Chap. 1 Introduction

Sec. 1.1 Some Basic Mathematical Models; Direction Fields

HW p. 7: #1, 11, 22, 31   Due: Friday, August 27

Def: An (ordinary) differential equation relates the derivatives of an unknown function of a single real variable.

Examples of ODEs: (Ask class for some examples)

$$\vec{F} = m\vec{a} \qquad \text{e.g.} \qquad -mg = m\frac{dv}{dt}$$ (models projectile motion if air resistance is neglected)

$$y' = 3x^2$$

$$y\frac{dy}{dx} + 4xy = x^2$$

Examples of equations that are not ODEs:

$$(x^3+5)' = 3x^2 \qquad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$x^3 + y^3 = 6xy$$

(3-D Wave Equation ... PDE)

Differential equations are useful in modeling physical phenomena.

Ex1 (#24, p.8) A certain drug is being administered intravenously to a hospital patient. Fluid containing 5 mg/cm^3 of the drug enters the patient's bloodstream at a rate of 100 cm^3/hr. The drug is absorbed by body tissues or otherwise leaves the bloodstream at a rate proportional to the amount present, with a rate constant of 0.4/hr.

(a) Assuming that the drug is always uniformly distributed throughout the bloodstream, write a differential equation for the amount of the drug that is present in the bloodstream at any time.

(b) How much of the drug is present in the bloodstream after a long time.

Solution. (a) Let $A = A(t)$ be the amount (in mg) of the drug in the patient's bloodstream at time t (in hours).

$$\begin{matrix}\text{Net rate of} \\ \text{change of } A\end{matrix} = \begin{matrix}\text{Net rate of} \\ \text{inflow of } A\end{matrix} - \begin{matrix}\text{Net rate of} \\ \text{outflow of } A\end{matrix}$$

$$\frac{dA}{dt} = \left(\frac{5\,mg}{\cancel{cm^3}}\right)\left(\frac{100\,\cancel{cm^3}}{hr}\right) - \left(\frac{0.4}{hr}\right)(A\,mg)$$

$$\boxed{\frac{dA}{dt} = 500 - 0.4A}$$ (units of both sides are mg/hr.)

(b) After a long time the inflow and outflow rates will reach equilibrium; i.e. $\frac{dA}{dt} \to 0$ as $t \to \infty$. The equilibrium amount of the drug in the patient's bloodstream will satisfy

$$0 = 500 - 0.4A$$

$$\Rightarrow \quad A = \frac{500}{0.4} = \boxed{1250\,mg}$$

We will learn how to solve first order ODEs (like Ex 1(a)) in Secs. 1.2 and 2.1-2.2. Meanwhile, it is useful to study geometrically the behavior of solutions to first order ODEs $y' = f(t,y)$ using its slope field (or direction field).

Ex 2 (Similar to #31, p. 10) (a) Draw a slope field for $(+)$ $y' = \overbrace{1 - ty}^{f(t,y)}$.

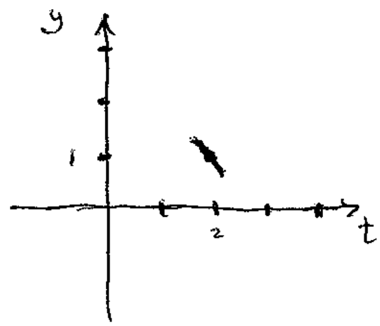
(b) Based on the slope field determine the behavior of the solution $y = y(t)$ as $t \to \infty$.

If the behavior depends on the initial value of y at $t=0$, describe this dependency.

Solution: (a) A solution to (†) passing through the point (2,1) in the ty-plane would have slope

$$y' = f(2,1) = 1-(2)(1) = -1$$

there.



If we plot the slopes for solutions to (†) at a grid of points in the ty-plane, we obtain a slope field (or direction field) of (†).

(Show Maple plot of slope field at this point.)

(b) From the slope field, it appears that $y(t) \to 0$ as $t \to \infty$ (This is independent of the initial value of y at $t=0$.)

3. $\dfrac{dy}{dx} = 1 - xy$

(a) $y(0) = 0$ **(b)** $y(-1) = 0$

(c) $y(2) = 2$ **(d)** $y(0) = -4$

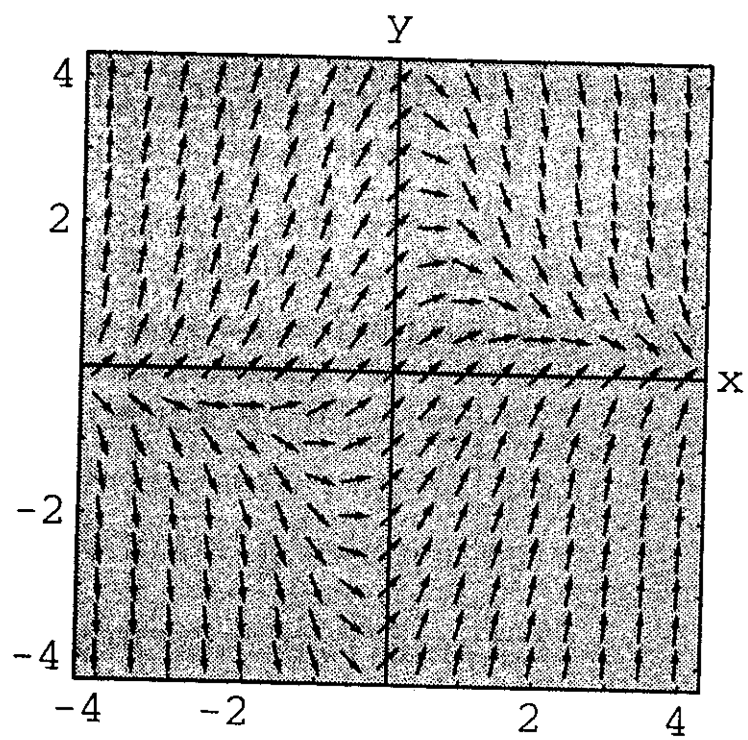


FIGURE 2.12 Direction field for Problem 3