NAME KEY

Math 12
Test 3
Summer 2013

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $\frac{dy}{dx} = \frac{1}{y^2(2x-5)^6}$ if $y = 0$ when $x = 2$.

$$y^2\,dy = \frac{1}{(2x-5)^6}\,dx$$

$$\int y^2\,dy = \int (2x-5)^{-6}\,dx$$

$$\frac{1}{3}y^3 = \frac{1}{2}\int u^{-6}\,du = \frac{-1}{10}u^{-5} + C$$

$$= \frac{-1}{10}(2x-5)^{-5} + C$$

$u = 2x-5$
$du = 2\,dx$
$\frac{1}{2}du = dx$

If $x=2$, $y=0$, so

$$0 = \frac{-1}{10}(4-5)^{-5} + C$$

$$0 = \frac{-1}{10}\frac{1}{(-1)^5} + C$$

$$0 = \frac{1}{10} + C$$

$$C = -\frac{1}{10}$$

$$\frac{1}{3}y^3 = \frac{-1}{10}(2x-5)^{-5} - \frac{1}{10}$$

$$y^3 = -\frac{3}{10}(2x-5)^{-5} - \frac{3}{10}$$

$$y = \sqrt[3]{-\frac{3}{10}(2x-5)^{-5} - \frac{3}{10}}$$

2. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^2\ln(3x)$

$$f'(x) = 2x\ln(3x) + x^2\left(\frac{1}{3x}\right)(3)$$

(b) $f(x) = \sqrt{e^{x^2-6}+4x} = \left(e^{x^2-6}+4x\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left(e^{x^2-6}+4x\right)^{-1/2}\left(e^{x^2-6}(2x)+4\right)$$

3. Suppose you want to have $15,000 three years from now in order to purchase a car. How much will you need to invest in one lump sum now at an annual rate of 6% compounded monthly in order to reach your goal?

$$B = P\left(1+\frac{r}{K}\right)^{Kt}$$

$$15000 = P\left(1+\frac{0.06}{12}\right)^{12\cdot 3}$$

$$15000 = P(1.005)^{36}$$

$$P = \frac{15000}{(1.005)^{36}} \approx \$12{,}534.67$$

4. Suppose that the number of hamburgers sold by a national fast-food chain grows exponentially. If 4 billion were sold by 2005 and 12 billion had been sold by 2010, in what year will sales reach the 25 billion mark?

$$B = Pe^{rt}$$

2005	$t=0$	$B=4$ billion
2010	$t=5$	$B=12$ billion
?	$t=?$	$B=25$ billion

① $4 = Pe^0$, $P=4$, so $B=4e^{rt}$

② $12 = 4e^{5r}$

$3 = e^{5r}$

$\ln 3 = 5r$

$r = \frac{\ln 3}{5} \approx 0.2197$

so now $B \doteq 4e^{0.2197t}$

③ $25 \doteq 4e^{0.2197t}$

$6.25 \doteq e^{0.2197t}$

$\ln 6.25 \doteq 0.2197t$

$t \approx \frac{\ln 6.25}{0.2197}$

$t \approx 8.34$ years

this will be partway through 2013.

5. a) If $\log_3 x = 2\log_3 6 - \log_3 7$, find x. Your solution should be a rational number.

$$\log_3 x = \log_3 36 - \log_3 7$$
$$\log_3 x = \log_3 \frac{36}{7}$$
$$x = \frac{36}{7}$$

b) If $\log_3 x = 6$, $\log_3 y = \frac{1}{2}$, and $\log_3 z = 3$, calculate $\log_3 \frac{\sqrt{x}}{y^2 z}$.

$$\log_3 \frac{\sqrt{x}}{y^2 z} = \log_3 \sqrt{x} - (\log_3 y^2 + \log_3 z)$$
$$= \frac{1}{2}\log_3 x - 2\log_3 y - \log_3 z$$
$$= \frac{1}{2}(6) - 2\left(\frac{1}{2}\right) - 3 = 3 - 1 - 3 = -1$$

6. Evaluate the following integrals:

a) $\int 5e^{3x}\,dx = \frac{5}{3}e^{3x} + C$

b) $\int \frac{3x - \sqrt{x}}{x^2}\,dx = \int (3x^{-1} - x^{-3/2})\,dx$
$$= 3\ln|x| + 2x^{-1/2} + C$$

c) $\int 2xe^x\,dx$

$u = 2x$ $\quad dv = e^x dx$

$du = 2dx$ $\quad v = e^x$

$$= 2xe^x - \int 2e^x dx = 2xe^x - 2e^x + C$$

7. Find all maxima, minima and inflection points of $f(x) = xe^x$. (If there are none, say so.) Also give the intervals where f is increasing, decreasing, concave up, and concave down. Find all asymptotes (or say there are none). Then carefully sketch the graph of f. Be sure to label the asymptotes, extrema, and inflection points.

$$f'(x) = e^x + xe^x$$
$$= e^x(1+x) = 0$$

CN: $x = -1$

$$\xrightarrow[\quad -1 \quad]{-\quad\quad +} f'$$

$$f(-1) = -1e^{-1} = \frac{-1}{e}.$$

$$f''(x) = e^x + e^x + xe^x$$
$$= e^x(2+x) = 0$$

IN: $x = -2$

$$\xrightarrow[\quad -2 \quad]{-\quad\quad +} f''$$

$$f(-2) = -2e^{-2} = \frac{-2}{e^2}$$

$$\lim_{x \to -\infty} xe^x = ? \qquad -100e^{-100}$$

↓ 0 fast

$$\lim_{x \to \infty} xe^x = \infty$$

dec down | dec up | inc up → combo

$-2 \quad -1$

Results

inc on $(-1, \infty)$

dec on $(-\infty, -1)$

min at $(-1, \frac{-1}{e})$

conc up on $(-2, \infty)$

conc down on $(-\infty, -2)$

inf pt $(-2, \frac{-2}{e^2})$

VA: none, defined for all x

HA: $y = 0$ (happens on left)