Read the directions carefully. You must show all your work to get partial credit.

Closed book, closed notes.
1. State the order of the given ODE. Determine whether the equation is linear or nonlinear. If the equation is nonlinear, state why.

   a. \( t^5 y^{(4)} - t^3 y''' + 6y = 0 \)
   
   4th order  
   Linear

   b. \( \frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3} \)
   
   2nd order  
   Nonlinear - degree on \( \frac{dy}{dx} \) is not 1.

   c. \( (1-x)y'' - 4xy' + 5y = \cos 4x \)
   
   2nd order  
   Linear

2. Determine a region in the xy-plane for which \( (4-y^2)y' = x^2 \) would have a unique solution whose graph passes through \((x_0, y_0)\) in the region.

   \[
   \frac{dy}{dx} = f(x, y) = \frac{x^2}{4-y^2} \quad x \text{ cont on IR} \\
   \frac{\partial f(x, y)}{\partial y} = \frac{(4-y^2)(x^2)' - (x^2)(4-y^2)'}{(4-y^2)^2} \\
   = -2x^2 y \quad \frac{1}{(4-y^2)^2} \quad x \text{ cont on IR} \\
   \text{Region: } x \text{ cont on IR} \\
   y \text{ cont everywhere except } -2, 2.
3. Find all the critical points for \( \frac{dy}{dx} = y^2(y^2 - 4) \) and classify each as either asymptotically stable, semi-stable, or unstable. Draw the phase plane and sketch the typical solution curves in the xy-plane determined by the graphs of the equilibrium solutions.

\[
y^2(y^2 - 4) = 0
\]
\[
y = 0 \quad y^2 - 4 = 0
\]
\[
y = \pm 2.
\]

<table>
<thead>
<tr>
<th>Int</th>
<th>TV</th>
<th>Sign</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\infty,-2)</td>
<td>-3</td>
<td>+</td>
<td>↑</td>
</tr>
<tr>
<td>(-2,0)</td>
<td>-1</td>
<td>-</td>
<td>↓</td>
</tr>
<tr>
<td>(0,2)</td>
<td>1</td>
<td>-</td>
<td>↓</td>
</tr>
<tr>
<td>(2,\infty)</td>
<td>3</td>
<td>+</td>
<td>↑</td>
</tr>
</tbody>
</table>

+ \( y = 2 \) unstable
+ \( y = 0 \) semi-stable
+ \( y = -2 \) asymptotically stable