Improved Technique for Extracting Parameters of Low-Loss Dielectrics on Printed Circuit Boards

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Abstract—The paper is devoted to a methodology and an improved technique of characterization of low-loss dielectrics on printed circuit boards. The technique is based on measuring S-parameters and recalculating them into complex propagation constant. Phase correction is proposed to assure that the phase constant passes through zero at zero frequency. An effect of dielectric loss upon a dielectric constant is considered in the analytical model for dielectric parameter extraction. Dielectric and conductor losses are separated using a model, which includes surface roughness of conductors. Network asymmetry is taken into account in the model. Extracted parameters for frequency-dispersive dielectrics satisfy Kramers-Krönig causality relations. The proposed model allows for extracting dielectric constant and dissipation factor with an increased accuracy.

I. INTRODUCTION

There are different ways of measuring dielectric properties of materials: plane-wave techniques, waveguide/transmission line methods, cavity methods, and capacitive methods. Each method has its own advantages and limitations. One can choose a method depending on frequency range, accuracy, test equipment applicability, necessity of measuring “in situ”, etc. For printed circuit boards (PCBs) used for high-speed digital design, it is important to obtain their dielectric properties over a wide frequency range from VHF band (~50 MHz) to SHF band (up to 35 GHz), since permittivity and loss of substrate dielectrics in reality are frequency-dependent. Cavity methods are narrowband in principle, so to characterize dielectrics one needs multiple test fixtures or multiple load/unload resonator measurements for different central frequencies [1-3]. Impedance analyzer methods are also limited to comparatively low frequencies (below 2 GHz) [4]. Wideband study of dielectric behavior can be conducted using time-domain reflectometer (TDR) [5-8] or vector network analyzer (VNA) measurements [6-9]. These methods are good for non-destructive extraction of dielectric properties on PCBs. However, at present, VNA methods of characterization have comparatively broader dynamic range and may provide higher accuracy [7, 9], while TDR methods are cheaper.

The objective of this work is to develop an accurate technique for extracting dielectric constant (DK, real part of complex permittivity) and dissipation factor (DF, loss tangent) of low-loss substrate dielectrics from S-parameter measurements using a VNA. Specifically, in this work the Agilent precision network analyzer (PNA) E8364B was used to test single-ended stripline geometries. The calibration of the PNA over the specified frequency ranges was done using the TRL (“Through-Reflect-Line”) procedure on auxiliary traces of the designed length on the board (Fig. 1).

Fig. 1. A picture of a PCB with TRL calibration test lines

The methodology presented herein is based on obtaining complex propagation constant of TEM mode in stripline geometry from S-parameters, which is a known approach (e.g., [10]). However, the improved extraction model takes into account interrelation between real and imaginary parts of permittivity, two-port network asymmetry in the reciprocal case, separation of conductor and dielectric loss on the line, as well as the effect of surface roughness upon conductor loss. The method is then applied to measuring dielectric properties of numerous PCBs with different conductor surface roughness profiles and varying resin contents. It is shown that dielectric constant of a board is predictable based on the knowledge of its resin content.

II. ANALYTICAL MODEL

The dielectric parameter extraction procedure is based on measuring S-parameters; their causality and passivity check, using Link Path Analyzer [11]; and conversion of S-parameters to ABCD transmission matrix parameters. Then the complex propagation constant \( \gamma = \alpha + j\beta \) is calculated from the ABCD parameters, where \( \alpha \) is a loss parameter, and \( \beta \) is a phase constant.

Loss parameter \( \alpha \) is comprised of dielectric and conductor loss \( \alpha = \alpha_d + \alpha_c \). One of the ways of separating these two contributions is applying the curve-fitting optimization procedure based on the genetic algorithm [12]. This is possible due to different behavior of dielectric and conductor loss with frequency. Dielectric loss in [12] is proportional to
frequency \( (a_d \sim \omega) \), while conductor loss is proportional to the square root of frequency \( (a_c \sim \sqrt{\omega}) \), when not taking into account surface roughness. In the new model, the dielectric loss is modeled as \( a_d \sim a \omega + b \omega^2 \), which is discussed in Section II.C.

As for conductor loss, it is obtained from computations based on an analytical model for stripline geometry. It includes skin-effect loss in a smooth conductor plus an additional term that depends on a surface roughness model. Since surface roughness is taken into account, conductor loss deviates from the square root of frequency dependence. This is discussed in Section II.D.

A. Interrelation between Real and Imaginary Parts of Permittivity in the Extraction Procedure

Real and imaginary parts of complex permittivity of a dielectric substrate can be calculated using the known approximate expressions [13],

\[
e^r = \frac{\epsilon_0}{\omega} \left( \beta^2 \right) \quad \text{and} \quad e^r = \frac{\epsilon_0}{\omega} \left( 2a_d \beta \right),
\]

which are valid only for dielectrics with extremely low loss. From (1) it follows that if DK is constant, this also causes the DF to be almost constant.

In an improved extraction procedure, the rigorous formulas are used instead of the approximate ones,

\[
\beta = \frac{\omega}{c} \sqrt{e^r + \epsilon^r \cdot \cos \left( \frac{\delta}{2} \right)};
\]

\[
a_d = \frac{\omega}{c} \sqrt{e^r + \epsilon^r \cdot \sin \left( \frac{\delta}{2} \right)}.
\]

For comparatively small loss \((\tan \delta < 0.1)\), cosine and sine functions can be approximated as the first terms in their Taylor expansions: \(\cos \left( \frac{\delta}{2} \right) \approx 1\) and \(\sin \left( \frac{\delta}{2} \right) \approx \frac{\delta}{2}\). Then the system of two equations (2) and (3) for \(e^r\) and \(e^r\) and known \(\beta\) and \(a_d\) can be easily solved.

\[
e^r = A \left( 1 - \frac{B}{A + B} \right) \quad \text{and} \quad e^r = \sqrt{\frac{A^2 - B}{A + B}},
\]

where \(A = \frac{\beta^2 \epsilon_0^2}{a_d^2}\) and \(B = \frac{4c^2 \epsilon_0^2}{a_d^2}\).

Fig. 2 shows behavior of the extracted DK in two cases: using (1) and (4). When using (1), the DK is independent of loss. When using (4), the value of \(e^r\) is affected by the loss part \(e^r\), and it is slightly lower. However, the deviation is small \((-0.025\%))\), since PCB dielectrics are low-loss ones.

The proposed algorithm may include higher-order Taylor series terms, if considering high-loss dielectrics \((\tan \delta > 0.1)\). However, the system of equations would be of a higher order and more cumbersome. In practice, loss tangents of actual PCB materials never approach this limit, with typical values of 0.015-0.030 for various FR4 laminates, and being as low as 0.002 for specialized low-loss materials.

B. Two-port Network Asymmetry

Asymmetry of the test network may be an important issue affecting accuracy of DK and DF extraction. As for reciprocity, our numerous measurements have confirmed that test structures that do not contain any anisotropic or magnetic materials are almost perfectly reciprocal.

In simplified extraction models, network symmetry \((S_{11} = S_{22})\) is enforced, and the complex propagation constant is calculated enforcing symmetry using only \(A\)-parameter of the ABCD matrix [13],

\[
yl = \text{arccosh}(A).
\]

However, in reality, inequality \(S_{11} \neq S_{22}\) might be substantial, especially as frequency increases, and the assumption of symmetry might lead to an error in the extraction results.

Fig. 2. Behavior of DK with and without effect of loss

Fig. 3. Asymmetrical test line structure and its equivalent model

An asymmetry can be due to the differences in the network port structures or due to asymmetric position of the line on the board, as is shown schematically in Fig. 3. Such a structure can be modeled as a cascade of impedances \(Z_1\), \(Z_2\), and \(Z_3\). The impedances \(Z_1\) and \(Z_3\) may not be equal, and they are attached to the main test line with characteristic impedance \(Z_2\). Then the resultant ABCD matrix is a product of three partial matrices:

\[
[ABCD] = \begin{bmatrix}
\frac{Z_2}{Z_1} & 0 & \cosh(yl) & jZ_2\sinh(yl) \\
0 & \frac{Z_2}{Z_1} & \frac{1}{\sqrt{Z_2}} \sinh(yl) & \cosh(yl) \\
\frac{Z_2}{Z_1} & 0 & \sqrt{Z_2} & 0 \\
0 & \frac{Z_2}{Z_1} & \sqrt{Z_2} & 0
\end{bmatrix}
\]

(6)
The first and third matrices in (6) correspond to the impedance discontinuities. Then the complex propagation constant should be calculated as

$$\gamma l = \text{arccosh} \sqrt{A \cdot D}.$$ (7)

Figs. 4 and 5 demonstrate the difference in the extracted values of DK and DF in two cases, respectively: when symmetry was enforced, and when asymmetry was taken into account. It should be mentioned that the boards under test were almost symmetrical, so the resultant difference is very small (on the order of the third significant digit). However, the equation (7) should be used in cases of highly-asymmetrical structures.

![Fig. 4. Comparison of extraction of dielectric constant in the algorithm with enforced symmetry and with asymmetry.](image)

Fig. 4. Comparison of extraction of dielectric constant in the algorithm with enforced symmetry and with asymmetry

![Fig. 5. Comparison of extraction of loss tangent in the algorithm with enforced symmetry and with asymmetry.](image)

Fig. 5. Comparison of extraction of loss tangent in the algorithm with enforced symmetry and with asymmetry

C. Phase Correction and Causality

Complex propagation constant $\gamma$ is obtained from the measured scattering matrix parameters. The phase constant $\beta$ is calculated from the unwrapped phases of S-parameters. The phase of $\gamma$ obtained directly from measurements does not span from exactly $-\pi$ to $+\pi$, so the unwrapped $\beta$ would have “staircase” behavior. When extrapolated to zero frequency, $\beta$ does not reach zero. These problems with phase are solved by making $\beta$ pass through zero at zero frequency. Slope of beta is calculated as

$$m = \frac{\sum_{n=2}^{N} (\beta_n - \beta_{n-1})}{\sum_{n=2}^{N} (f_n - f_{n-1})}.$$ (8)

where $N$ is the total number of frequency points. This slope is multiplied by frequency to obtain $\beta = m \cdot f$. The results for DK and DF without phase correction and with phase correction are shown Fig. 6. If there is no phase correction, DK “blows up” as frequency goes to zero. Similar increase is seen in the DF curve. DK with phase correction seems almost constant. However, indeed it is decreasing very slightly, since causality was built-in the extraction procedure with $\omega$ and $\omega^2$ terms. Consequently, DF linearly increases with frequency. In the figure, the solid line corresponds to the case without phase correction, while the dashed line, which is almost constant. The difference between the mean values of DK in these two cases is less than 2%. This demonstrates that the phase correction can be an acceptable approximation for the frequency range of interest. Phase correction compensates an error in measuring phase in frequency domain, which should ideally start from zero.

However, if the material under test is frequency-dispersive, the constant DK does not reflect the correct nature of permittivity. Indeed, constant DK and DF lead to violation of Kramers-Krönig causality relations [14]. Frequency-independent DK must correspond to zero loss. However, the extracted DF is non-zero, which contradicts causality. Besides, our experimental analysis of many PCB dielectrics in a wide frequency range (up to 35 GHz) has shown that DF is not constant, but increases almost linearly with frequency.

![Fig. 6. Extracted DK (a) and DF (b) for FR-4 dielectric when Debye dependence is taken into account.](image)

Fig. 6. Extracted DK (a) and DF (b) for FR-4 dielectric when Debye dependence is taken into account

This trend of DF that seems reasonable from the causality point of view can be achieved assuming that a PCB substrate
behaves as a dispersive Debye-like material (for simplicity, a single-term Debye material [15]),

$$
\varepsilon = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j \omega \tau_s} - j \frac{\sigma_s}{\omega \varepsilon_0},
$$

where $\varepsilon_s$ and $\varepsilon_\infty$ are the static and high-frequency permittivities, $\tau_s$ is the relaxation time, and $\sigma_s$ is the equivalent d.c. conductivity of a dielectric. From (9), it can be derived that the dielectric loss behaves as $\alpha_d \sim \omega + \omega^2$. If conductor surface is considered smooth, the conductivity loss is proportional to the square root of frequency ($\alpha_c \sim \sqrt{\omega}$).

An improved model takes into account Debye dependence and causality. In the model the total loss $\alpha_T$ is curve-fitted as $\alpha_T = a\sqrt{\omega} + b\omega + c\omega^2$, using an accurate non-linear least-squares curve-fit algorithm, when surface roughness is not considered.

An example of the extracted DK and DF for one of the PCB dielectrics of FR-4 type is shown in Fig. 6. This dielectric is different from the one whose characteristics are shown in Figs. 4 and 5. As a result of applying the Debye model, the DF increases linearly. As Fig. 6(b) shows, this increase is from 0.0245 at 50 MHz to 0.0284 at 34 GHz.

**D. Conductor Surface Roughness Factor**

Surface roughness of metal conductors in microstrip and stripline structures is typically taken into account using well-known Hammerstad’s equation [16],

$$
\alpha_c = \alpha_{c_0} \left[ 1 + \frac{2}{\pi} \arctan \left( \frac{1.4}{\lambda} \right) \right],
$$

where $\alpha_{c_0}$ is the attenuation constant in metal with smooth surface, $\delta$ is skin-depth, and $\lambda$ is the RMS roughness value. The formula (10) applies only for conductors with RMS roughness less than $\lambda \sim 3 \mu m$, as at higher $\lambda$ the arctangent function “saturates”. However, practical PCB might have much higher surface roughness.

It should be mentioned that surface roughness of the trace is more significant for the total conductor loss than the surface roughness on the reference planes, since surface currents on the reference planes of a stripline are negligibly smaller than the current density distribution on the signal trace [17]. For this reason, only surface roughness on the trace has been considered in the present model.

In this study, the approach [18, 19] for calculating conductivity loss in stripline geometries has been adopted. This approach uses approximation of surface roughness as a periodic function of coordinates and expansion of this function as a Fourier series

$$
\alpha_c = \alpha_{c_0} + \Delta \alpha_c,
$$

where the first term

$$
\alpha_{c_0} = \frac{\beta_0 \eta_0}{4 \pi \omega} \delta
$$

is conductivity loss in a perfectly smooth conductor.

According to [19],

$$
\Delta \alpha_c = \frac{\beta_0 \eta_0}{4 \pi \omega} \left[ \sum_{n=1}^{\infty} H_n \left( 1 - \frac{1}{\sqrt{2} \left( n^2 \delta^4 + 4 - n^2 \delta^2 \right)^{1/2}} \right) \right]
$$

is an additional conductor loss term due to roughness. This second term $\Delta \alpha_c$, in contrast to $\alpha_{c_0}$, is not necessarily proportional to a square root of frequency. In (12) and (13), $\beta_0$ and $\eta_0$ are the propagation constant and the characteristic impedance of free space, respectively; $\rho$ is the perimeter of the stripline cross-section; $Z_s$ is the wave impedance of the stripline under test; $\delta = \frac{\sigma}{\sqrt{\mu \rho}}$ is the skin depth at a given frequency, where $\sigma$ is a conductivity of the trace (for standard foil copper it is $\sigma = 5.818 \times 10^7$ S/m), $f$ (Hz) is frequency, and $\mu = \mu_0 \mu_r$ is the permeability of copper, which is almost the same as that for vacuum, since copper is a diamagnetic metal ($\mu_r \approx 1$); $H_n$ is the magnitude of the $n$th harmonic of the surface roughness function. In (13), $s = 2\pi / \Lambda$, where $\Lambda$ is the mean period of surface roughness function, which is assumed to be the same along all three dimensions. This is basically the average distance between two neighboring “peaks” on the rough surface.

An evaluation of the peak-to-valley amplitude of roughness $\Lambda$ and periodicity $\Lambda$ can be done only if micrographs or surface electron microscopy (SEM) images of trace surfaces or cross-sections are available. Fig. 7, for example, shows a cross-section of a copper stripline trace. This picture was obtained using an optical microscope. The cross-section of the trace is closer to a copper stripline trace. This picture is the manufacturer’s SEM of a trace surface at 60-degree inclination view. Surface roughness may be on the order of a few micrometers (worst case) down to hundreds of nanometers (best case). The surface shown is that resulting from the inner-layer adhesion promotion (oxide alternative) process.

![Fig. 7. Cross-sectional micrographic view of the stripline trace](image)

![Fig. 8. Manufacturer’s micrograph of a trace surface at 60° view](image)
The effective values of $A$ and $A$ need to be proportionally weighted to the top and bottom widths, according to Fig. 7, as

$$A_{eff} = A_{top} \left( \frac{W_{top}}{W_{top}+W_{bottom}} \right) + A_{bottom} \left( \frac{W_{bottom}}{W_{top}+W_{bottom}} \right)$$ \hspace{1cm} (14)

$$A_{eff} = A_{top} \left( \frac{W_{top}}{W_{top}+W_{bottom}} \right) + A_{bottom} \left( \frac{W_{bottom}}{W_{top}+W_{bottom}} \right)$$ \hspace{1cm} (15)

It should be mentioned that the most accurate way of taking into account surface roughness based on micrographs is proposed in [20], where the 2D power spectral density (or the corresponding correlation function) of a surface with random deviation from median level are obtained through image processing. However, this is not practical in many cases, when it is impossible to get detailed mapping of the surface roughness on traces of the boards.

In the present model, based on [18, 19], it is assumed for simplicity that roughness on a conductor (copper) surface is close to the periodic sawtooth function. Then the magnitude of the $n^{th}$ harmonic of the surface roughness function can be calculated as

$$H_n = A \left( -1 \right)^{n-1} / (n \pi)$$ \hspace{1cm} (16)

where $A$ is the mean peak-to-valley magnitude of the surface roughness function.

Figs. 9 and 10 present calculated roughness factor

$$r = \frac{a_n}{a_{0}} = \frac{a_{0} + \Delta a_{c}}{a_{0}}$$ \hspace{1cm} (17)

as a function of frequency for different amplitudes of surface roughness $A$ and different spatial period of the sawtooth function $A$. The surface roughness term $\Delta a_{c}$ leads to the deviation of the conductivity loss $a_{c}$ from the smooth case $\sim \sqrt{\alpha}$ behavior.

Fig. 11 shows that when conductor surfaces are considered perfectly smooth (but in reality they are not), the extracted DF for identical materials, but different roughness profiles turn out to substantially deviate from each other. The deviation between the resultant DF values has reduced from 36% (Fig. 11) to 5% (Fig. 12), when the surface roughness model described by equations (11)-(13) has been incorporated in the extraction algorithm. This means that surface roughness plays an important role in extraction of DF. Testing different surface roughness functions in the extraction model has shown that the results do not depend much on the choice of the surface roughness function. However, the extracted conductor loss significantly depends on the estimated values of $A$ and $\Lambda$. This is reasonable for different samples of the same material.

An alternative “redistribution” approach for separating conductor and dielectric loss has been proposed in [21], but it can be applied only if at least two boards with the same dielectric properties and different types of conductor foil are available.
III. STUDY OF RESIN CONTENT EFFECT UPON DIELECTRIC CONSTANT

The objective of this Section is to retrieve a function to predict the DK value, if resin content percentage is known. This study was done experimentally with numerous test boards whose resin contents varied in a wide range from 40 % (low) to 70 % (high).

![Dielectric constant - Linear fit](image)

Fig. 13. Change in dielectric constant with respect to resin weight percentage

In Fig. 13, the two lower straight lines are the laminate manufacturers’ data for two different types of fiber-glass-filled epoxy resin materials with varying resin content [22]. The slopes of these two lines are approximately the same. The upper line is obtained in the present experimental study and connects data with the lowest and highest known resin contents. The resultant slope is identical to those of the other two lines. It is clear that DK of the same type of material from the same manufacturer can be predicted based on the resin content data, provided that fiberglass with the same DK is used throughout.

IV. CONCLUSION

An improved technique for extracting dielectric properties (dielectric constant and loss tangent) of low-loss PCBs using a VNA is presented. Phase correction to assure that the phase constant passes through zero at zero frequency has been applied, and the results were compared to those without phase correction. Kramers-Kröning causality has been achieved during the extraction procedure by accurate curve-fitting. An effect of loss in dielectric upon a dielectric constant has been taken into account. The new model accounts for possible asymmetry of the test network (provided reciprocity). Conductor and dielectric loss were separated, and conductor surface roughness was modeled. Application of the proposed extraction procedure to the experimental analysis of many different PCBs with fiber-glass-filled epoxy-resin-based substrates from the same manufacturer has proved that the DK value can be predicted as a function of the resin content.

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REFERENCES