4}y^{3/4} - \lambda = 0$

$F_y = \frac{27}{4}x^{1/4}y^{-1/4} - \lambda = 0$

$F_\lambda = -x - y + 60000 = 0$

$\lambda = \frac{9}{4}x^{-3/4}y^{+3/4} = \frac{27}{4}x^{1/4}y^{-1/4}$

$\frac{9}{\cancel{4}}y = \frac{27}{\cancel{4}}x$

$y = 3x$

$-x - 3x + 60000 = 0$

$4x = 60000$

$x = 15000, \; y = 45000.$

6. Solve $\int_{-\infty}^{-2} \frac{1}{(x+1)^3}\,dx$.

$$\int_{-\infty}^{-2} (x+1)^{-3}\,dx = \lim_{n\to-\infty} \int_n^{-2} (x+1)^{-3}\,dx$$

$$= \lim_{n\to-\infty} \left[\frac{(x+1)^{-2}}{-2}\right]_n^{-2}$$

$$= \lim_{n\to-\infty} \left(\frac{(-1)^{-2}}{-2} - \frac{(n+1)^{-2}}{-2}\right)$$

$$= \lim_{n\to-\infty} \left[\frac{-1}{2} + \frac{1}{2(n+1)^2}\right]$$

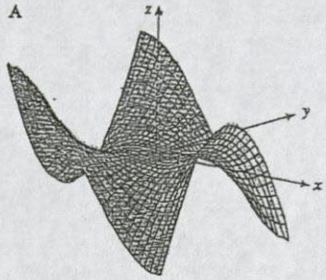
$$= -1/2$$

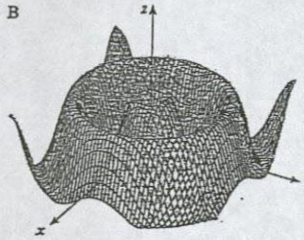
square large negative, we get a big positive # in denom. term $\rightarrow 0$

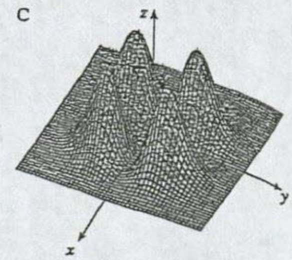
7. Match the level curves with the corresponding surface graphs below:

Surface A matches Level Curve VI
Surface B matches Level Curve III
Surface C matches Level Curve II

Surface D matches Level Curve IV
Surface E matches Level Curve I
Surface F matches Level Curve V

A


B


C


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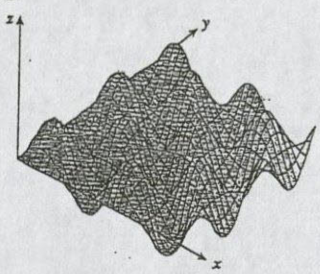
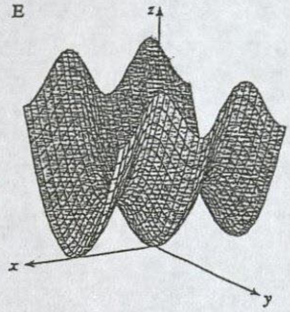
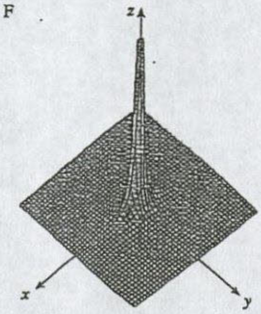
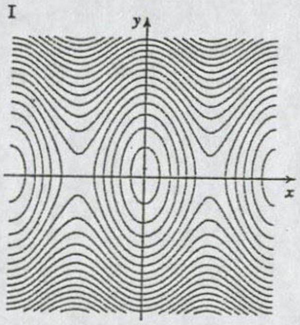
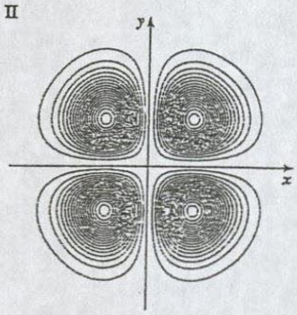
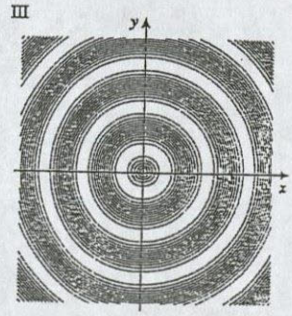


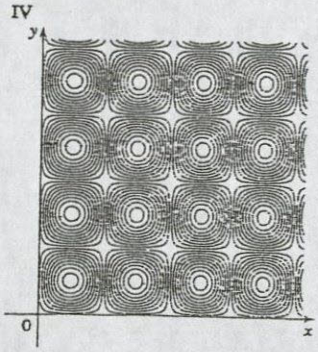
E


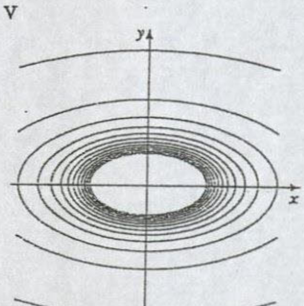
F


I


II


III


IV


V


VI