

NAME \_\_\_\_\_

Math 12  
Test 4  
Spring 2013

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region bounded by the curves  $y = x$ ,  $y = \frac{1}{x^2}$ ,  $y = 0$ , and  $x = 2$ . Be sure to sketch a graph first!

2. Suppose  $z = xe^{x-y} - ye^{y-x}$ . Show that  $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = e^{x-y} - e^{y-x}$ .

3. Find and classify the critical points of  $f(x, y) = x^3 - 3xy + y^2 + y - 5$ .

4. The demand functions for two products are given by  $D_1 = \frac{100}{p_1 \sqrt{p_2}}$  and

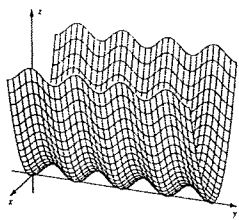
$D_2 = \frac{500}{p_2 \sqrt[3]{p_1}}$ , where  $p_1$  and  $p_2$  are the respective prices of the products. Are the

two products competitive, complementary, or neither? (show work, and remember that prices are positive numbers) Give an example of two products that might behave in this way.

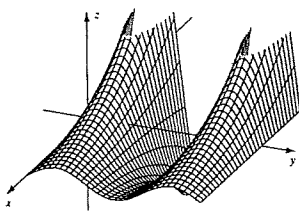
5. When  $x$  units of labor and  $y$  units of capital are invested, a manufacturer's total production is given by the function  $Q = 5x^{\frac{1}{5}}y^{\frac{4}{5}}$  units. Each unit of labor costs \$22 and each unit of capital costs \$66. If exactly \$23,760 is to be spent on production, determine the number of units of labor and the number of units of capital that should be invested to maximize production.

6. Solve  $\int_0^{\infty} 5e^{-x} dx$ .

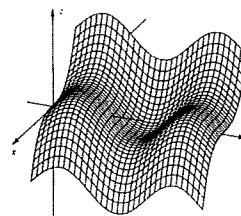
7. Match the surfaces (top 6 graphs) with the lettered level curves (bottom 6 graphs).



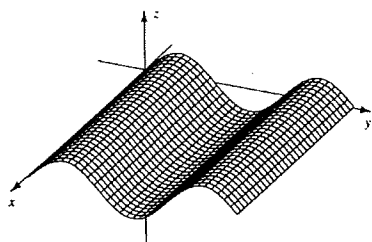
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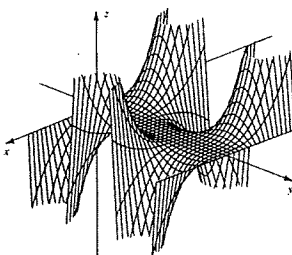
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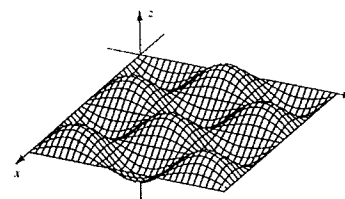
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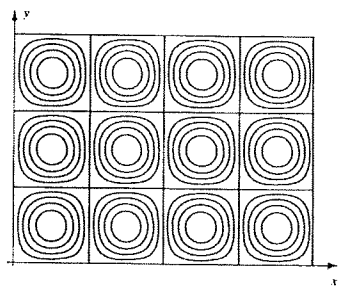
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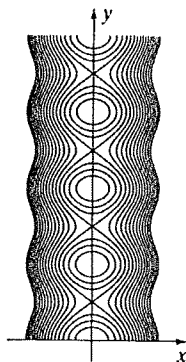
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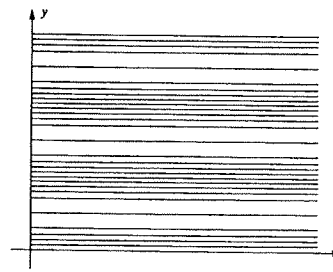
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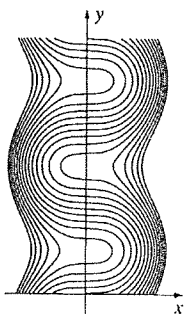
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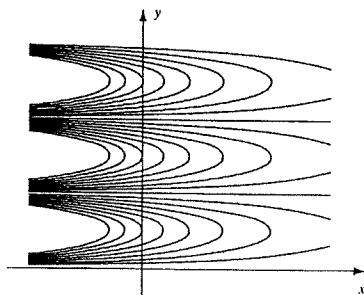
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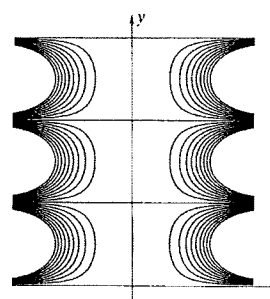
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D



E



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