NAME KEY

Math 12
Test 2
Fall 2010

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find all intervals of increase and decrease for $f(x)=\dfrac{x^2}{x^2-4}$. Then find all extrema.

$$f'(x)=\frac{(2x)(x^2-4)-(x^2)(2x)}{(x^2-4)^2}=\frac{2x^3-8x-2x^3}{(x^2-4)^2}=\frac{-8x}{[(x+2)(x-2)]^2}$$

increasing on $(-\infty,-2)\cup(-2,0)$
decreasing on $(0,2)\cup(2,\infty)$

$f(0)=0$

maximum $(0,0)$
no minima

CN: $x=0,2,-2$

f': $+$ on $(-\infty,-2)$, $+$ on $(-2,0)$, $-$ on $(0,2)$, $-$ on $(2,\infty)$

2. Calculate the following limits.

a) $\displaystyle\lim_{x\to-\infty}\frac{x^3-3x+5}{2x+3}=\lim_{x\to-\infty}\frac{x^3}{2x}=\lim_{x\to-\infty}\frac{x^2}{2}=\infty$

b) $\displaystyle\lim_{x\to\infty}\frac{x(2x-3)}{7-x^2}=\lim_{x\to\infty}\frac{2x^2-3x}{-x^2+7}=-2$

c) $\displaystyle\lim_{x\to\infty}\left(2+\frac{1}{x^2}\right)=\lim_{x\to\infty}\left(\frac{2x^2}{x^2}+\frac{1}{x^2}\right)=\lim_{x\to\infty}\frac{2x^2+1}{x^2}=2$

3. Suppose that at price p, demand for a certain product is given by $q(p)=\sqrt{144-2p}$ when price is a positive value less than \$72.

a) Find the price elasticity of demand when price is \$60.

$$E(p)=\frac{p}{q}\cdot q'=\left(\frac{p}{\sqrt{144-2p}}\right)\left(\frac{1}{2}\right)(144-2p)^{-1/2}(-2)=\frac{-p}{144-2p}$$

$$E(60)=\frac{-60}{144-120}=\frac{-60}{24}=\frac{-10}{4}=\frac{-5}{2}$$

b) Is demand elastic or inelastic at this price? Write a sentence in plain English that explains your answer from (a).

$|E(60)|=5/2>1$, so demand is elastic. If price goes up 1% (from \$60 to \$60.60), demand will go down 2.5%

c) Give an example of a product in the correct price range that might behave this way.

elastic → luxury item. \$60...
maybe a nice bottle of wine, fancy dinner...

4. Differentiate the following functions. Do NOT simplify!

a) $f(x)=\left(\frac{x^2+1}{x^2-1}\right)^3$

$$f'(x)=3\left(\frac{x^2+1}{x^2-1}\right)^2\left(\frac{(2x)(x^2-1)-(x^2+1)(2x)}{(x^2-1)^2}\right)$$

b) $f(x)=(2x-5)^4(8x^2-5)^{-3}$

$$f'(x)=4(2x-5)^3(2)(8x^2-5)^{-3}+(2x-5)^4(-3)(8x^2-5)^{-4}(16x)$$

5. Find the absolute maximum and minimum points on the graph of $f(x) = -3x^4 + 8x^3 - 10$ on the interval $[1,3]$.

$f'(x) = -12x^3 + 24x^2$

$= -12x^2(x - 2)$

outside of interval

Critical numbers: $x = 0, x = 2$

Endpoints: $x = 1, x = 3$

$f(1) = -3 + 8 - 10 = -5$

$f(2) = -48 + 64 - 10 = 16$

$f(3) = -243 + 216 - 10 = -37$

absolute max $(2, 16)$

absolute min $(3, -37)$

6. Sketch the graph of a function $f(x)$ so that all conditions below are satisfied. Be sure your graph is big enough so I can see it and it is properly labeled.

a) $f(x)$ is defined for all x except $x = 2$.

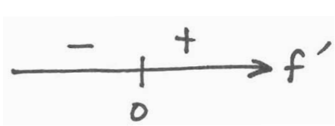
b) $f'(x) < 0$ when $x < 0$, but $f'(x) \geq 0$ otherwise.

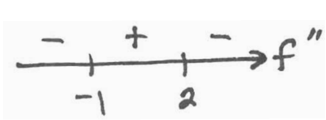
c) $f''(x) < 0$ when $x < -1$ and when $x > 2$, but $f''(x) \geq 0$ otherwise.

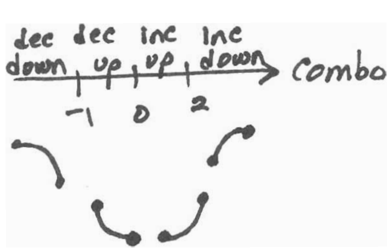
d) $\lim_{x \to -\infty} f(x) = -1$.

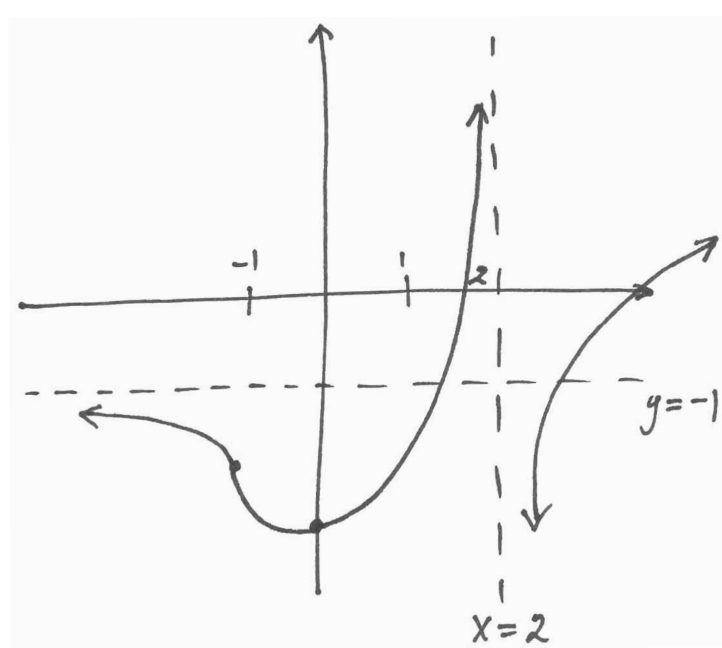
Hole or asymp at $x = 2$

HA $y = -1$, happens on left side









7. Find the equation of the line tangent to $(xy^2+1)^4 = 90x - 9y$ at the point (1,1).

$$4(xy^2+1)^3\left((1)(y^2)+(x)(2yy')\right) = 90 - 9y'$$

$x=1, y=1$, so

$$4(2)^3(1+2y') = 90 - 9y'$$

$$32 + 64y' = 90 - 9y'$$

$$73y' = 58$$

$$y' = 58/73 = m$$

Line: $y - 1 = \frac{58}{73}(x-1)$

8. A store expects to sell 800 bottles of perfume this year. The perfume costs the store owner \$20 per bottle, there is an ordering fee of \$10 per shipment, and the cost of storing the perfume is 40¢ per bottle per year. The perfume is consumed at a constant rate through to the year, and each shipment arrives just as the preceding shipment is used up.

a) How many bottles should the store order in each shipment so that cost is minimized? 200 bottles

b) How often should the store order the perfume? 4 times per year

Cost = product cost + shipping cost + storage cost

$$C = (800)(20) + (800x^{-1})(10) + \left(\frac{x}{2}\right)(0.40)$$

$$C = 16000 + 8000x^{-1} + .2x$$

x = # bottles/shipment

$\frac{800}{x} = 800x^{-1} =$ # shipments

$\frac{x}{2}$ = avg # bottles in storage

$$C' = -8000x^{-2} + .2 = 0$$

$$\frac{8000}{x^2} = \frac{1}{5}$$

$$40000 = x^2$$

$$\pm 200 = x$$

Is $x = 200$ the number giving min cost?

Method ①

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -200 & & 200 & \end{array} \quad C'$$

min

method ②

$$C'' = 16000x^{-3}$$

$$C''(200) = 16000/200^3 > 0$$

min.