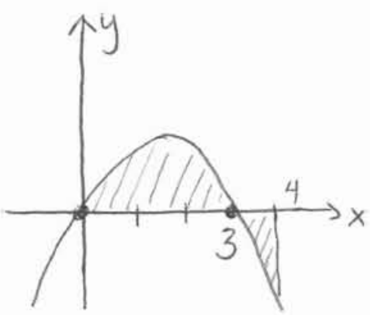
NAME Key

Math 12
Test 4
Fall 2011

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region between $y = 3x - x^2$ and the x-axis, from $x = 0$ to $x = 4$. Be sure to sketch a graph first!

$y = x(3-x) \quad (0,0)\ (3,0)$



$$A = \int_0^3 [(3x-x^2) - 0]\,dx + \int_3^4 [0-(3x-x^2)]\,dx$$

$$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3 + \left[-\frac{3}{2}x^2 + \frac{1}{3}x^3\right]_3^4$$

$$= \left[\left(\frac{27}{2} - 9\right) - (0-0)\right] + \left[\left(-24 + \frac{64}{3}\right) - \left(-\frac{27}{2} + 9\right)\right]$$

$$= \frac{27}{2} - 9 - 24 + \frac{64}{3} + \frac{27}{2} - 9$$

$$= 27 - 18 - 24 + 64/3 = -15 + 64/3$$

$$= 19/3$$

2. Find all four second-order partial derivatives of $f(x, y) = ye^x - x\ln y$. Do NOT simplify.

$f_x = ye^x - \ln y$

$f_y = e^x - xy^{-1}$

$f_{xx} = ye^x$

$f_{xy} = e^x - \frac{1}{y}$

$f_{yy} = 0 + xy^{-2} = \frac{x}{y^2}$

$f_{yx} = e^x - \frac{1}{y}$

3. Find and classify the critical points of $f(x,y) = x^3 - y^3 + 6xy$.

$f_x = 3x^2 + 6y = 0 \rightarrow x^2 + 2y = 0 \rightarrow y = \frac{-x^2}{2}$

$f_y = -3y^2 + 6x = 0 \rightarrow -y^2 + 2x = 0 \rightarrow$

$-\left(\frac{-x^2}{2}\right)^2 + 2x = 0$

$-\frac{x^4}{4} + 2x = 0$

$x^4 - 8x = 0$

$x(x^3 - 8) = 0$

$x = 0, \; x = 2$

$y = 0 \quad y = -2$

$$\left.\begin{array}{l} f_{xx} = 6x \\ f_{yy} = -6y \\ f_{xy} = 6 \end{array}\right\} D(x,y) = -36xy - 36$$

crit pts $(0,0), (2,-2)$

$D(0,0) = -36 < 0$. $(0,0)$ gives a saddle pt

$D(2,-2) = -36(-4) - 36 > 0$

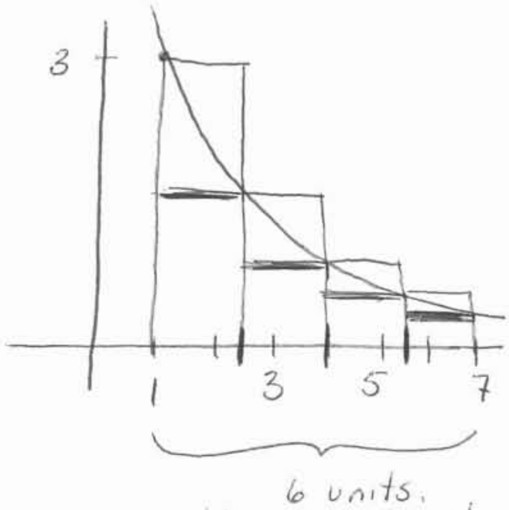
$f_{xx}(2,-2) = 6(2) > 0$, $(2,-2)$ gives a min

4. Suppose two products have demand equations $D_1 = 500 + \frac{10}{p_1 + 2} - 5p_2$ and $D_2 = 400 - 2p_1 + \frac{7}{p_2 + 3}$, where p_1 and p_2 are the respective prices of the products. Are the products competitive, complementary, or neither? Give an example of two products that might behave this way.

$$\left.\begin{array}{l} \frac{\partial D_1}{\partial p_2} = -5 < 0 \\ \frac{\partial D_2}{\partial p_1} = -2 < 0 \end{array}\right\}$$

products are complementary, like hot dogs & hot dog buns

5. Using four rectangles, ***estimate*** the area under the curve $y = \frac{3}{x}$ between $x = 1$ and $x = 7$. Then find the ***exact*** area.



width of rectangle $= \frac{6}{4} = \frac{3}{2}$

using lower ones:

$$A \approx R_1 + R_2 + R_3 + R_4$$
$$\approx \frac{3}{2} f\left(\frac{5}{2}\right) + \frac{3}{2} f(4) + \frac{3}{2} f\left(\frac{11}{2}\right) + \frac{3}{2} f(7)$$
$$\approx \frac{3}{2}\left(\frac{6}{5}\right) + \frac{3}{2}\left(\frac{3}{4}\right) + \frac{3}{2}\left(\frac{6}{11}\right) + \frac{3}{2}\left(\frac{3}{7}\right)$$
$$\approx \frac{9}{5} + \frac{9}{8} + \frac{9}{11} + \frac{9}{14} \approx 4.386$$

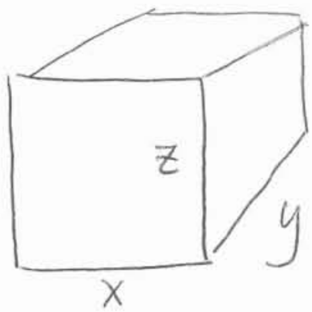
(uppers works, too)

$$A = \int_1^7 \frac{3}{x}\,dx = 3\ln|x| \Big|_1^7 = 3\ln 7 - 3\ln 1$$
$$= 3\ln 7 \approx 5.838$$

6. Calculate $\int_4^\infty e^{-x/2}dx$.

$$\int_4^\infty e^{-\frac{1}{2}x}\,dx = \lim_{n\to\infty} \int_4^n e^{-\frac{1}{2}x}\,dx$$
$$= \lim_{n\to\infty} -2e^{-\frac{1}{2}x} \Big|_4^n$$
$$= \lim_{n\to\infty} \left(\underbrace{-2e^{-\frac{1}{2}n}}_{0} + 2e^{-2}\right)$$
$$= \frac{2}{e^2}$$

7. Suppose a rectangular container with volume 288 cubic feet is to be built. If the bottom of the container costs \$5 per square foot and the top and sides each cost \$3 per square foot to construct, find the minimum cost of the container.



$V = 288 = xyz$ (constraint)

$\text{Cost} = 5xy + 3xz + 3xz + 3yz + 3yz + 3xy$

$= 8xy + 6xz + 6yz$ (minimize)

$F(x, y, z, \lambda) = 8xy + 6xz + 6yz - \lambda(xyz - 288)$

$F_x = 8y + 6z - \lambda yz = 0$

$F_y = 8x + 6z - \lambda xz = 0$

$F_z = 6x + 6y - \lambda xy = 0$

$F_\lambda = -xyz + 288 = 0$

$8xy + 6xz - \lambda xyz = 0$

$8xy + 6yz - \lambda xyz = 0$

$6xz - 6yz = 0$

$x = y$

$8xy + 6yz - \lambda xyz = 0$

$6xz + 6yz - \lambda xyz = 0$

$8xy - 6xz = 0$

$2x(4y - 3z) = 0$

$4y = 3z$

$z = \frac{4}{3}y$

$-(y)(y)(\frac{4}{3}y) + 288 = 0$

$288 = \frac{4}{3}y^3$

$216 = y^3$

$\boxed{6 = y \quad x = 6 \quad z = 8}$

minimum cost is $8(6)(6) + 6(6)(8) + 6(6)(8) =$

$288 + 288 + 288 = \$864$