

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $\frac{dy}{dx} = \frac{1}{y^2(2x-5)^6}$ if $y = 0$ when $x = 2$.

$$y^2 dy = \frac{1}{(2x-5)^6} dx$$

$$\int y^2 dy = \int (2x-5)^{-6} dx$$

$$\frac{1}{3} y^3 = \frac{1}{2} \int u^{-6} du = -\frac{1}{10} u^{-5} + C$$

$$= -\frac{1}{10} (2x-5)^{-5} + C$$

$$u = 2x-5$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$\text{if } x=2, y=0, \text{ so}$$

$$0 = -\frac{1}{10} (4-5)^{-5} + C$$

$$0 = -\frac{1}{10} \frac{1}{(-1)^5} + C$$

$$0 = \frac{1}{10} + C$$

$$C = -\frac{1}{10}$$

$$\frac{1}{3} y^3 = -\frac{1}{10} (2x-5)^{-5} - \frac{1}{10}$$

$$y^3 = -\frac{3}{10} (2x-5)^{-5} - \frac{3}{10}$$

$$y = \sqrt[3]{-\frac{3}{10} (2x-5)^{-5} - \frac{3}{10}}$$

2. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^2 \ln(3x)$

$$f'(x) = 2x \ln(3x) + x^2 \left(\frac{1}{3x}\right)(3)$$

(b) $f(x) = \sqrt{e^{x^2-6} + 4x} = (e^{x^2-6} + 4x)^{1/2}$

$$f'(x) = \frac{1}{2} (e^{x^2-6} + 4x)^{-1/2} (e^{x^2-6} (2x) + 4)$$

3. Suppose you want to have \$15,000 three years from now in order to purchase a car. How much will you need to invest in one lump sum now at an annual rate of 6% compounded monthly in order to reach your goal?

$$B = P \left(1 + \frac{r}{k} \right)^{kt}$$

$$15000 = P \left(1 + \frac{0.06}{12} \right)^{12 \cdot 3}$$

$$15000 = P (1.005)^{36}$$

$$P = \frac{15000}{(1.005)^{36}} \approx \$12,534.67$$

4. Suppose that the number of hamburgers sold by a national fast-food chain grows exponentially. If 4 billion were sold by 2005 and 12 billion had been sold by 2010, in what year will sales reach the 25 billion mark?

$$B = Pe^{rt}$$

2005	$t=0$	$B = 4$ billion
2010	$t=5$	$B = 12$ billion
?	$t=?$	$B = 25$ billion

$$\begin{aligned} \textcircled{3} \quad 25 &= 4e^{0.2197t} \\ 6.25 &= e^{0.2197t} \\ \ln 6.25 &= 0.2197t \\ t &\approx \frac{\ln 6.25}{0.2197} \end{aligned}$$

$$\textcircled{1} \quad 4 = Pe^0, \quad P=4, \quad \text{so } B = 4e^{rt}$$

$$\textcircled{2} \quad 12 = 4e^{5r}$$

$$3 = e^{5r}$$

$$\ln 3 = 5r$$

$$r = \frac{\ln 3}{5} \approx 0.2197$$

$$\text{so now } B = 4e^{0.2197t}$$

$$t \approx 8.34 \text{ years}$$

This will be partway through 2013.

5. a) If $\log_3 x = 2\log_3 6 - \log_3 7$, find x . Your solution should be a rational number.

$$\log_3 x = \log_3 36 - \log_3 7$$

$$\log_3 x = \log_3 \frac{36}{7}$$

$$x = \frac{36}{7}$$

- b) If $\log_3 x = 6$, $\log_3 y = \frac{1}{2}$, and $\log_3 z = 3$, calculate $\log_3 \frac{\sqrt{x}}{y^2 z}$.

$$\log_3 \frac{\sqrt{x}}{y^2 z} = \log_3 \sqrt{x} - (\log_3 y^2 + \log_3 z)$$

$$= \frac{1}{2} \log_3 x - 2 \log_3 y - \log_3 z$$

$$= \frac{1}{2} (6) - 2 \left(\frac{1}{2}\right) - 3 = 3 - 1 - 3 = -1$$

6. Evaluate the following integrals:

a) $\int 5e^{3x} dx = \frac{5}{3} e^{3x} + C$

b) $\int \frac{3x - \sqrt{x}}{x^2} dx = \int (3x^{-1} - x^{-3/2}) dx$
 $= 3 \ln|x| + 2x^{-1/2} + C$

c) $\int 2xe^x dx$

$$u = 2x$$

$$du = 2dx$$

$$dv = e^x dx$$

$$v = e^x$$

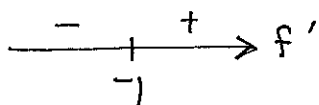
$$\rightarrow = 2xe^x - \int 2e^x dx = 2xe^x - 2e^x + C$$

7. Find all maxima, minima and inflection points of $f(x) = xe^x$. (If there are none, say so.) Also give the intervals where f is increasing, decreasing, concave up, and concave down. Find all asymptotes (or say there are none). Then carefully sketch the graph of f . Be sure to label the asymptotes, extrema, and inflection points.

$$f'(x) = e^x + xe^x$$

$$= e^x(1+x) = 0$$

CN: $x = -1$

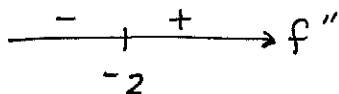


$$f(-1) = -1e^{-1} = -\frac{1}{e}$$

$$f''(x) = e^x + e^x + xe^x$$

$$= e^x(2+x) = 0$$

IN: $x = -2$



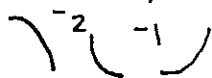
$$f(-2) = -2e^{-2} = -\frac{2}{e^2}$$

$$\lim_{x \rightarrow -\infty} xe^x = ? \quad -100e^{-100}$$

↓
o fast

$$\lim_{x \rightarrow \infty} xe^x = \infty$$

dec down | dec up | inc up → combo



Results

inc on $(-1, \infty)$

dec on $(-\infty, -1)$

min at $(-1, -\frac{1}{e})$

conc up on $(-2, \infty)$

conc down on $(-\infty, -2)$

inf pt $(-2, -\frac{2}{e^2})$

VA: none, defined for all x

HA: $y = 0$ (happens on left)

