Sec. 12. Solutions of Some ODEs
HW p.15: 1, 7, 12 Due: Monday, August 30 Schaum's pp.2-3

Ex 1 | Consider the ODE

\( (*) \quad \frac{dA}{dt} = 500 - 0.4A \)

(from Ex 1 in Sec. 1.1) describing the amount of drug in a patient's bloodstream. Separating variables gives

\[ \frac{dA}{500 - 0.4A} = dt \]

Integrating both sides yields

\[ \frac{1}{0.4} \ln |500 - 0.4A|_c = \int \frac{dA}{500 - 0.4A} = \int dt = t + c_1, \]

Solving for \( A \):

\[ \ln |500 - 0.4A| = -0.4t + c_3 \quad \quad (c_3 = -0.4(c_1 - c_2)) \]

\[ 500 - 0.4A = c_4 e^{-0.4t} \quad \quad (c_4 = \pm e^{c_3}) \]

\[ \frac{1}{0.4} \left( \frac{500 - c_4 e^{-0.4t}}{0.4} \right) = A \]

\[ 1250 - ke^{-0.4t} = A(t) \]

(Where \( k \) is an arbitrary constant)

\[ k = \frac{c_4}{0.4} \]

This is the general solution of \( (*) \).
Graphs of particular solutions to \( \frac{dA}{dt} = 500 - 0.4A \).

\[
A(t) = 1250 - 1250e^{-0.4t}
\]

is the general solution of \( \frac{dA}{dt} = 500 - 0.4A \).

Ex 2 | Solve the initial value problem \( \frac{dA}{dt} = 500 - 0.4A \), \( A(0) = 0 \).

Solu: By previous work, the general solution of the ODE is \( A(t) = 1250 - 1250e^{-0.4t} \) where \( k \) is an arbitrary constant. We need to choose the constant \( k \) so \( A(0) = 0 \). That is,

\[
0 = A(0) = 1250 - 1250e^{-0.4\cdot0} = 1250 - k
\]

Therefore we should choose \( k = 1250 \). The solution is

\[
A(t) = 1250 - 1250e^{-0.4t}
\]