1-2 IVP Examples.

Let \( y(x) = \frac{1}{x^2+c} \) be a one-parameter family of solutions of the 1st order DE \( y'+2xy^2 = 0 \). Find a solution of the 1st order IVP consisting of this DE and a given IC.

1. \( y(2) = \frac{1}{3} \)
   \[ y(2) = \frac{1}{3} = \frac{1}{2^2+c} \]
   \[ c+4 = 3 \]
   \[ c = -1 \]
   So \( y(x) = \frac{1}{x^2-1} \)

2. \( y(-2) = \frac{1}{\sqrt{2}} \)
   \[ y(-2) = \frac{1}{\sqrt{2}} = \frac{1}{(-2)^2+c} \]
   \[ c+4 = 2 \]
   \[ c = -2 \]
   So \( y(x) = \frac{1}{x^2-2} \)

Let \( x(t) = c_1 \cos t + c_2 \sin t \) be a two-parameter family of solutions of \( x'' + x = 0 \). Find a particular solution using the given IC.

3. \( x(0) = -1 \); \( x'(0) = 8 \)
   \[ x(0) = c_1 \cos(0) + c_2 \sin(0) = -1 \]
   \[ c_1 = -1 \]
   \[ x(t) = -\cos t + c_2 \sin t \]
   \[ x'(t) = \sin t + c_2 \cos t \]
   \[ x'(0) = \sin(0) + c_2 \cos(0) = 8 \]
   \[ c_2 = 8 \]
   \[ x(t) = -\cos t + 8 \sin t \]

4. \( x(\frac{\pi}{2}) = 0 \); \( x'(\frac{\pi}{2}) = 1 \)
   \[ x(\frac{\pi}{2}) = c_1 \cos(\frac{\pi}{2}) + c_2 \sin(\frac{\pi}{2}) = 0 \]
   \[ c_1 = 0 \]
   \[ c_1 + c_2 = 0 \Rightarrow c_2 = 0 \]
   \[ x(t) = c_1 \cos t \]
   \[ x'(t) = -c_1 \sin t \]
   \[ x'(\frac{\pi}{2}) = -c_1 \sin(\frac{\pi}{2}) = 1 \]
   \[ -c_1 = 1 \Rightarrow c_1 = -1 \]
   \[ x(t) = -\cos t \]
Determine by inspection at least two solutions of the given 1st order ODE.

5. \(xy' = 2y \quad y(0) = 0\)

\[
\begin{align*}
\frac{dy}{dx} &= 2y \\
\frac{dx}{x} &= 1 \\
\end{align*}
\]

Note \(y = 0\) is a solution since.

\(\text{LHS: } 0. \quad \text{(trivial solution)}\)

\(\text{RHS: } 0.\)

Also \(y = x^2\) is a solution since

\(\text{LHS: } y' = 2x\)

\(\text{RHS: } 2y = 2x^2 = 2x.\)

\(\frac{x}{x}\)

6. \(y' = 3y^{2/3} \quad y(0) = 0\)

\(y = 0\) is a solution since

\(\text{LHS: } y' = 0\)

\(\text{RHS: } 3(0)^{2/3} = 0\)

Also \(y = x^3\) is a solution since

\(\text{LHS: } y' = 3x^2\)

\(\text{RHS: } 3y^{2/3} = 3(x^3)^{2/3} = 3x^2\)
Determine a region of the xy-plane for the given DE would have a unique solution whose graph passes through the point \((x_0, y_0)\) in the region.

7. \(\frac{dy}{dx} = \frac{\sqrt{xy}}{y} = f(x,y)\)

\[\begin{align*}
\text{x: cont on [0,\infty)} \\
\text{y: cont on [0,\infty)}.
\end{align*}\]

\(\frac{\partial f(x,y)}{\partial x} = \frac{1}{\sqrt{y}} \quad \text{x cont on [0,\infty)}\)

\(\frac{\partial f(x,y)}{\partial y} = \frac{2}{\sqrt{y}} \quad \text{y: cont on (0,\infty)}.
\]

Region where \(x \geq 0\)

\(y > 0\).

8. \(\frac{dy}{dx} = \frac{x^2}{1+y^3} = f(x,y)\)

\[\begin{align*}
\text{x cont everywhere.} \\
\text{y cont everywhere except y=-1}
\end{align*}\]

\(\frac{\partial f(x,y)}{\partial x} = (1+y^3)(x^2)' - x^2(1+y^3)'\)

\(\frac{\partial f(x,y)}{\partial y} = \frac{(1+y^3)^2}{(1+y^3)^2}
\]

\(= -\frac{3x^2y^2}{(1+y^3)^2} \quad \text{x cont everywhere} \quad \text{y cont (-\infty,-1) U (-1,\infty)}\)

Region where \(x\) is anywhere

\(-\infty < y < -1\) or \(-1 < y < \infty\).