NAME KEY

Math 12
Test 1
Fall 2013

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 8 of the following 9 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 12 points, and you get 4 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = \frac{2}{3x-4}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{3(x+h)-4} - \frac{2}{3x-4}}{h} = \lim_{h \to 0} \frac{\frac{2(3x-4) - 2(3x+3h-4)}{[3(x+h)-4](3x-4)}}{h}$$

$$= \lim_{h \to 0} \frac{6x-8-6x-6h+8}{(3x+3h-4)(3x-4)} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-6h}{h(3x+3h-4)(3x-4)}$$

$$= \lim_{h \to 0} \frac{-6}{(3x+3h-4)(3x-4)} = \frac{-6}{(3x-4)^2}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim_{x \to 2^+} \frac{x-1}{x^2-3x+2} = \lim_{x \to 2^+} \frac{x-1}{(x-1)(x-2)} = \lim_{x \to 2^+} \frac{1}{x-2} = \infty$

plug in, get $\frac{1}{4-6+2} = \frac{1}{0}$, use chart

x	$f(x)$
3	1
2.5	2
2.1	10
2.01	100
2^+	∞

(b) $\lim_{x \to 2} \frac{x^2-3x}{x+1} = \frac{4-6}{3} = \frac{-2}{3}$

(c) $\lim_{x \to 2} \frac{x^3-8}{2-x} = \lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{2-x} = \lim_{x \to 2} -(x^2+2x+4)$

plug in, get $\frac{8-8}{2-2} = \frac{0}{0}$
not working...

$= -(4+4+4)$

$= -12$

3. An efficiency study of the morning shift at a packaging plant indicates that an average worker arriving on the job at 8:00 am will have packed a total of $Q(t) = -t^3 + 9t^2 + 12t$ boxes ready for shipping t hours later.

a) Using marginal analysis, ***estimate*** how many boxes the worker will pack between 9:00 am and 10:00 am.

$Q'(t) = -3t^2 + 18t + 12$. At 9:00, $t=1$, and at 10:00, $t=2$.

#boxes $\approx Q'(1) = -3 + 18 + 12 = 27$ boxes

b) Find the ***exact*** number of boxes the worker actually packs between 9:00 am and 10:00 am.

$$\begin{aligned} \#\text{ boxes} &= Q(2) - Q(1) \\ &= (-8 + 36 + 24) - (-1 + 9 + 12) \\ &= 52 - 20 \\ &= 32 \text{ boxes} \end{aligned}$$

4. Find the equation of the line parallel to $3y - 5x + 10 = 0$ that goes through the point $(-4,-2)$.

$$3y = 5x - 10$$
$$y = \underbrace{\tfrac{5}{3}}_{m}x - \tfrac{10}{3}$$

using $m = 5/3$ and the point $(-4,-2)$, our line is

$y + 2 = \frac{5}{3}(x + 4)$ ← fine to stop here

$$y = \tfrac{5}{3}x + \tfrac{20}{3} - \tfrac{6}{3}$$
$$y = \tfrac{5}{3}x + \tfrac{14}{3}$$

5. Find the equation of the line tangent to the graph of $f(x) = \dfrac{3x^2 + 2x}{\sqrt{x}}$ at the point where $x = 1$.

point: $x = 1$, $y = f(1) = \frac{3+2}{1} = 5$ $(1, 5)$.

slope: $f(x) = (3x^2 + 2x)(x^{-1/2}) = 3x^{3/2} + 2x^{1/2}$

$$f'(x) = \tfrac{9}{2}x^{1/2} + x^{-1/2}$$
$$m = f'(1) = \tfrac{9}{2} + 1 = \tfrac{11}{2}$$

line $y - 5 = \frac{11}{2}(x - 1)$ ← fine to stop here

$$y = \tfrac{11}{2}x - \tfrac{11}{2} + \tfrac{10}{2}$$
$$y = \tfrac{11}{2}x - \tfrac{1}{2}$$

6. Find y' for the following functions (do not simplify) :

a) $y = \dfrac{x^2+4}{(2x-1)(x^2+3x-2)}$

$$y' = \frac{(2x)(2x-1)(x^2+3x-2) - (x^2+4)[(2)(x^2+3x-2) + (2x-1)(2x+3)]}{[(2x-1)(x^2+3x-2)]^2}$$

can instead multiply out, $y = \dfrac{x^2+4}{2x^3+6x^2-4x-x^2-3x+2} = \dfrac{x^2+4}{2x^3+5x^2-7x+2}$

and then do quotient rule.

b) $y = \dfrac{-x^2}{16} + \dfrac{2}{x} - \sqrt[3]{x^2} - \dfrac{1}{x} + \dfrac{1}{3x^2} + x^{-2}$

$$y = -\frac{1}{16}x^2 + 2x^{-1} - x^{2/3} - x^{-1} + \frac{1}{3}x^{-2} + x^{-2}$$

$$y' = -\frac{1}{8}x - 2x^{-2} - \frac{2}{3}x^{-1/3} + x^{-2} - \frac{2}{3}x^{-3} - 2x^{-3}$$

7. A bus company uses the following pricing structure when charging groups to charter their buses. Groups containing no more than 40 people will be charged a fixed amount of \$2400 (40 times \$60). In groups containing between 40 and 80 people everyone will pay \$60 minus 50 cents for each person in excess of 40. The company's lowest fare of \$40 per person will be offered to groups that have 80 people or more. Express the bus company's revenue as a function of the size of the group.

Let x = # people in group.

$$\text{Revenue} = R(x) = \begin{cases} 2400 & x \le 40 \\ \left(60 - \frac{1}{2}(x-40)\right)(x) & 40 < x < 80 \\ 40x & x \ge 80 \end{cases}$$

not picky about the =, doesn't matter since R is same either way.

8. Consider the graph of the function $f(x)$ given below.

a) Find $\lim_{x\to 2} f(x)$. $= 4$

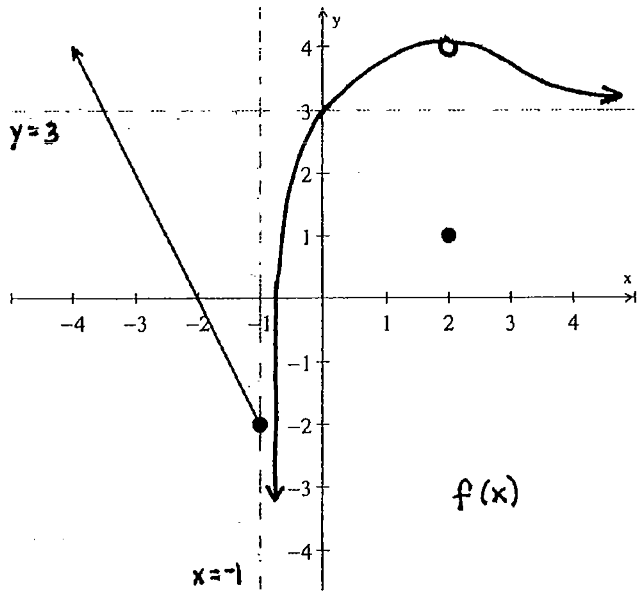
b) Find $\lim_{x\to -2} f(x)$. $= 0$

c) Find $\lim_{x\to \infty} f(x)$. $= 3$

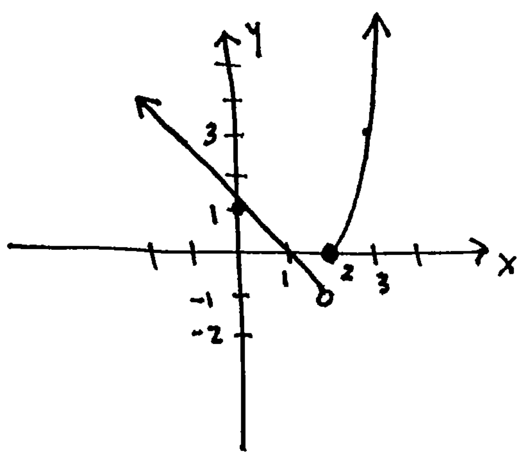
d) Find $\lim_{x\to -1^-} f(x)$. $= -2$

e) Find $\lim_{x\to -1^+} f(x)$. $= -\infty$

f) Find $\lim_{x\to -1} f(x)$. DNE



9. Sketch the graph of $f(x) = \begin{cases} 1-x & if \quad x<2 \\ x^2-2x & if \quad x \geq 2 \end{cases}$. Fully describe the continuity of this function.



From the graph, we see that f is continuous on $(-\infty, 2) \cup (2, \infty)$ (continuous everywhere except $x=2$).

Another way, not looking at the graph, is to see that the pieces of $f(x)$ are polynomials, so f is continuous everywhere except maybe at $x=2$. Then consider $x=2$.

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (1-x) = -1$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^2-2x) = 0$

Not same, so $\lim_{x\to 2} f(x)$ DNE, and f is NOT continuous at $x=2$.