Follow the directions carefully.
Show all of your clearly in pencil.
1. Find the general solution to the following DE.
\[ x \frac{dy}{dx} - y = x^2 \sin(x). \]

Find the largest interval I over which the general solution is defined.

\[ \frac{dy}{dx} - \frac{y}{x} = \sin(x) \]

\[ \text{IF: } e^{-\frac{1}{x} \int dx} = e^{-\ln x} = e^{-x^{-1}} = x^{-1} \]

\[ x^{-1} \left( \frac{dy}{dx} - \frac{y}{x} = x \sin(x) \right) \]

\[ x^{-1} \frac{dy}{dx} - \frac{y}{x^2} = \sin(x) \]

\[ \frac{d}{dx} \left[ x^{-1} y \right] = \sin(x) \]

\[ \int \frac{d}{dx} \left[ x^{-1} y \right] dx = \int \sin(x) dx \]

\[ x^{-1} y = -\cos(x) + C \]

\[ y(x) = CX - x \cos(x) \]

\[ x \frac{dy}{dx} - y = x^2 \sin(x) \]

\[ \frac{dy}{dx} - \frac{y}{x} = x \sin(x) \]

\[ P(x): (-\infty, 0) \cup (0, \infty) \]

\[ Q(x): \mathbb{R} \]

\[ I: x > 0 \]
2. Suppose a cold beer at 40°F is placed into a warm room at 70°F. Suppose 10 mins later, the temperature of the beer is 48°F. Use Newton's law of cooling to find the temperature 25 mins after the beer was placed in the room.

\[\frac{dT}{dt} = K(T - T_m) = K(T - 70)\]

\[T(0) = 40°F\]
\[T(10) = 48°F\]

\[\int \frac{dT}{T-70} = \int kdt\]

\[\ln |T-70| = kt + C\]

\[T - 70 = c_1 e^{kt}\]

\[T(t) = 70 + c_1 e^{kt}\]

\[T(0) = 40 = 70 + c_1\]

\[\Rightarrow c_1 = -30\]

\[T(t) = 70 - 30e^{kt}\]

\[T(10) = 48 = 70 - 30e^{10k}\]

\[-30e^{10k} = -22\]

\[e^{10k} = \frac{11}{15}\]

\[k = \frac{\ln \left( \frac{11}{15} \right)}{10} \approx -0.031\]

So \[T(t) = 70 - 30e^{-0.031t}\]

Then \[T(25) = 70 - 30e^{-0.031(25)}\]

\[\approx 56.18°F\]