NAME KEY

Math 12
Test 1
Summer 2014

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the ***definition*** of the derivative, find $f'(x)$ if $f(x)=\sqrt{x-4}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h-4}-\sqrt{x-4}}{h} \cdot \frac{\sqrt{x+h-4}+\sqrt{x-4}}{\sqrt{x+h-4}+\sqrt{x-4}}$$

$$= \lim_{h\to 0} \frac{(x+h-4)-(x-4)}{h\left(\sqrt{x+h-4}+\sqrt{x-4}\right)} = \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h-4}+\sqrt{x-4}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h-4}+\sqrt{x-4}} = \frac{1}{\sqrt{x-4}+\sqrt{x-4}} = \frac{1}{2\sqrt{x-4}}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim_{x\to 3} \frac{x^2+2x+1}{x+3} = \frac{3^2+2(3)+1}{3+3} = \frac{16}{6} = \frac{8}{3}$

(b) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4} \frac{(x-4)(\sqrt{x}+2)}{x-4}$

$$= \lim_{x\to 4}(\sqrt{x}+2) = 2+2 = 4$$

(c) $\lim_{x\to 2^+} \frac{1}{\sqrt{x^2-4}} = \infty$

fill in, get $\frac{1}{0}$, so use a chart.

x	y
3	$\frac{1}{\sqrt{5}} \approx 0.447$
2.5	$\frac{1}{\sqrt{2.25}} \approx 0.667$
2.1	$\frac{1}{\sqrt{0.41}} \approx 1.562$
2.01	$\frac{1}{\sqrt{0.0401}} \approx 4.994$
2.001	$\frac{1}{\sqrt{0.004001}} \approx 15.809$

2 $\qquad\qquad$ ∞

3. The total cost of producing x packages of cookies is $C(x)=\frac{1}{20}x^2+3x+33$ dollars. All x packages will be sold if the price is set at $p(x)=\frac{1}{5}(45-x)$ dollars per package.

a) Find an equation for profit when x packages of cookies are produced and sold.
b) ***Estimate*** the profit gained from the production and sale of the 11th package.
c) Find the ***actual*** profit from the 11th package.

a) Profit = Revenue − Cost

$= \text{price} \cdot \text{quantity} - \text{cost}$

$P(x) = \frac{1}{5}(45-x)(x) - \left(\frac{1}{20}x^2+3x+33\right)$

$= 9x - \frac{1}{5}x^2 - \frac{1}{20}x^2 - 3x - 33$

$= \frac{-1}{4}x^2 + 6x - 33$

b) marginal Profit $= P'(x) = \frac{-1}{2}x + 6$

Profit from 11th package $\approx P'(10) = -5+6 = \$1$

c) Actual profit from 11th package $= P(11) - P(10)$

$= \left(\frac{-1}{4}(121) + 6(11) - \cancel{33}\right) - \left(\frac{-1}{4}(100) + 6(10) - \cancel{33}\right)$

$= -\frac{121}{4} + 66 + 25 - 60 = \$0.75$

4. Find $f'(x)$ (do not simplify!) if :

a) $f(x)=\dfrac{x^2-3x+2}{2x^2-5x+1}$

$$f'(x) = \frac{(2x-3)(2x^2-5x+1) - (x^2-3x+2)(4x-5)}{(2x^2-5x+1)^2}$$

b) $f(x)=-\dfrac{x^2}{16}+\dfrac{2}{x}-x^{\frac{3}{2}}+\dfrac{1}{3x^2}+\dfrac{x}{3}$ $= \frac{-1}{16}x^2 + 2x^{-1} - x^{3/2} + \frac{1}{3}x^{-2} + \frac{1}{3}x$

$f'(x) = \frac{-1}{8}x - 2x^{-2} - \frac{3}{2}x^{1/2} - \frac{2}{3}x^{-3} + \frac{1}{3}$

5. Find the equation of the line tangent to the graph of the function $f(x) = (3x+1)(2x^2-4)(5x^3+2x-1)$ at the point where $x = 0$.

Method ① : (multiply out) $f(x) = (6x^3+2x^2-12x-4)(5x^3+2x-1)$

$f'(x) = (18x^2+4x-12)(5x^3+2x-1) + (6x^3+2x^2-12x-4)(15x^2+2)$

Method ② : $f(x) = [(3x+1)(2x^2-4)](5x^3+2x-1)$

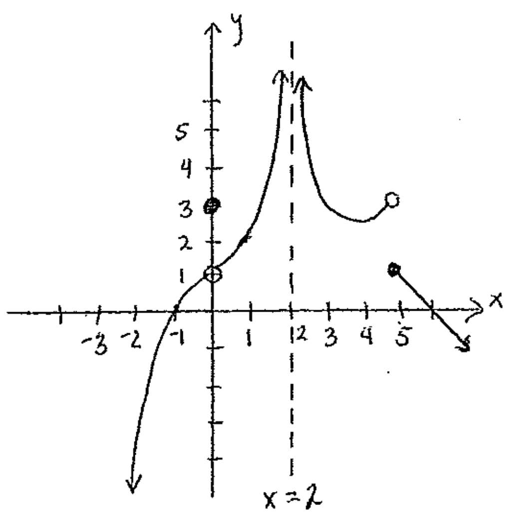
$f'(x) = [3(2x^2-4) + (3x+1)(4x)](5x^3+2x-1) + [(3x+1)(2x^2-4)](15x^2+2)$

Slope : $m = f'(0) = -12(-1) + (-4)(2) = 4$

Point : $x=0,\ y = 1 \cdot -4 \cdot -1 = 4 \quad (0,4)$

Line : $y - 4 = 4(x-0)$ or $y = 4x+4$

6. Consider the graph of the function $f(x)$ given below.



(a) For what values of x is $f(x)$ discontinuous? $x = 0,\ x = 2,\ x = 5$

(b) Find $\lim_{x \to -1} f(x)$. $= 0$

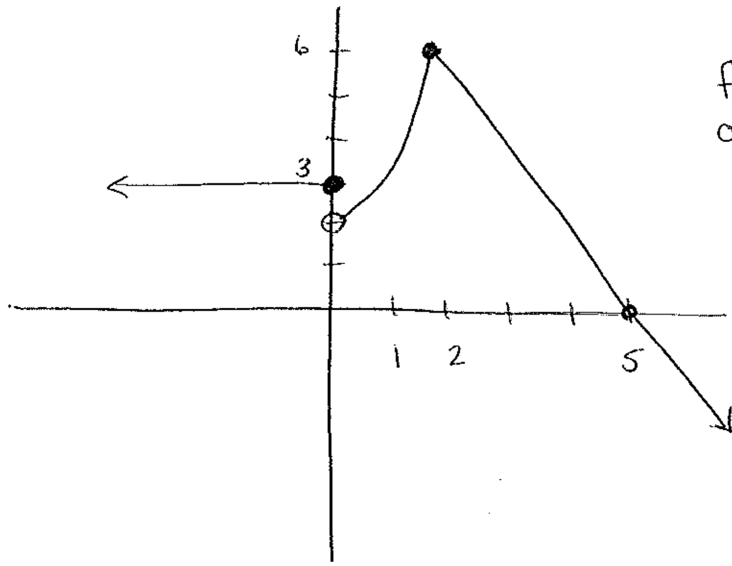
(c) Find $\lim_{x \to 0} f(x)$. $= 1$

(d) Find $\lim_{x \to 2} f(x)$. $= \infty$

(e) Find $\lim_{x \to 5^-} f(x)$. $= 3$

(f) Find $\lim_{x \to 5^+} f(x)$. $= 1$

7. Carefully graph the function $f(x)=\begin{cases} 3 & \text{if } x \le 0 \\ x^2+2 & \text{if } 0 < x < 2 \\ -2x+10 & \text{if } 2 \le x \end{cases}$. Does this function have any discontinuities, and if so where?



f is discontinuous only at $x=0$.

8. A bakery can produce small wedding cakes at a cost of \$80 apiece. Sales figures indicate that if the cakes are sold for x dollars each, approximately $300-x$ cakes will be sold during the May-September wedding season. Find an equation for ***profit***, and determine the price and number of cakes that will maximize profit. What will be the maximum profit?

Profit = Revenue − cost.

x = price per cake

$P(x)$ = price · quantity − total cost

$P(x) = x(300-x) - 80(300-x)$

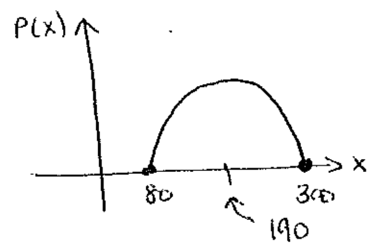
$= 300x - x^2 - 24000 + 80x$

$= -x^2 + 380x - 24000$

parabola, opens down. max profit is at the vertex.

$= -(x^2 - 380x + 24000)$

$= -(x-300)(x-80)$



$x = 190$ will give max. profit.

price = \$190

number of cakes = $300 - 190 = 110$ cakes

max profit = $P(190) = -(-110)(110)$

$= \$12{,}100$