Mathematics 315 | Midterm Exam Spring 2014 | Name:______________________

1.(30 pts.) Define each the following terms, phrases, or symbols.

(a) A sequence $\{f_n\}_{n=1}^{\infty}$ of real functions converges to $f$ pointwise on a set $E$.

(b) A sequence $\{f_n\}_{n=1}^{\infty}$ of real functions converges to $f$ uniformly on a set $E$.

(c) If $f$ is a bounded real function on $[a,b]$, $P$ is a partition of $[a,b]$, and $\alpha$ is an increasing real function on $[a,b]$, define $U(f,P,\alpha)$, $L(f,P,\alpha)$, $\overline{\int_a^b} f d\alpha$, and $\underline{\int_a^b} f d\alpha$.

(d) The bounded real function $f$ is Riemann-Stieltjes integrable with respect to the increasing real function $\alpha$ on $[a,b]$.

(e) The real function $\alpha$ is of bounded variation on $[a,b]$.

(f) Define the Riemann-Stieltjes integral of a continuous function $f$ with respect to a function $\alpha$ of bounded variation on $[a,b]$.

(g) The Fourier coefficients $a_0(f)$, $a_n(f)$, and $b_n(f)$ $(n = 1,2,3,...)$ of a Riemann integrable function $f$ on the interval $[-\pi,\pi]$.

(h) $(X,N)$ is a real normed linear space.

(i) $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in a real normed linear space $(X,N)$.

(j) $(X,N)$ is a real Banach space.

2.(30 pts.) Give statements for each of the following.

(a) Lebesgue's theorem about the "smoothness" of monotone functions.

(b) Jordan's theorem relating increasing functions and functions of bounded variation.

(c) Holder's inequality and conditions under which it is guaranteed to hold.

(d) The Weierstrass M-Test.

(e) Weierstrass' theorem on the approximation of continuous real functions on a closed, bounded interval.

3.(50 pts.) In each of the following cases, give an example or tell why such an example is impossible.

(a) A set of measure zero which is uncountable.

(b) A countable set which is not of measure zero.

(c) A function which is differentiable on $[0,1]$ but not of bounded variation on $[0,1]$.

(d) A function which is Lipschitz continuous on $[0,1]$ but not of bounded variation on $[0,1]$.

(e) A uniformly convergent sequence of differentiable functions on $[0,1]$ whose limit function is not differentiable on $[0,1]$.

(f) A uniformly convergent sequence of differentiable functions on $[0,1]$ whose limit function is not continuous on $[0,1]$.

(g) A norm $N$ on the vector space $C[0,1]$ such that $(C[0,1],N)$ is not a Banach space.

(h) A norm $N$ on the vector space $C[0,1]$ such that $(C[0,1],N)$ is a Banach space.