Read the directions carefully.
Please neatly in pencil. You must show all work to get full credit.

LEAD Sessions - Wednesdays
CSF CSB
6:00 - 7:30

Test 1 - 27 Jan
1.1 - 2.3
Give the order to the following ODEs. Determine whether the equation is linear or non-linear. If the equation is non-linear, explain why.

a. \( \frac{d^3y}{dx^3} - (\frac{dy}{dx})^4 + y = 0 \)

3rd order
Nonlinear since \( \frac{dy}{dx} \) is raised to the 4th power.

dx

b. \((1+x)y'' - 4xy' + 5y = \cos(x)\)

2nd order
Linear

c. \( \frac{d^2R}{dt^2} = -K \)

2nd order
Nonlinear since degree on \( R \) is not one.

d. \( t^5 (y^{(4)}) - t^3 y''' + 6y = 0 \)

4th order
Linear

e. \( \frac{d^2y}{dx^2} = \sqrt{1+(\frac{dy}{dx})^3} \)

2nd order
Nonlinear since the degree on \( \frac{dy}{dx} \) is not equal to one.

dx
2. Let \( x(t) = c_1 \cos(t) + c_2 \sin(t) \) be a two-parameter family of solutions to the differential equation \( x'' + x = 0 \). Find a particular solution using the given initial conditions.

a. \( x(0) = -1 \); \( x'(0) = 8 \)

\[
x(t) = c_1 \cos(t) + c_2 \sin(t)
\]

\[
x(0) = c_1 \cos(0) + c_2 \sin(0) = c_1 (1) + c_2 (0) = -1
\]

\[
\Rightarrow c_1 = -1
\]

\[
x(t) = -\cos(t) + c_2 \sin(t)
\]

\[
x'(t) = \sin(t) + c_2 \cos(t)
\]

\[
x'(0) = \sin(0) + c_2 \cos(0) = 0 + c_2 = 8
\]

\[
\Rightarrow c_2 = 8
\]

So \( x(t) = -\cos(t) + 8\sin(t) \)

b. \( x(\frac{\pi}{2}) = 0 \); \( x'(\frac{\pi}{2}) = 1 \)

\[
x(\frac{\pi}{2}) = c_1 \cos(\frac{\pi}{2}) + c_2 \sin(\frac{\pi}{2}) = c_1 (0) + c_2 (1) = 0
\]

\[
\Rightarrow c_2 = 0
\]

\[
x(t) = c_1 \cos(t)
\]

\[
x'(t) = -c_1 \sin(t)
\]

\[
x'(\frac{\pi}{2}) = -c_1 \sin(\frac{\pi}{2}) = -c_1 (1) = 1
\]

\[
\Rightarrow c_1 = -1
\]

\[
x(t) = -\cos(t)
\]

3. **Bonus (2 pts):** Where have you seen the answers to these questions worked out before? You’ve seen the answers doing the homework or checking my site!