Section 5.1 – Antidifferentiation: The Indefinite Integral

4. $\int \sqrt{t}\,dt = \int t^{1/2}\,dt = \frac{2}{3} t^{3/2} + C.$

6. $\int 3e^x dx = 3\int e^x\,dx = 3e^x + C.$

8. $\int (x^{1/2} - 3x^{2/3} + 6)\,dx = \frac{2}{3} x^{3/2} - \frac{9}{5} x^{5/3} + 6x + C$

12. $\int (\sqrt{x^3} - \frac{1}{2\sqrt{x}} + \sqrt{2})\,dx = \int (x^{3/2} - \frac{1}{2} x^{-1/2} + \sqrt{2})\,dx = \frac{2}{5} x^{5/2} - x^{1/2} + \sqrt{2}\,x + C$

16. $\int \frac{x^2 + 3x - 2}{\sqrt{x}}\,dx = \int (x^{3/2} + 3x^{1/2} - 2x^{-1/2})\,dx = \frac{2}{5} x^{5/2} + 2x^{3/2} - 4x^{1/2} + C$

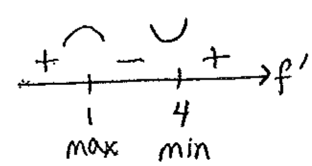
20. $\int x(2x+1)^2\,dx = \int x(4x^2 + 4x + 1)\,dx = \int (4x^3 + 4x^2 + x)\,dx$

$$= x^4 + \frac{4}{3}x^3 + \frac{1}{2}x^2 + C.$$

22. Tangent to $f(x)$ has slope $f'(x) = 3x^2 + 6x - 2$, graph goes through $(0,6)$:

$$f(x) = \int (3x^2 + 6x - 2)\,dx$$
$$= x^3 + 3x^2 - 2x + C$$
$$f(0) = 6 = C, \text{ so } f(x) = x^3 + 3x^2 - 2x + 6.$$

24. min at $x = 1$, max at $x = 4$.
Try $f'(x) = (x-1)(x-4) = (x^2 - 5x + 4)$



Then we adjust this to make the sign work.
Really we need $f'(x) = k \cdot (x-1)(x-4)$ where k is any negative constant. Using $k = -1$, $f'(x) = -x^2 + 5x - 4$, so

$$f(x) = \int (-x^2 + 5x - 4)\,dx$$
$$f(x) = -\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x$$

(using $C = 0$. Any value for C will work.)

Section 5.1 - cont

28. In t years, value is increasing at a rate of $V'(t)$ dollars per year. Find an expression for the amount by which V will increase during the next 5 years.

Amt of increase $= V(5) - V(0)$, using $V(t) = \int V'(t)\,dt$
(notice C's will cancel from $V(5)$ and $V(0)$).

34. $R'(q) = 100q^{-1/2}$ $\qquad P(16) = \$520$. Find $P(25)$.
$C'(q) = 0.4q$

$P = R - C, \quad P' = R' - C'$

$P' = 100q^{-1/2} - 0.4q$

$P = \int (100q^{-1/2} - 0.4q)\,dq = 200q^{1/2} - 0.2q^2 + K$

$P(16) = 520 = 200\sqrt{16} - 0.2(16)^2 + K = 800 - 51.2 + K$

$520 = 748.8 + K$

$-228.8 = K$

$P(q) = 200q^{1/2} - 0.2q^2 - 228.8$

$P(25) = 200(5) - 0.2(625) - 228.8 = 1000 - 125 - 228.8 = \$646.20$

38. $h'(t) = 0.2t^{2/3} + \sqrt{t}$ ft/yr. $h(0) = 2$. Find $h(27)$.

$h(t) = \int (0.2t^{2/3} + t^{1/2})\,dt = 0.2\left(\frac{3}{5}t^{5/3}\right) + \frac{2}{3}t^{3/2} + C$

$h(t) = 0.12t^{5/3} + \frac{2}{3}t^{3/2} + C$

$h(0) = 2 = 0 + 0 + C$

$h(t) = 0.12t^{5/3} + \frac{2}{3}t^{3/2} + 2$

$h(27) = 0.12(3)^5 + \frac{2}{3}(27)^{3/2} + 2 \approx 29.16 + 93.5307 + 2$

≈ 124.7 ft.