MTH 204
Quiz 6
10 Oct 2008

Follow the directions carefully.
Please write neatly in pencil.
You must show all your work
to receive full credit. This quiz
is closed book, closed notes, but
you may use your homework
solutions. If you get stuck, feel
free to ask me for help.

LEAD - Thursdays
5-7 CSFG 5D
mst.edu/~njwzn7d.
Consider the DE $x^2 y'' - 4xy' = x^5$, $x > 0$

a. Classify the DE.
   2nd order, linear, VC, NH (Cauchy-Euler)

b. What method(s) can you use to solve this DE?
   Methods: 5 MUE -> "not in this form"
   (VOP)

c. Note that $y_1(x) = c_1 + c_2 x^5$. Find the general solution
1. Solve $x^2 y'' - 4xy' = 0$
   Assume $y(x) = x^m$
   $\Rightarrow y_1(x) = c_1 y_1 + c_2 y_2 = c_1 + c_2 x^5$
2. Put DE in std form
   $y'' - \frac{4}{x} y' = x^3$
3. Find $W(y_1, y_2)$
   $W(y_1, y_2) = \begin{vmatrix} x^5 & 1 \\ 5x^4 & 0 \end{vmatrix} = 5x^4 - 0 = 5x^4$
4. Assume $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$
   "5" Plug $y_p$ into DE
   6a. Cramer's Rule
   $W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y'_2 \end{vmatrix} = 0 - x^8 = -x^8$
   $W_2 = \begin{vmatrix} y_1 & 0 \\ y'_1 & f(x) \end{vmatrix} = x^3 - 0 = x^3$
   $W(1, y_1, y_2) = 5x^4$
   So $u_1'(x) = \frac{-8}{W} = -\frac{x^8}{1 - x^4}$
   $u_1(x) = -\frac{1}{5} x^4$
and \( u_2'(x) = \frac{\omega_2}{\omega(x_1, x_2)} = \frac{x^3}{5x^4} = \frac{1}{5x} \)

7. Integrate \& plug in:

\[ u_1(x) = -\frac{1}{5} \int x^4 \, dx = -\frac{1}{5} x^5 + C_1. \]

\[ u_2(x) = \frac{1}{5} \int \frac{dx}{x} = \frac{1}{5} \ln x + C_2. \]

\[ y_p(x) = u_1(x) y_1(x) + u_2(x) y_2(x) = -\frac{1}{5} x^5 (1) + \left( \frac{1}{5} \ln x \right) x^5. \]

(\text{absorbed into } y_2)

8. Check if any terms are absorbed,

\[ y_p(x) = \frac{x^5 \ln x}{5}. \]

9. GS:

\[ y(x) = y_h(x) + y_p(x) = C_1 + C_2 x^5 + \frac{x^5 \ln x}{5}. \]

\[ \text{Bonus (3pts)}: \] A 16 lb weight stretches a spring 8 ft. Initially the weight starts from rest 2 ft below the equilibrium position, and the subsequent motion in a medium offers a damping force numerically equal to \( \frac{1}{2} \) the instantaneous velocity. Set up, but don't solve the IVP when there is an external force of \( g(t) = 10 \cos(3t) \),

\[ m = \frac{1}{32} = \frac{1}{2} \text{ slugs} \]

\[ B = \frac{1}{2} \]

\[ mg = \frac{1}{3} K \]

\[ \Rightarrow K = 6 \text{ lb/ft} \]

\[ \text{IVP:} \]

\[ \frac{1}{2} y'' + \frac{1}{2} y' + 6y = 10 \cos(3t) \]

\[ y(0) = 2 \]

\[ y'(0) = 0, \]