1. (33 pts.) Solve $u_t - ku_{xx} = 0$ in the upper half-plane $-\infty < x < \infty$, $0 < t < \infty$, subject to the initial condition $u(x, 0) = e^{-x^2}$ for $-\infty < x < \infty$.

A solution to $u_t - ku_{xx} = 0$ in the upper half-plane, satisfying $u(x, 0) = \varphi(x)$ for all real $x$, is given by

$$u(x,t) = \frac{1}{\sqrt{4\pi kt}} \int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4kt}} \varphi(y) dy.$$  

Therefore, a solution to our problem is

$$u(x,t) = \frac{1}{\sqrt{4\pi kt}} \int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4kt}} - \frac{1}{4t} \int_{-\infty}^{\infty} \frac{y^2 - 2xy + x^2 + 4ty^2}{4t} dy$$

$$= e^{-\frac{x^2}{4t}} \int_{-\infty}^{\infty} e^{-\frac{(1+4t)y^2 - 2y^2}{4t}} dy \quad (*)$$

We need to complete the square in $y$ in the exponent of the integrand. Thus,

$$- \frac{(1+4t)y^2 - 2y^2}{4t} = - \left[ (1+4t)y^2 - 2 \left( \frac{x}{1+4t} \right) y + \left( \frac{x}{1+4t} \right)^2 \right] + \frac{x^2}{4t(1+4t)}$$

Consequently, if we substitute this in $(*)$, we obtain

$$u(x,t) = e^{-\frac{x^2}{4t}} \int_{-\infty}^{\infty} e^{-\frac{(1+4t)y^2 - x^2}{4t}} dy. \quad \text{Let } p = \frac{\sqrt{1+4t}y - \frac{x}{\sqrt{1+4t}}}{\sqrt{1+4t}}$$

Then

$$dp = \sqrt{1+4t} \, dy,$$

so

$$u(x,t) = e^{-\frac{x^2}{4t}} \left( \frac{1}{1+4t} \right) \int_{-\infty}^{\infty} e^{-p^2} dp = \frac{1}{\sqrt{4\pi t}} \left( \frac{x^2}{1+4t} \right) \int_{-\infty}^{\infty} e^{-p^2} dp.$$  

But $\int_{-\infty}^{\infty} e^{-p^2} dp = \sqrt{\pi}$, so

$$u(x,t) = e^{-\frac{x^2}{1+4t}} \sqrt{\frac{1}{\sqrt{4\pi t}}}.$$
2.(34 pts.) Find a solution to the damped wave equation (1)
\[ u_{tt} - u_{xx} + 2u_t = 0 \quad \text{in} \quad 0 < x < \pi, \quad 0 < t < \infty, \]
satisfying (2)-(3)
\[ u_t(0,t) = 0 = u_t(\pi, t) \quad \text{for} \quad t \geq 0, \]
(4)
\[ u(x,0) = 0 \quad \text{for} \quad 0 \leq x \leq \pi, \]
and
(5)
\[ u_t(x,0) = \frac{1}{2} + \frac{1}{2}\cos(2x) \quad \text{for} \quad 0 \leq x \leq \pi. \]

(You may find useful the facts that \( y'' + 2y' + n^2 y = 0 \) has general solution \( y_0(t) = c_1 + c_2 e^{-2t} \) if \( n = 0 \), \( y_1(t) = c_1 e^{-t} + c_2 te^{-t} \) if \( n = 1 \), and \( y_n(t) = e^{-t} \left( c_1 \cos \left( t \sqrt{n^2 - 1} \right) + c_2 \sin \left( t \sqrt{n^2 - 1} \right) \right) \) if \( n = 2, 3, 4, \ldots \))

Bonus (10 pts.): Show that the solution to (1)-(2)-(3)-(4)-(5) is unique.

We use the method of separation of variables. We seek nontrivial solutions of the form \( u(x,t) = \Xi(x)T(t) \) to the homogeneous portion of the problem: (1)-(2)-(3)-(4). Substituting in (1) yields
\[ \Xi(x)T''(t) - \Xi''(x)T(t) + 2\Xi(x)T'(t) = 0. \]

or
\[ \Xi''(x) = -\left( \frac{T''(t) + 2T'(t)}{T(t)} \right) = \text{constant} = \lambda. \]

Substituting in (2) and (3) gives \( \Xi'(0)T(t) = 0 = \Xi'(\pi)T(t) \) for \( t \geq 0 \), while substituting in (4) yields \( \Xi'(x)T(0) = 0 \) for \( 0 \leq x \leq \pi \). In order that \( u(x,t) = \Xi(x)T(t) \) is not zero, we must have \( \Xi'(0) = 0 = \Xi'(\pi) \) and \( T'(0) = 0 \). Thus we have the coupled system of ODE's and B.C.'s:
\[ \left\{ \begin{array}{l}
\Xi''(x) + \lambda \Xi(x) = 0, \\
\Xi(0) = 0 = \Xi(\pi), \\
T''(t) + 2T'(t) + \lambda T(t) = 0, \\
T'(0) = 0.
\end{array} \right. \]

By Sec. 4.2, the eigenvalues and eigenfunctions of the eigenvalue problem (in the presence of Neumann boundary conditions) are \( \lambda_n = \left( \frac{n\pi}{\ell} \right)^2 = \left( \frac{n\pi}{\pi} \right)^2 = n^2 \) and
\[ \Xi_n(x) = \cos \left( \frac{n\pi x}{\ell} \right) = \cos \left( \frac{n\pi x}{\pi} \right) = \cos(nx) \quad \text{where} \quad n = 0, 1, 2, 3, \ldots \]
in the \( T \)-equations produces \( T_n''(t) + 2T_n'(t) + n^2 T_n(t) = 0 \), \( T_n(0) = 0 \). By the useful facts (see above), the general solution of the \( T \)-differential equation is:
\[ T_0(t) = c_1 + c_2 e^{-2t} \quad \text{if} \quad n = 0, \quad T_1(t) = c_1 e^{-t} + c_2 t e^{-t} \quad \text{if} \quad n = 1, \quad \text{and} \]
\[ T_n(t) = e^{-t} \left[ c_1 \cos \left( t \sqrt{n^2 - 1} \right) + c_2 \sin \left( t \sqrt{n^2 - 1} \right) \right] \quad \text{if} \quad n = 2, 3, 4, \ldots \]
We apply the boundary condition $T_n'(0) = 0$ and find that, up to a constant factor,

$$T_0(t) = 1 - e^{-2t}, \quad T_1(t) = t e^t, \quad \text{and} \quad T_n(t) = e^{t \sin(t/\sqrt{n-1})} \quad \text{if} \quad n = 2, 3, 4, \ldots$$

Applying the superposition principle to the homogeneous problem (1)-(2)-(3)-(4), we see that, for any integer $N \geq 0$ and any choice of constants $A_0, A_1, \ldots, A_N$,

$$u(x,t) = \sum_{n=0}^{N} A_n \delta_n(x) T_n(t) = A_0 (1 - e^{-2t}) + A_1 \cos(x) e^{-t} + \sum_{n=2}^{N} A_n \cos(nx) e^{-t} \sin(t/\sqrt{n-1})$$

solves (1)-(2)-(3)-(4). Then

$$u_t(x,t) = 2A_0 e^{-2t} + A_1 \cos(x) (1-t) e^{-t} + \sum_{n=2}^{N} A_n \cos(nx) e^{-t} \left[ \sqrt{n-1} \cos(t/\sqrt{n-1}) - \sin(t/\sqrt{n-1}) \right],$$

so to satisfy (5) we must have

$$\frac{1}{2} + \frac{1}{2} \cos(2x) = u_t(x,0) = 2A_0 + A_1 \cos(x) + \sum_{n=2}^{N} A_n \sqrt{n-1} \cos(nx) \quad \text{for all} \quad 0 \leq x \leq \pi.$$}

By inspecting coefficients, we have $\frac{1}{2} = 2A_0$, $\frac{1}{2} = A_1 \sqrt{1-1}$, and all other $A_n = 0$. Thus,

$$u(x,t) = \frac{1}{4} (1 - e^{-2t}) + \frac{1}{2 \sqrt{3}} \cos(2x) e^{-t} \sin(t/\sqrt{3})$$

is a solution to (1)-(2)-(3)-(4)-(5).

**Bonus:** Let $u = v(x,t)$ be any solution to (1)-(2)-(3) and consider its energy function

$$E(t) = \int_0^\pi \left[ \frac{1}{2} v_t^2(x,t) + \frac{1}{2} v_x^2(x,t) \right] dx \quad \text{for} \quad 0 \leq t < \infty.$$ Then

$$E'(t) = \int_0^\pi \left[ \frac{1}{2} v_t v_t + \frac{1}{2} v_x v_x \right] dx$$

$$= \int_0^\pi \left[ v_t v_t + v_t v_t \right] dx$$

(integrate by parts on the second integrand term)

$$= \int_0^\pi v_t v_t (x,t) dx + v_t v_t (x,t) dx$$

(over)

$$- \int_0^\pi v_t v_t (x,t) dx$$

by (2)-(3)
So \[ E'(t) = \int_0^\pi \left[ \frac{\partial}{\partial t} \left( v_t(x,t) \right) \right] dx = -2 \int_0^\pi \left[ v_t(x,t) \right]^2 dx \leq 0. \]

That is, \( E = E(t) \) is a decreasing function on \( t \geq 0 \). Now suppose that \( u = w(x,t) \) is another solution to (1)-(2)-(3)-(4)-(5) and consider

\[ v(x,t) = u(x,t) - w(x,t) \quad \text{for} \quad 0 \leq x \leq \pi, \quad 0 \leq t < \infty, \]

where \( u = u(x,t) \) is the solution to (1)-(2)-(3)-(4)-(5) we obtained earlier in this problem. Then \( v \) solves

\[
\begin{align*}
1' \quad & v_{tt} - v_{xx} + 2v_t = 0 \quad \text{in} \quad 0 < x < \pi, \quad 0 < t < \infty, \\
2-3' \quad & v_x(0,t) = 0 = v_x(\pi,t) \quad \text{for} \quad t \geq 0, \\
4'-5' \quad & v(x,0) = 0 = v_t(x,0) \quad \text{for} \quad 0 \leq x \leq \pi.
\end{align*}
\]

By the preceding work, the energy function \( E(t) = \int_0^\pi \left[ \frac{1}{2} v_t^2(x,t) + \frac{1}{2} v_x^2(x,t) \right] dx \) for \( v \) on \( t > 0 \) is decreasing. Therefore, for \( t > 0 \),

\[ 0 \leq E(t) \leq E(0) = \int_0^\pi \left[ \frac{1}{2} v_t^2(x,0) + \frac{1}{2} v_x^2(x,0) \right] dx = 0 \]

by (5') and differentiation of (4'). The vanishing theorem then guarantees that

\[ \frac{1}{2} v_t^2(x,t) + \frac{1}{2} v_x^2(x,t) = 0 \quad \text{for} \quad 0 \leq x \leq \pi \quad \text{and} \quad 0 \leq t < \infty. \]

Therefore,

\[ v_t(x,t) = 0 = v_x(x,t) \quad \text{for} \quad 0 \leq x \leq \pi \quad \text{and} \quad 0 \leq t < \infty. \]

So \( v(x,t) = 0 \) in the strip \( 0 \leq x \leq \pi, \quad 0 \leq t < \infty \). But (4') implies \( v(x,0) = 0 \) for \( 0 \leq x \leq \pi \)

so \( v(x,t) = 0 \) in the strip. That is, \( u(x,t) = w(x,t) \) for all \( 0 \leq x \leq \pi, 0 \leq t < \infty \),

thus showing that there is only one solution to (1)-(2)-(3)-(4)-(5).
3. (33 pts.) Use Fourier transform methods to derive the formula

\[ u(x,t) = \frac{1}{2} \int_0^\infty \int_0^{\infty} f(s, \tau) d\tau ds \]

for a solution to the inhomogeneous wave equation in the \( xt \)-plane,

\[ u_t - u_{xx} = f(x, t), \tag{1} \]

satisfying homogeneous initial conditions: \( u(x, 0) = 0 \) and \( u_t(x, 0) = 0 \) for \( -\infty < x < \infty \).

Bonus (10 pts.) Compute a solution to \( u_t - u_{xx} = xt \) in the \( xt \)-plane which satisfies homogeneous initial conditions.

Let \( u = u(x,t) \) be a solution to (1) for \( -\infty < x < \infty, -\infty < t < \infty \), satisfying \( -\infty < x < \infty \). We take the Fourier transform of both sides of (1) with respect to \( x \) to obtain

\[ \mathcal{F}(u_{tt})(\xi) - \mathcal{F}(u_{xx})(\xi) = \mathcal{F}(f)(\xi, t), \]

\[ \frac{\partial^2}{\partial t^2} \mathcal{F}(u)(\xi) - (i \xi)^2 \mathcal{F}(u)(\xi) = \mathcal{F}(f)(\xi, t), \]

\[ \frac{\partial^2}{\partial t^2} \mathcal{F}(u)(\xi) + \xi^2 \mathcal{F}(u)(\xi) = \mathcal{F}(f)(\xi, t), \tag{2} \]

The general solution of the second-order ODE \( \frac{\partial^2}{\partial t^2} \mathcal{F}(u)(\xi) + \xi^2 \mathcal{F}(u)(\xi) = \mathcal{F}(f)(\xi, t) \) in \( t \) (with parameter \( \xi \)) is

\[ \mathcal{F}(u)(\xi) = U_h(\xi, t) + U_p(\xi, t) \]

where \( U_h(\xi, t) = c_1(\xi) \cos(\xi t) + c_2(\xi) \sin(\xi t) \) is the general solution of the associated homogeneous equation of (2), \( \frac{\partial^2}{\partial t^2} \mathcal{F}(u)(\xi) + \xi^2 \mathcal{F}(u)(\xi) = 0 \), and \( U_p(\xi, t) \) is a particular solution of (2). We use variation of parameters,

\[ U_p(\xi, t) = u_1(\xi, t) \cos(\xi t) + u_2(\xi, t) \sin(\xi t), \]

to find a particular solution of (2). Here

\[ u_1(\xi, t) = \int_0^t -\frac{F_y}{W} \, d\tau \quad \text{and} \quad u_2(\xi, t) = \int_0^t \frac{F_y}{W} \, d\tau \]

where \( y_1 = \cos(\xi t), \ y_2 = \sin(\xi t) \) form a fundamental set of solutions (OVER)
of the associated homogeneous equation of (5), $F = \hat{f}(s, t)$ is the forcing function of (4), and

$$W = W(y_1, y_2)(s) = \begin{vmatrix} y_1(s, t) & y_2(s, t) \\ \frac{\partial}{\partial t} y_1(s, t) & \frac{\partial}{\partial t} y_2(s, t) \end{vmatrix} = \begin{vmatrix} \cos(s) & \sin(s) \\ -\sin(s) & \cos(s) \end{vmatrix} = \frac{1}{s}$$

is the Wronskian of $y_1, y_2$. Therefore

$$F(y)(s) = c_1(s) \cos(s) + c_2(s) \sin(s) + \cos(s) \int_0^t -\frac{\hat{f}(s,t) \sin(s)}{s} \, dt + \sin(s) \int_0^t \frac{\hat{f}(s,t) \cos(s)}{s} \, dt$$

We use (2) to identify $c_1(s)$ and $c_2(s)$. First, (2) implies

$$0 = F(u(., o))(s) = c_1(s) + 0 + 0 + 0 = c_1(s).$$

Next, observe that

$$\frac{\partial}{\partial t} F(u)(s) = -\frac{3}{s} c_1(s) \sin(s) + \frac{3}{s} c_2(s) \cos(s) + 3 \sin(s) \int_0^t -\frac{\hat{f}(s,t) \sin(s)}{s} \, dt$$

$$+ \cos(s) \left[ -\frac{\hat{f}(s,t) \sin(s)}{s} \right] + 3 \cos(s) \int_0^t \frac{\hat{f}(s,t) \cos(s)}{s} \, dt$$

$$+ \sin(s) \left[ \frac{\hat{f}(s,t) \cos(s)}{s} \right].$$

By (3) we have

$$0 = F(u(., o))(s) = \frac{\partial}{\partial t} F(u)(s) \bigg|_{t=0} = 0 + 3 c_1(s) + 0 + 0 + 0$$

$$c_2(s) = 0.$$ Thus

$$F(u)(s) = \cos(s) \int_0^t -\frac{\hat{f}(s,t) \sin(s)}{s} \, dt + \sin(s) \int_0^t \frac{\hat{f}(s,t) \cos(s)}{s} \, dt.$$
\[
\begin{align*}
= \int_0^t \frac{\hat{f}(\xi, \tau)}{\xi} \left[ \sin(\xi t) \cos(\xi \tau) - \cos(\xi t) \sin(\xi \tau) \right] d\tau \\
= \int_0^t \frac{\hat{f}(\xi, \tau) \sin(\xi (t-\tau))}{\xi} d\tau,
\end{align*}
\]

Using formula A in the table of Fourier transforms with \( b = t-\tau \), we see that
\[
\frac{\sin(\xi (t-\tau))}{\xi} = \sqrt{\frac{\pi}{2}} \hat{f}(\xi \tau) \text{(*)},
\]
so substituting in (*),
\[
\hat{F}(u)(\xi) = \int_0^t \hat{f}(\xi \tau) \sqrt{\frac{\pi}{2}} \hat{f}(\xi (-t+\tau), \tau) \left( \frac{1}{\sqrt{2\pi}} \right) d\tau
\]
\[
= \int_0^t \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\pi}{2}} \hat{f}(\xi \tau) \hat{f}(\xi (-t+\tau), \tau) \left( \frac{1}{\sqrt{2\pi}} \right) d\tau.
\]

where we have used the convolution property \( \frac{1}{\sqrt{2\pi}} \hat{g} \ast \hat{h}(\xi) = \hat{g}(\xi) \hat{h}(\xi) \). But interchanging the order of integration yields
\[
\hat{F}(u)(\xi) = \frac{i}{\sqrt{2\pi}} \int_0^t \hat{f}(\xi \tau) \hat{f}(\xi (-t+\tau), \tau) \left( \frac{1}{\sqrt{2\pi}} \right) d\tau
\]
\[
= \frac{i}{\sqrt{2\pi}} \int_0^t \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left( \hat{f}(\xi \tau) \hat{f}(\xi (-t+\tau), \tau) \right) (x) e^{-ix} dx d\tau
\]
\[
= \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left( \frac{i}{2} \int_0^t \hat{f}(\xi \tau) \hat{f}(\xi (-t+\tau), \tau) (x) dx \right) e^{-ix} d\tau
\]
\[
= \hat{F} \left( \frac{i}{2} \int_0^t \hat{f}(\xi \tau) \hat{f}(\xi (-t+\tau), \tau) (x) dx \right) (\xi)
\]

By the uniqueness theorem for Fourier transforms, it follows that for all real \( x \) and \( t \)
\[
\begin{align*}
u(x, t) &= \frac{i}{2} \int_0^t \int_{-\infty}^{\infty} \hat{f}(\xi \tau) \hat{f}(\xi (-t+\tau), \tau) (x-s) ds d\tau \\
&= \frac{i}{2} \int_0^t \int_{-\infty}^{\infty} \hat{f}(\xi \tau) \hat{f}(\xi (-t+\tau), \tau) (x-s) ds d\tau.
\end{align*}
\]
But \[ X_{(-t-T), t-T} (x-s) = \begin{cases} 
1 & \text{if } -t-T < x-s < t-T, \\
0 & \text{otherwise,} 
\end{cases} \]

\[ = \begin{cases} 
1 & \text{if } x-(t-T) < s < x+(t-T), \\
0 & \text{otherwise,} 
\end{cases} \]

\[ = X_{x-(t-T), x+t-T} (s) \tag{\star} \]

for all real \( x \) and \( s \), and all \( 0 < T < t \). Consequently, (II) and (\star) imply

\[ u(x,t) = \frac{1}{2} \int_0^t \int_{-\infty}^{\infty} f(s,T) X_{x-(t-T), x+t-T} (s) \, ds \, dT \]

\[ = \frac{1}{2} \int_0^t \int_{x-(t-T)}^{x+t-T} f(s,T) \, ds \, dT. \]

**Bonus:** Applying the formula above with \( f(x,t) = xt \) yields a solution to \( u_{tt} - u_{xx} = xt \) in the \( xt \)-plane, subject to homogeneous initial conditions \( u(x,0) = 0 = u_t (x,0) \) for \(-\infty < x < \infty\). Therefore

\[ u(x,t) = \frac{1}{2} \int_0^t \int_{x-(t-T)}^{x+t-T} sT \, ds \, dT = \frac{1}{2} \int_0^t \frac{TS^2}{2} \bigg|_{s=x-(t-T)}^{s=x+t-T} \, dT = \frac{1}{4} \int_0^t \left[ (x+t-T)^2 - (x-(t-T))^2 \right] dT \]

\[ = \frac{1}{4} \int_0^t (x-T)^2 dt = \frac{1}{2} \left[ \frac{2}{3} x^3 \right]_T^t = \frac{1}{2} \left[ \frac{2}{3} x^3 \right]_0^t = \frac{x t^3}{6}, \]
### A Brief Table of Fourier Transforms

\[ f(x) \quad \hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} \, dx \]

<table>
<thead>
<tr>
<th>A. ( f(x) )</th>
<th>( \hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} , dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 1 ] if ( -b &lt; x &lt; b ), ( 0 ) otherwise.</td>
<td>( \frac{2}{\sqrt{\pi}} \sin(b\xi) )</td>
</tr>
<tr>
<td>B. ( f(x) )</td>
<td>( \frac{e^{-i\xi x} - e^{-i\xi \xi}}{i\xi \sqrt{2\pi}} )</td>
</tr>
<tr>
<td>[ 1 ] if ( c &lt; x &lt; d ), ( 0 ) otherwise.</td>
<td>( \frac{\pi}{\xi} e^{-i\xi</td>
</tr>
<tr>
<td>C. ( \frac{1}{x^2 + a^2} ) ((a &gt; 0))</td>
<td>( \frac{-1 + 2e^{-ib\xi} - e^{-2ib\xi}}{\xi^2 \sqrt{2\pi}} )</td>
</tr>
<tr>
<td>( x ) if ( 0 &lt; x \leq b ), ( 2b - x ) if ( b &lt; x &lt; 2b ), ( 0 ) otherwise.</td>
<td>( \frac{1}{(a + i\xi) \sqrt{2\pi}} )</td>
</tr>
<tr>
<td>D. ( f(x) )</td>
<td>( \frac{e^{(a-i\xi)x} - e^{(a+i\xi)b}}{(a-i\xi) \sqrt{2\pi}} )</td>
</tr>
<tr>
<td>( e^{-i\xi x} ) if ( x &gt; 0 ), ( 0 ) otherwise.</td>
<td>( \frac{\sqrt{2}}{\pi} \sin(b(\xi - a)) )</td>
</tr>
<tr>
<td>E. ( f(x) )</td>
<td>( \frac{i\xi e^{(a+i\xi)x} - e^{i(a+i\xi)b}}{i(\xi - a) \sqrt{2\pi}} )</td>
</tr>
<tr>
<td>( e^{ax} ) if ( b &lt; x &lt; c ), ( 0 ) otherwise.</td>
<td>( \frac{1}{\sqrt{2a}} e^{-e^{2x} + (a)\xi} )</td>
</tr>
<tr>
<td>F. ( f(x) )</td>
<td>( \begin{cases} 0 &amp; \text{if }</td>
</tr>
<tr>
<td>( e^{ax} ) if ( -b &lt; x &lt; b ), ( 0 ) otherwise.</td>
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<tr>
<td>G. ( f(x) )</td>
<td></td>
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<tr>
<td>( e^{ax} ) if ( c &lt; x &lt; d ), ( 0 ) otherwise.</td>
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<tr>
<td>H. ( f(x) )</td>
<td></td>
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<tr>
<td>( e^{-ax^2} ) ((a &gt; 0))</td>
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</tr>
<tr>
<td>I. ( f(x) )</td>
<td></td>
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<tr>
<td>J. ( \frac{\sin(ax)}{x} ) ((a &gt; 0))</td>
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Math 325
Exam II
Fall 2010

mean: 72.2
standard deviation: 16.1
number taking exam: 37

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<th>Undergraduate Letter Grade</th>
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