Sec 1.2. Solutions of Some ODEs

HW p.15: # 1, 7, 12   Due: Monday, August 30   Schaum's pp. 2-3

Ex 1 | Consider the ODE

$$(*) \qquad \frac{dA}{dt} = 500 - 0.4A$$

(from Ex 1 in Sec. 1.1) describing the amount of drug in a patient's bloodstream. Separating variables gives

$$\frac{dA}{500 - 0.4A} = dt$$

Integrating both sides yields

$$\frac{1}{-0.4} \ln|500 - 0.4A| + c_2 = \int \frac{dA}{500 - 0.4A} = \int dt = t + c_1$$

Solving for A:

$$\ln|500 - 0.4A| = -0.4t + c_3 \qquad (c_3 = -0.4(c_1 - c_2))$$

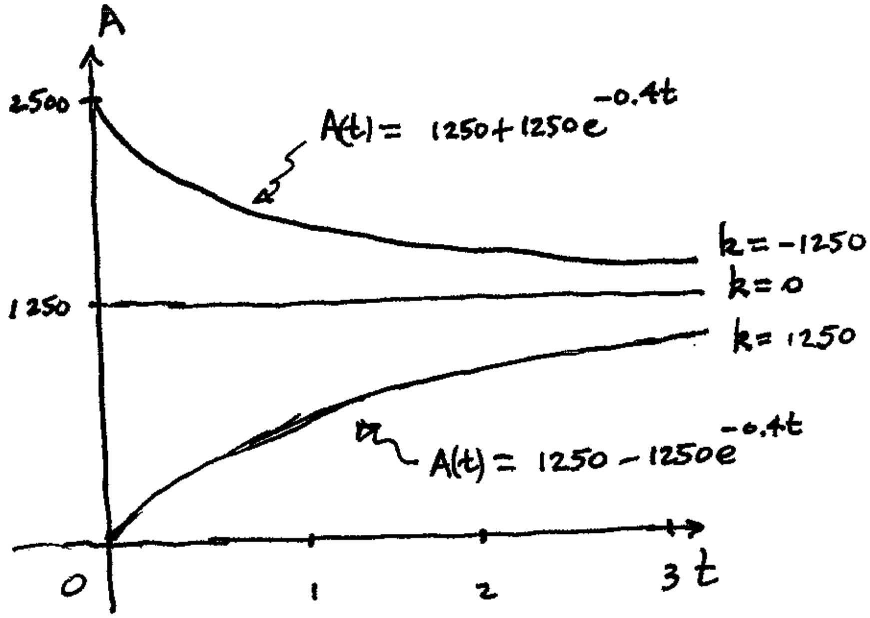
$$500 - 0.4A = c_4 e^{-0.4t} \qquad (c_4 = \pm e^{c_3})$$

$$\frac{1}{0.4}\left(500 - c_4 e^{-0.4t}\right) = A$$

$$\boxed{1250 - ke^{-0.4t} = A(t)} \qquad \text{(where } k \text{ is an arbitrary constant)}$$

$$\left(k = \frac{c_4}{0.4}\right)$$

This is the general solution of (*).



Graphs of particular solutions to (*) $\frac{dA}{dt} = 500 - 0.4A$.

[$A(t) = 1250 - ke^{-0.4t}$ is the general solution of (*).]

Ex 2 | Solve the initial value problem $\frac{dA}{dt} = 500 - 0.4A$, $A(0) = 0$.

Soln: By previous work, the general solution of the ODE is $A(t) = 1250 - ke^{-0.4t}$ where k is an arbitrary constant. We need to choose the constant k so $A(0) = 0$. That is,

$$0 = A(0) = 1250 - ke^{(-0.4)(0)} = 1250 - k$$

Therefore we should choose $k = 1250$. The solution is

$$\boxed{A(t) = 1250 - 1250e^{-0.4t}}.$$