You have 50 minutes to complete this test. You must show all work to receive full credit. Work any 6 of the following 7 problems. Clearly CROSS OUT the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region bounded by the curves $y = 4x$ and $y = x^3 + 3x^2$. Be sure to sketch a graph first!

Intersection points:
- $(0, 0)$
- $(-4, -16)$
- $(3, 12)$

$$A = \int_{-4}^{0} (x^3 + 3x^2 - 4x) \, dx + \int_{0}^{3} (x^3 - (x^3 + 3x^2)) \, dx$$

$$= \left[ \frac{1}{4} x^4 + x^3 - 2x^2 \right]_{-4}^{0} + \left[ 2x^2 - \frac{1}{4} x^4 - x^3 \right]_{0}^{3}$$

$$= \left[ 0 - \left( \frac{256}{4} - 64 - 32 \right) \right] + \left[ (18 - \frac{81}{4} - 27) - 0 \right]$$

$$= - (64 - 64 - 32) + (18 - \frac{81}{4} - 27)$$

$$= 32 + \frac{-81}{4} + 9 = 23 - \frac{81}{4} = \frac{11}{4}$$

2. Find all four second-order partial derivatives of $f(x, y) = x^2 ye^x + 2x^2 y^2$. Do NOT simplify.

$$f_x = (2x)(ye^x) + (x^2)(ye^x) + 6x^2 y^2$$

$$f_y = x^2 e^x + 4x^3 y$$

$$f_{xx} = (2)(ye^x) + (2x)(ye^x) + (2x)(ye^x) + (x^2)(ye^x) + 12x y^2$$

$$f_{xy} = 2xe^x + x^2 e^x + 12 x^2 y$$

$$f_{yy} = 4x^3$$

$$f_{yx} = 2xe^x + x^2 e^x + 12 x^2 y$$
3. Find and classify the critical points of \( f(x, y) = x^3 + y^2 - 6xy + 9x + 5y + 2 \).

\[
\begin{align*}
  f_x &= 3x^2 - 6y + 9 = 0 \\
  f_y &= 2y - 6x + 5 = 0 \\
  f_{xx} &= 6x \\
  f_{yy} &= 2 \\
  f_{xy} &= -6 \\
  D &= f_{xx}f_{yy} - (f_{xy})^2 \\
  &= 12x - 36 \\
  D(2, \frac{3}{2}) &= 24 - 36 < 0 \\
  (2, \frac{3}{2}) &\text{ gives a saddle point} \\
  D(4, \frac{19}{2}) &= 48 - 36 > 0 \\
  f_{xx}(4, \frac{19}{2}) &= 6(4) > 0 \\
  (4, \frac{19}{2}) &\text{ gives a minimum} \\
\end{align*}
\]

4. Suppose product A and product B are competitive.

a) If the price of product A goes up, the demand for product A will go \underline{down}.

b) If the price of product A goes up, the demand for product B will go \underline{up}.

c) Two products that might behave this way are \underline{Coke} and \underline{Pepsi}.

 Suppose product A and product B are complementary.

d) If the price of product A goes up, the demand for product A will go \underline{down}.

e) If the price of product A goes up, the demand for product B will go \underline{down}.

f) Two products that might behave this way are \underline{hot dogs} and \underline{hot dog buns}.
According to postal regulations, the girth (distance around) plus the length of parcels sent by 4th class mail may not exceed 108 inches. What is the largest possible volume of a rectangular parcel with two square sides that can be sent by 4th class mail?

To maximize volume,

\[ 108 = 4x + y \quad \text{constraint} \]

\[ V = x^2 y \quad \text{optimize this} \]

\[
F(x, y, \lambda) = f(x, y) - \lambda(g(x, y) - 108)\\
F_x = 2xy - 4\lambda = 0 \\
F_y = x^2 - \lambda = 0 \\
F_\lambda = -4x - y + 108 = 0
\]

\[ 2xy - 4x^2 = 0 \quad \rightarrow \quad 2x(y - 2x) = 0 \]

or \[ y = 2x \]

\[ -4x - 2x + 108 = 0 \]

\[ -6x + 108 = 0 \]

\[ 108 = 6x \]

\[ x = \frac{108}{6} = 18 \text{ inches} \]

\[ y = 2x = 36 \text{ inches} \]

Largest possible volume is

\[ V = x^2 y \]

\[ V = (18)^2 (36) = 10,368 \text{ in}^3 \]
5. On a single plane, sketch and label 3 level curves of the surface \( z = xy \).

\[
\begin{align*}
\underline{z = 0} &: \quad x = 0 \text{ or } y = 0, \text{ curve is the coordinate axes.} \\
\underline{z = \frac{1}{2}} &: \quad \frac{1}{2} = xy \quad (1, \frac{1}{2}), (\frac{1}{2}, 1) \\
\underline{z = \frac{1}{2}} &: \quad \frac{1}{2} = xy \quad (1, \frac{1}{2}), (\frac{1}{2}, 1) \\
\end{align*}
\]

6. Calculate \( \int_1^\infty e^{-x} \, dx \).

\[
\begin{align*}
\int_1^\infty e^{-x} \, dx &= \lim_{n \to \infty} \int_1^n e^{-x} \, dx \\
&= \lim_{n \to \infty} \left[ e^{-x} \right]_1^n \\
&= \lim_{n \to \infty} \left( e^{-1} - e^{-n} \right) \\
&= 1 - e^{-1} \\
&= 0.3678794.
\end{align*}
\]