Sec. 1.3 Classification of DEs.

HW p.24: # 4, 7, 17, 19 Due: Mon., Aug. 30 Schaum's pp. 1-3

We have already discussed the difference between <u>ordinary</u> differential equations and <u>partial</u> differential equations. (See lecture on Sec. 1.1)

<u>Def</u>: The <u>order</u> of an ODE is the order of the highest derivative that appears in the equation.

<u>Ex 1</u> Determine the order of the following ODEs.

Eqn.	order?	linear?	homogeneous
$y' = 3t^2$	1	Yes	No
$yy' + 4ty = t^2$	1	No	Not Applicable
$\frac{dy}{dt} = \sqrt{1 + \left(\frac{d^3y}{dt^3}\right)^{2}}$	3	No	NA
$t^2y'' + 5y = \tan(t)$	2	Yes	No
$t^2y'' - 4ty' + 4y = 0$	2	Yes	Yes

<u>Def</u>: An n^{th} order ODE is <u>linear</u> if it has the form

$$a_n(t)y^{(n)} + a_{n-1}(t)y^{(n-1)} + \dots + a_0(t)y = g(t)$$

where $a_n(t), a_{n-1}(t), \dots, a_0(t)$, and $g(t)$ are given functions of t. If $g(t) \equiv 0$ then the linear equation is called <u>homogeneous</u>.

Ex 2 Determine which equations in example 1 are linear and of those, which are homogeneous.

Def: A *solution* of an ODE on an open interval I is a function $y = y(t)$ which possesses the required number of derivatives on I and "satisfies" the ODE at all points in I.

Ex 3 (#20, p.25) Determine the values of the constant r for which the differential equation

$$(*) \qquad t^2 y'' - 4ty' + 4y = 0$$

has solutions of the form $y(t) = t^r$ for $t > 0$.

Solution: Suppose $y(t) = t^r$ solves $(*)$ on the interval $0 < t < \infty$. Then $y'(t) = rt^{r-1}$ and $y''(t) = r(r-1)t^{r-2}$. Substituting these expressions into $(*)$ yields

$$t^2 r(r-1)t^{r-2} - 4trt^{r-1} + 4t^r = 0$$

for all $0 < t < \infty$. Simplifying we have

$$r(r-1)t^r - 4rt^r + 4t^r = 0$$

or

$$t^r \left[r(r-1) - 4r + 4 \right] = 0 .$$

Hence

$$r^2 - 5r + 4 = 0$$

or

$$(r-4)(r-1) = 0$$

So $\boxed{r = 4 \text{ or } r = 1}$. I.e. $y_1(t) = t^4$ and $y_2(t) = t$ are solutions of $(*)$.