You have 50 minutes to complete this test. You must show all work to receive full credit. Work any 6 of the following 7 problems. Clearly cross out the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region between the curves $y = 2x - x^2$ and $y = x^2 - 4$. Be sure to sketch a graph first!

2. Find the minimum value (the smallest possible value for $f$) of $f(x, y) = x^2 + 2y^2 - xy$ subject to the constraint $2x + y = 22$. 
3. Find and classify the critical points of \( f(x, y) = x^3 + y^2 - 6xy + 9x + 5y + 2 \).

4. Suppose \( p_1 \) and \( p_2 \) are the prices of two products. Also suppose

\[
D_1(p_1, p_2) = 3000 + \frac{400}{p_1 + 3} + 50p_2 \quad \text{and} \quad D_2(p_1, p_2) = 2000 - 100p_1 + \frac{500}{p_2 + 4}
\]

are the demand functions for the two products (quantities). Are these two products competitive, complementary, or neither? (show work!) Give an example of two products that might behave in this way.
5. Using four rectangles, estimate the area between the curve $f(x) = x^2$ and the $x$-axis between $x = 1$ and $x = 3$.

6. Calculate $\int_1^\infty \frac{1}{x^2} \, dx$. 
Suppose $z = x^2 + 2xy^2 + \frac{2y}{3x}$. Compute all four second-order partial derivatives.