NAME KEY

Math 12
Test 1
Fall 2013

You have 50 minutes to complete this test. You must show all work to receive full credit. Work any 8 of the following 9 problems. Clearly Cross Out the problem you do not wish me to grade. Each problem is worth 12 points, and you get 4 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find \( f'(x) \) if \( f(x) = \frac{2}{3x - 4} \).

\[
\begin{align*}
    f'(x) &= \lim_{{h \to 0}} \frac{f(x+h)-f(x)}{h} \\
    &= \lim_{{h \to 0}} \frac{2}{3(x+h) - 4} - \frac{2}{3x - 4} \\
    &= \lim_{{h \to 0}} \frac{2(3x-4) - 2(3x+3h-4)}{(3(x+h)-4)(3x-4)} \\
    &= \lim_{{h \to 0}} \frac{-6}{(3x+3h-4)(3x-4)} \\
    &= \frac{-6}{(3x-4)^2}
\end{align*}
\]

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it’s undefined" and "denominator is zero" are not sufficient explanations).

(a) \( \lim_{{x \to 2^+}} \frac{x-1}{x^2-3x+2} = \lim_{{x \to 2^+}} \frac{x-1}{(x-1)(x-2)} = \lim_{{x \to 2^+}} \frac{1}{x-2} = \infty \)

plug in, get \( \frac{1}{4-6+2} = \frac{1}{0} \), use chart

(b) \( \lim_{{x \to 2^+}} \frac{x^2-3x}{x+1} = \frac{4-2}{3} = \frac{-2}{3} \)

(c) \( \lim_{{x \to 2}} \frac{x^3-8}{2-x} = \lim_{{x \to 2}} \frac{(x-2)(x^2+2x+4)}{2-x} = \lim_{{x \to 2}} -\frac{(x^2+2x+4)}{x-2} \\
\text{plug in, get} \ \frac{2}{2-2} = 0 \not \text{working} \ldots \)

not working...

\( \frac{-(4+4+4)}{2} = -12 \)
3. An efficiency study of the morning shift at a packaging plant indicates that an average worker arriving on the job at 8:00 am will have packed a total of \( Q(t) = -t^3 + 9t^2 + 12t \) boxes ready for shipping \( t \) hours later.

a) Using marginal analysis, estimate how many boxes the worker will pack between 9:00 am and 10:00 am.

\[
Q'(t) = -3t^2 + 18t + 12. \quad \text{At } 9:00 \text{ am, } t=1, \text{ and at } 10:00 \text{ am, } t=2.
\]

\[
\# \text{boxes} \approx Q'(1) = -3 + 18 + 12 = 27 \text{ boxes}
\]

b) Find the exact number of boxes the worker actually packs between 9:00 am and 10:00 am.

\[
\# \text{boxes} = Q(2) - Q(1)
\]

\[
= (-8 + 36 + 24) - (-1 + 9 + 12)
\]

\[
= 58 - 20
\]

\[
= 32 \text{ boxes}
\]

4. Find the equation of the line parallel to \( 3y - 5x + 10 = 0 \) that goes through the point \((-4, -2)\).

\[
3y = 5x - 10
\]

\[
y = \frac{5}{3}x - \frac{10}{3}
\]

\( m \)

\[
y + 2 = \frac{5}{3} (x + 4) \quad \text{Fine to stop here}
\]

\[
y = \frac{5}{3}x + \frac{20}{3} - \frac{6}{3}
\]

\[
y = \frac{5}{3}x + \frac{14}{3}
\]

5. Find the equation of the line tangent to the graph of \( f(x) = \frac{3x^2 + 2x}{\sqrt{x}} \) at the point where \( x = 1 \).

\( \text{Point: } x=1, \quad y = f(1) = \frac{3+2}{1} = 5 \quad (1, 5) \).

\( \text{Slope: } \quad f(x) = (3x^2 + 2x)(x^{-\frac{1}{2}}) = 3x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}\)

\[
f'(x) = \frac{9}{2}x^{\frac{1}{2}} + x^{-\frac{3}{2}}
\]

\[
m = f'(1) = \frac{9}{2} + 1 = \frac{11}{2}
\]

\( \text{Line: } y - 5 = \frac{11}{2} (x - 1) \quad \text{Fine to stop here}
\]

\[
y = \frac{11}{2}x - \frac{11}{2} + \frac{10}{2}
\]

\[
y = \frac{11}{2}x - \frac{1}{2}
\]
6. Find $y'$ for the following functions (do not simplify):

a) $y = \frac{x^2 + 4}{(2x-1)(x^2 + 3x - 2)}$

$$y' = \frac{(2x)(2x-1)(x^2 + 3x - 2) - (x^2 + 4)[(2)(x^2 + 3x - 2) + (2x-1)(2x+3)]}{[(2x-1)(x^2 + 3x - 2)]^2}$$

Can instead multiply out,

$$y = \frac{x^2 + 4}{2x^3 - 6x^2 + 8x - 2} = \frac{x^2 + 4}{2x^3 + 5x^2 - 7x + 2}$$

and then do quotient rule.

b) $y = \frac{-x^2}{16} + \frac{2}{x} - \frac{3\sqrt{x} - 1}{x} + \frac{1}{3x^2 + x^{-2}}$

$$y = -\frac{1}{16}x^2 + 2x^{-1} - x^{3\sqrt{3}} - x^{-1} + \frac{1}{3}x^{-2} + x^{-2}$$

$$y' = -\frac{1}{8}x - 2x^{-2} - \frac{2}{3}x^{-\sqrt{3}} + x^{-2} - \frac{2}{3}x^{-3} - 2x^{-3}$$

7. A bus company uses the following pricing structure when charging groups to charter their buses. Groups containing no more than 40 people will be charged a fixed amount of $2400 (40 times $60). In groups containing between 40 and 80 people everyone will pay $60 minus 50 cents for each person in excess of 40. The company’s lowest fare of $40 per person will be offered to groups that have 80 people or more. Express the bus company’s revenue as a function of the size of the group.

Let $x = \# \text{ people in group}$.

$$\text{Revenue} = R(x) = \begin{cases} 2400 & x \leq 40 \\ (60 - \frac{1}{2}(x-40))(x) & 40 < x \leq 80 \\ 40x & x \geq 80 \end{cases}$$

↑ not picky about the =, doesn't matter since R is same either way.
8. Consider the graph of the function \( f(x) \) given below.

a) Find \( \lim_{x \to 2^-} f(x) = 4 \)

b) Find \( \lim_{x \to 2^+} f(x) = 0 \)

c) Find \( \lim_{x \to \infty} f(x) = 3 \)

d) Find \( \lim_{x \to 1^-} f(x) = -2 \)

e) Find \( \lim_{x \to 1^+} f(x) = -\infty \)

f) Find \( \lim_{x \to 1} f(x) \). DNE

9. Sketch the graph of \( f(x) = \begin{cases} 
1-x & \text{if } x < 2 \\
x^2 - 2x & \text{if } x \geq 2 
\end{cases} \). Fully describe the continuity of this function.

From the graph, we see that \( f \) is continuous on \((-\infty, 2) \cup (2, \infty)\) (continuous everywhere except \( x = 2 \)).

Another way, not looking at the graph, is to see that the pieces of \( f(x) \) are polynomials, so \( f \) is continuous everywhere except maybe at \( x = 2 \). Then consider \( x = 2 \).

\( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (1-x) = -1 \)

\( \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 - 2x) = 0 \)

Not same, so \( \lim f(x) \) DNE, and \( f \) is NOT continuous at \( x = 2 \).