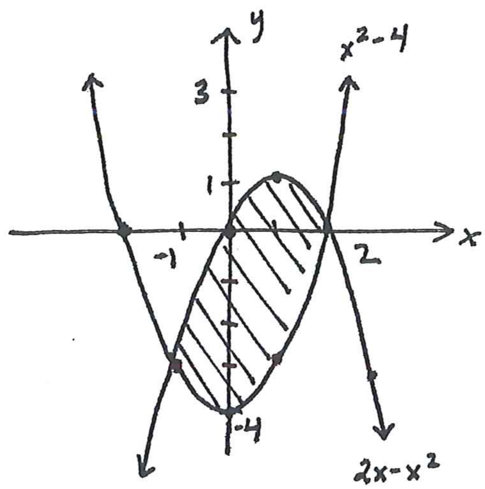
NAME *KEY*

Math 12
Test 4
Spring 2014

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region between the curves $y = 2x - x^2$ and $y = x^2 - 4$. Be sure to sketch a graph first!



$2x - x^2 = x^2 - 4$

$0 = 2x^2 - 2x - 4$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2,\ x = -1$

intersection points: $(2,0), (-1,-3)$

$$A = \int_{-1}^{2} [(2x - x^2) - (x^2 - 4)]\,dx$$

$$= \int_{-1}^{2} (-2x^2 + 2x + 4)\,dx = -\frac{2}{3}x^3 + x^2 + 4x \Big|_{-1}^{2}$$

$$= \left(-\frac{16}{3} + 4 + 8\right) - \left(\frac{2}{3} + 1 - 4\right) = 9$$

2. Find the minimum **value** (the smallest possible value for f) of $f(x,y) = x^2 + 2y^2 - xy$ subject to the constraint $2x + y = 22$.

$F(x,y,\lambda) = x^2 + 2y^2 - xy - \lambda(2x + y - 22)$

$F_x = 2x - y - 2\lambda = 0 \longrightarrow 8y - 2\lambda - y - 2\lambda = 0$

$F_y = 4y - x - \lambda = 0 \rightarrow x = 4y - \lambda$

$F_\lambda = -2x - y + 22 = 0$

$7y = 4\lambda$

$y = \frac{4}{7}\lambda$

$x = \frac{16}{7}\lambda - \lambda$

$x = \frac{9}{7}\lambda$

$-2\left(\frac{9}{7}\lambda\right) - \left(\frac{4}{7}\lambda\right) = -22$

$-\frac{22}{7}\lambda = -22$

$\lambda = 7$

$\left.\begin{matrix} y = 4 \\ x = 9 \end{matrix}\right\} \longrightarrow$ so $f(9,4) = 81 + 32 - 36$

$= 77 \leftarrow$ minimum value

3. Find and classify the critical points of $f(x,y)=x^3+y^2-6xy+9x+5y+2$.

$f_x = 3x^2-6y+9 \longrightarrow x^2-2y+3=0$

$f_y = 2y-6x+5$

$y = \frac{-x^2-3}{-2} = \frac{x^2+3}{2}$

$f_{xx} = 6x$

$f_{yy} = 2$

$f_{xy} = -6$

$x^2+3-6x+5=0$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

use to get y-values

$x=2, \quad y=7/2$

$x=4 \quad y=19/2$

$D(x,y) = 12x-36$

$D(2, 7/2) = 24-36 < 0$, so $(2, 7/2)$ gives a saddle point

$D(4, 19/2) = 48-36 > 0$. $f_{xx}(4, 19/2) = 6(4) > 0$, so

$(4, 19/2)$ gives a minimum.

4. Suppose p_1 and p_2 are the prices of two products. Also suppose $D_1(p_1,p_2)=3000+\dfrac{400}{p_1+3}+50p_2$ and $D_2(p_1,p_2)=2000-100p_1+\dfrac{500}{p_2+4}$ are the demand functions for the two products (quantities). Are these two products competitive, complementary, or neither? (show work!) Give an example of two products that might behave in this way.

$\dfrac{\partial D_1}{\partial p_2} = 50 \qquad \dfrac{\partial D_2}{\partial p_1} = -100$

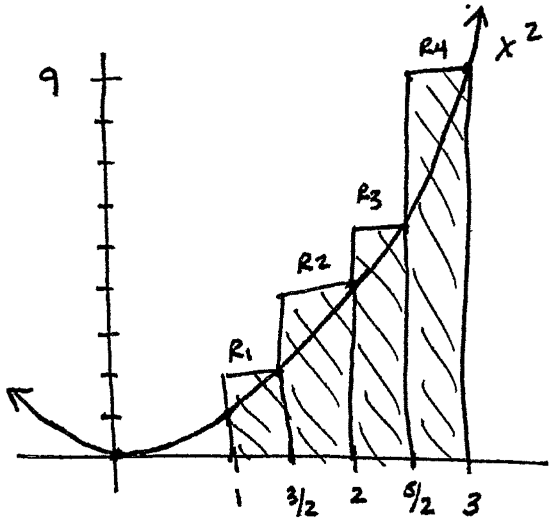
both >0 competitive

both <0 complementary.

In this case, the products are neither competitive nor complementary.

(Any two unrelated products can work as an example).

5. Using four rectangles, estimate the area between the curve $f(x) = x^2$ and the x-axis between $x = 1$ and $x = 3$.



$$A \approx R_1 + R_2 + R_3 + R_4$$

$$\approx \frac{1}{2}\left(\frac{3}{2}\right)^2 + \frac{1}{2}(2)^2 + \frac{1}{2}\left(\frac{5}{2}\right)^2 + \frac{1}{2}(3)^2$$

$$\approx \frac{9}{8} + \frac{16}{8} + \frac{25}{8} + \frac{36}{8}$$

$$\approx \frac{86}{8}$$

$$\approx \frac{43}{4}$$

alternately, using left-hand endpoints with rectangles below the function,

$$A \approx \frac{1}{2}(1)^2 + \frac{1}{2}\left(\frac{3}{2}\right)^2 + \frac{1}{2}(2)^2 + \frac{1}{2}\left(\frac{5}{2}\right)^2 = \frac{4}{8} + \frac{9}{8} + \frac{16}{8} + \frac{25}{8} = \frac{54}{8}$$

$$A \approx \frac{27}{4}$$

6. Calculate $\int_1^\infty \frac{1}{x^2}\,dx$.

$$\int_1^\infty x^{-2}\,dx = \lim_{n \to \infty} \int_1^n x^{-2}\,dx$$

$$= \lim_{n \to \infty} \left[\frac{x^{-1}}{-1}\right]_1^n$$

$$= \lim_{n \to \infty} \left[\frac{-1}{n} - (-1)\right]$$

$$= \lim_{n \to \infty} \left[\cancel{\frac{-1}{n}}^{\,0} + 1\right]$$

$$= 1$$

7. Suppose $z = x^2 + 2xy^2 + \dfrac{2y}{3x}$. Compute all four second-order partial derivatives (be sure to do each one separately).

Easiest to start with

$$z = x^2 + 2xy^2 + \frac{2}{3}x^{-1}y$$

$$z_x = 2x + 2y^2 - \frac{2}{3}x^{-2}y$$

$$z_y = 4xy + \frac{2}{3}x^{-1}$$

$$z_{xx} = 2 + \frac{4}{3}x^{-3}y$$

$$z_{xy} = 4y - \frac{2}{3}x^{-2}$$

$$z_{yy} = 4x$$

$$z_{yx} = 4y - \frac{2}{3}x^{-2}$$